Money and Credit in a Keynesian Model of Income Determination

by

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careful critiques of an earlier draft.
ABSTRACT

This paper formally integrates the theory of money and credit derived ultimately from Wicksell into the Keynesian theory of income determination, with assets allocated according to Tobinesque principles. The model deployed has much in common with the modern "endogenous money" school initiated by Kaldor which emphasises the essential role played by credit in any real life economy, since production takes time and the future is always uncertain. New ground is broken methodologically because all the propositions are justified by simulations of a rigorous (sixty equation) model, making it possible to pin down exactly why the results come out as they do. One conclusion of the paper is that there is no such thing as a supply of money distinct from the money which agents wish to hold, or find themselves holding. This finding is inimical, possibly in the end lethal, to the way macroeconomics is currently taught as well as to the neo-classical paradigm itself.

KEYWORDS: Macroeconomics, Stocks & Flows, Real Time, Balance Sheets, Inflation Accounting, Endogenous Money, Credit, Loans, Banks, Simulation, Asset Allocation
INTRODUCTION

It is well known that the Keynesian orthodoxy which ruled policy during the successful post war years was eclipsed by monetarism in the mid seventies. While the temporary ascendancy of the monetarist counter-revolution can largely be attributed to political factors, most Keynesians were intellectually ill prepared for the onslaught when it came. Indeed their models, formal or informal, often had no place in them for money at all.

But a few Keynesians, pre-eminently Nicholas Kaldor (1970), immediately pointed out that monetarists' monetary theory was itself defective, in particular because the concept of an exogenously determined "money supply", essential to the architecture of their model, was chimerical. And during the last twenty five years a new "endogenous money" (EM) school has grown up in the Keynesian tradition, well reviewed in, for example, Bellofiore (1992), Smithin (1994) and Deleplace and Nell (1996). The essence of the EM view is that money is generated by the creation of credit, a process essential to the functioning of the real world economy since production and distribution take time and the future is always uncertain. This view is not just a modern abreaction to monetarism; it has distinguished antecedents in the work of Wicksell (1889), Schumpeter (1910) and Robertson (1938) not to mention Keynes himself (when not writing the General Theory) as well as late Hicks (1989). It is no exaggeration to say that the EM view is potentially lethal not merely to monetarism, which has now been discredited, but to the neo-classical paradigm itself.

Some EM writers, for instance Graziani (1989), have outlined an extension of the theory of credit and money to cover national income determination and distribution. However none of them has yet set out a comprehensive, fully articulated, theoretical model which could provide the blueprint for an empirical representation of a whole economic system. There exists no macroeconomic textbook based on Kaldorian or EM ideas.

This paper takes a step in the right direction by incorporating EM ideas into a complete, if very much simplified, model of a whole economy. Writings on monetary theory commonly rely solely on a narrative method which puts a strain on the reader's imagination and makes disagreements difficult to resolve. The narratives in this paper will all describe simulations which
are grounded in a rigorous model which will make it possible to pin down exactly why the results come out as they do.

But the step taken here is very much a first step. Although the model to be deployed has more variables than is usual for a theoretical model, I am painfully aware that many drastic simplifications have been made; the economy is closed, there is no fixed investment, no fixed capital or equity, no borrowing by firms other than from banks and no borrowing at all by households, while wage inflation is exogenous. Yet I hope that, notwithstanding its shortcomings, this paper describes some key features of a monetary economy seen as a complete system, and adumbrates a new methodology.

Tables 1 and 2 below set out the transactions (flow) and balance sheet (stock) matrices which define the nominal variables of the model and describe the accounting relationships between them. The second section describes how each sector is motivated and what it does. The third section describes how the model works as a whole and illustrates the impact of various shocks using numerical simulations. The final section gives a formal account of the model and each of its sixty odd equations (referred to in the main text as A1, A2 etc.) is accompanied by a brief verbal explanation. All the terms are defined and given illustrative numbers in the appendix.
<table>
<thead>
<tr>
<th></th>
<th>Households</th>
<th>Firms:</th>
<th>Banks:</th>
<th>Govt.</th>
<th>Row Sum</th>
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<tr>
<td></td>
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<td>Capital</td>
<td>Current</td>
<td>Capital</td>
<td></td>
</tr>
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<td>Consumption</td>
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<td>+C</td>
<td></td>
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<td>-T</td>
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<tr>
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<td>-Fb</td>
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<td>-rb.Bs_i</td>
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<td>-B_i</td>
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<td>-AHh</td>
<td>+AH</td>
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<tr>
<td>∆stock of current</td>
<td>-ΔMn</td>
<td></td>
<td>+ΔMn</td>
<td></td>
<td>0</td>
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<td>deposits</td>
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<tr>
<td>∆stock of bills</td>
<td>-ΔBsp</td>
<td>-ΔBsb</td>
<td>+ΔBs</td>
<td></td>
<td>0</td>
</tr>
<tr>
<td>∆stock of bonds</td>
<td>-ΔBpb</td>
<td></td>
<td>+ΔBpb</td>
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<td>0</td>
</tr>
<tr>
<td>∆stock of loans</td>
<td></td>
<td>+ΔL</td>
<td>-ΔL</td>
<td></td>
<td>0</td>
</tr>
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<td>Column Sum</td>
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### TABLE 2

**BALANCE SHEETS**

<table>
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<tr>
<th></th>
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<th>Firms</th>
<th>Banks</th>
<th>Government</th>
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<tr>
<td><strong>Inventories</strong></td>
<td>+1</td>
<td></td>
<td></td>
<td>1</td>
</tr>
<tr>
<td><strong>Cash</strong></td>
<td>+Hp</td>
<td>+Hb</td>
<td>-H</td>
<td>0</td>
</tr>
<tr>
<td><strong>Demand deposits</strong></td>
<td>+Mn</td>
<td></td>
<td>-Mn</td>
<td>0</td>
</tr>
<tr>
<td><strong>Time deposits</strong></td>
<td>+M</td>
<td></td>
<td>-M</td>
<td>0</td>
</tr>
</tbody>
</table>
| **Bills**          | +Bsp       |       | +Bsb  | -Bs        | 0
| **Bonds**          | +B.pb      |       | +B.pb | 0          |
| **Loans**          |            | -L    | +L    | 0          |

| Column Sum         | V          | 0     | DG    | 1          |

**Footnote to Table 2**

V = Household wealth  
DG = Total government liabilities

“Bonds” are perpetuities each paying one unit of currency per period so the price of a bond (pb) is the reciprocal of the interest rate. A change in the value of the stock of bonds between two periods has two components

\[ B_{t}\cdot pb_{t} - B_{t-1}\cdot pb_{t-1} = \Delta B_{t}\cdot pb + \Delta pb_{t-1} \]

The first term on the RHS describes the value of transactions in bonds, the second describes the capital gain or loss resulting from a change in the bond price.

As these matrices show, the model has four sectors, firms, households, the government and banks. Each row and column of the flow matrix sums to zero on the principle that every flow comes from somewhere and goes somewhere. The financial balance of any sector - the gap between its income and expenditure reading vertically in Table 1 - is always equal to the total of its transactions in financial assets. Changes in the value of financial asset stocks, shown as levels,
in Table 2, include nominal capital gains and losses on bonds as well as flow transactions (A19 - A21). The change in the value of inventories in Table 1 includes both the value of their physical change and also stock appreciation (A17a). The stock of household wealth is given alternatively by the sum of all the financial assets (reading down column 1) or by the sum of government debt and inventories (reading horizontally at the foot of the table). To derive key functional relationships, many of the nominal flows in Table 1 will have to be "inflation accounted"; real personal disposable income, for instance, will be defined so that it equals real consumption plus the change in the real stock of wealth (A22-22a). The watertight accounting of the model implies that there will always be one equation which is logically implied by all the others.

THE MAIN INSTITUTIONAL AND BEHAVIORAL ASSUMPTIONS

The agents in this model comprise a variety of institutions which are distinctively motivated. The tendency of the system as a whole is governed by stock flow norms rather than the equilibrium (or disequilibrium) conditions postulated by neo-classical theory. There is no underlying assumption that all agents are maximising individuals.

A) FIRMS

"Firms" comprise distributive trades as well as producers narrowly defined. Manufacturing firms produce an infinite diversity of goods which intermediary traders stock, advertise, guarantee and market, holding prices constant in the short term. The whole productive chain is in a state of uncertainty about what the value of sales and profits will actually be. It is assumed that firms are operating within the normal range of outputs at which running costs per unit of output are constant, and that they base their decisions about production, prices and employment on the quantity they expect to sell profitably plus any adjustment to inventory levels.
they wish to achieve. Firms respond to quantity rather than to price signals. It is in response to realised sales and inventory levels that firms, certainly in the short term, decide whether or not to increase production or change prices.

Realised sales are determined by actual consumption and government expenditure, while realised profits are the residual between sales and costs, as shown in column 2 of the transactions matrix. It will be assumed that profits are all distributed to households - an assumption which carries the logical implication (as revealed in the transactions matrix) that bank loans to firms expand or contract, $ for $, with inventories. Firms require revolving finance from banks, not only because production and distribution take time while wages have to be paid in advance of sales being made, but also because they cannot know exactly what their sales are going to be and any shortfall requires a simultaneous addition to loans if the wage bill is to be met. It is unrealistic to suppose, as some EM writers do, that what is produced in one period will automatically be sold in the next.

B) HOUSEHOLDS

Real consumption is determined by the real stock of household wealth inherited from the previous period together with the expected flow of real disposable income. Underlying this assumption, as formally implied by the consumption function (A24), is the idea that, aggregated across the sector, wealth is accumulated at a particular rate and that there exists a desired long run wealth-income ratio. The short run Keynesian consumption function, which simply makes consumption some proportion, less than 1, of income still stalks the post Keynesian literature, for instance in Davidson (1994 pp.37-40). But this consumption function, since it has no sensible implication regarding wealth accumulation, makes it impossible to incorporate the theory of
credit, money and asset allocation into that of income determination in a coherent way

In the model presented in this paper, households aspire to apportion wealth they accumulate each period between the five assets available in proportions determined by their real rates of return, including the rate of inflation (A28-31). But they do this subject to their having enough cash and liquid assets to carry out transactions; and when unexpected things happen these assets move in correspondingly unexpected ways (A38-40). The way this has been modeled owes everything to the work of James Tobin.4

C) THE GOVERNMENT

The government's budget constraint, given by A42 and also by column 6 of the flow matrix, is simple and traditional. The government has several policy instruments at its disposal which together constitute most of the exogenous variables of the model. It determines the level of public expenditure, the tax rate, the nominal rate of interest on bills and bonds and the fractional reserve ratio with which banks must comply. Given its policy settings, the government (defined to include the central bank) has no direct control over its own deficit, nor over the size and composition of its own debt, including the quantity of bank reserves. The assumption that nominal bill and bond rates of interest are determined by government fiat is another way of saying that the government will exchange any quantity of securities for cash at the declared rate of interest and therefore that it always stands ready to act as lender of last resort (A45-48).

D) BANKS

As every row in the flow matrix sums to zero and as every column other than banks' asset transactions (column 5) also sums to zero, it follows logically that the column describing banks' asset transactions must sum to zero as well. This carries the important logical implication that, so
long as there is no default on debt, no configuration of behaviour whatever can unbalance the banks' consolidated balance sheet - that is, make the sum of their assets different from the sum of their liabilities. However banks' operations would become unprofitable if the interest they receive on loans and bills were to fall short of what they have to pay on their liabilities. Banks are price takers with regard to the interest rates they pay on bills and price makers with regard to the rates they charge on loans and pay on money. It is a key behavioural assumption that banks set these rates so that they make profits (A50-60). Flows of interest payments are not often discussed in the literature, although a model of the whole system cannot be solved unless they are explicitly included (as in A50).

We now have so many accounting identities and behavioural assumptions that all the banks' remaining transactions must be passive responses to the transactions of other sectors (A61-65). Given the other assumptions of the model, banks passively exchange any form of money (cash, demand and time deposits) for any other form. It is also implied that banks passively provide loans to firms on the security of inventories, which results in an addition to the money holdings of wage earners as wages become due and are paid. Banks automatically extinguish loans when cash or cheques are deposited by firms as sales are realised except to the extent that new loans, in an ongoing situation, will be needed to keep up the flow of production. The "supply" of money is a redundant concept - there is no such thing. Even the term "demand" for money strains language; for it badly describes a situation where people aim to keep their holdings of money within some normal range but where the sums they end up with are determined in large part by impulse purchases, windfalls and other unexpected events. It is unfortunate that the stock of money, measured ex post, should have become generally known as
"the money supply" - a term which invites the supposition that a supply exists independently from what people wish to hold.

As banks follow a rigid reserve requirement (A51), an element of flexibility is essential if they are to operate in the way described. This is to be found in banks' holdings of bills which, can always be exchanged for cash even if this involves, *in extremis*, borrowing from the government, perhaps out of the discount window at a penal rate of interest. Faced with a fall in the defensive belt of their bill holdings below a safe level, banks will raise the rate of interest on money (relative to that on bills) to whatever extent is necessary to get holders of government securities to sell them in exchange for time deposits. Such sales bring about an increase in banks' cash holdings, which can be used to buy bills or discharge debts to the central bank. If banks raise the rates on money, they must raise rates on loans as well if they are to stay in profit.

**BEHAVIOUR OF THE MODEL AS A WHOLE**

Although the model has neither an equilibrium nor a disequilibrium in the neo-classical sense, it does have a well defined steady state to which it will tend. Readers will remember from Blinder and Solow (1973) that, ignoring interest payments and inflation, the flow steady state of any stock flow model (in which all stocks and all flows are constant, and hence in which the average propensity to consume is equal to unity) will be given by government expenditure times the reciprocal of the tax rate since then government outlays are equal to government receipts.

\[ y^{**} = \frac{g}{\theta} \]

where \( y \) is output, \( g \) is government expenditure, \( \theta \) is the tax rate and the double star denotes a steady state.

The stock steady state is given by
1b) \[ v^* = \frac{a(1 - \theta)g}{\theta} \]

where \( v \) is wealth and \( a \) is the steady state ratio of wealth to disposable income (23a-c).

These formulae describing steady states are fundamentally at odds with the flow equilibrium to be found in old fashioned textbooks as well as in Davidson (op.cit.)

2a) \[ y^* = g, \quad \frac{1}{1 - a} \quad 0 < a < 1 \]

where \( a \) is the marginal propensity to consume and there is no argument in wealth. Equation 2a) cannot possibly be a steady state because it describes a situation in which wealth and government debt stocks are rising sine die.

There is an analogue to 1a) in the model presented here, identical in spirit, but slightly more complex because the government’s interest payments are treated separately from government expenditure proper and are generated by the (endogenous) stock of government debt inherited from the past. The flow steady state is

1) \[ y^* = g \frac{1 - \sigma r}{\theta - r(a - \sigma)} \]

where \( \sigma \) is the steady state inventory/output ratio (A5) and \( r \) is the real rate of interest (averaged across all types of government liability) on the real stock of government debt.

The stock steady state of the model is given by

2) \[ v^* = g \frac{a(1 - \theta - r\sigma)}{\theta - r(a - \sigma)} \]

The dynamics of the model are intrinsic; the speed at which the system, once shocked, moves towards its new steady state is governed by stock flow norms as outlined by Godley and Cripps (1983 pp.121-125)

SOME SIMULATIONS

In the simulations which follow, no significance should be attached to the magnitudes of...
the responses, only to their shapes, since the parameter values are arbitrary. The model contains a somewhat awkward splicing together of fast and slow processes. What are, in the real world, very fast processes (such as the response of banks to changes in government interest rates) have been assumed to play out in a time scale comparable with slow processes (such as the income multiplier) but this should not vitiate the essence of the analysis.

The first experiment shows how the model responds when, starting from a full steady state, the desired ratio of inventories to output makes a once for all jump, while inflation remains constant. This would occur if, for instance, the production period were to rise. The example is chosen because it is a variation on the ubiquitous, but possibly misleading, theme that "every loan creates a deposit".

Chart 1 shows the effects of the rise in the level of inventories on the main expenditure flows.

CHART 1

The shapes of these curves are what one would expect from conventional multiplier/accelerator analysis. Inventory accumulation rises and then tails away as the new stock/flow norm is reached. The aggregate income flow rises initially exactly in line with inventory accumulation, then rises further in response to multiplier effects which temporarily raise consumption, then tails way towards a new steady state. The new steady state is slightly below the old one and the reason for this can be inferred from the steady state described in equation 1) above. The ratio of wealth to disposable income is unchanged between the two steady states (by A23). But the share of inventories in wealth will be higher in the new steady state and hence the share of government liabilities in total wealth will be correspondingly lower; so the total flow of
interest payments from the government to the private sector will be lower and it is this which ultimately reduces the aggregate income flow.

CHARTS 1B AND 1C HERE

Chart 1B shows how wealth rises in response to the additional income flow and subsides again with it, reaching a new steady state (like income and for the same reason) slightly lower than it started out. It is only in the very first period of all, and then only because of inertia in the asset allocation process, that the stock of money rises by an equivalent amount. After the first period, while wealth goes on rising for a time, the normal process of asset allocation begins, so that holdings in deposit accounts fall back and holdings in time deposits and securities rise. In these immediately following periods, before interest rates have changed much, the fact that some of the new wealth is allocated by households to government securities means that banks find their stock of bills depleted. Chart 1B illustrates clearly why this is so, since the banks' consolidated balance sheet must sum to zero all the time. To restore their bill stock, banks have to raise the rate of interest on money relative to the bill rate, which is assumed to be unchanged throughout. And they go on raising it until the defensive belt is completely restored.

As we reach the new steady state, reverting to Table 1B, the structure of wealth holdings has been permanently altered. The fact that inventories are now a higher proportion of wealth means that the rate of interest on money has to be permanently higher relative to the bill rate. Furthermore, if banks are to maintain their profits, loan rates of interest must be raised pari passu with money rates, partly choking off the inventory change which started the whole thing off.

This increase in loan and money rates seems to be at odds with the "horizontalist" position taken in for example Moore (1988, pp. 57-63). Moore's key point is that the "demand " for
deposits expands automatically in line with the additional supply of loans so there is no need for the hierarchy of interest rates to change. But he ignores the fact that the addition to loans will alter the aggregate income stream, the consequential expenditure and tax payments which generate further changes in wealth stocks and so on until an altogether different steady state is reached.

In the second simulation experiment, the short and long interest rate are raised together in a step. The effect on the major flows is shown in Chart 2A.

CHART 2A

There are two transmission mechanisms at work. First the inventory/output ratio is reduced as a result of higher interest rates (by A5) and this has a temporary effect on inventory accumulation. My belief is that in the real world, as in this model, effects of this kind, which supposedly work through a direct impact on interest sensitive expenditure, are not large; certainly they are not easy to find empirically. The main impact works through the negative effect from higher interest rates on asset prices - in the present case on the price of perpetuities which account (in this model) for 30% of household wealth.

But while the immediate effects on wealth and hence on income and expenditure are substantial, it can be read off from equation 1) that the steady state real income flow is an increasing function of the real rate of interest. And this is what the simulation shows. Aggregate demand, given that fiscal and monetary policy do not change again, eventually recovers to a level higher than it was before the shock for the simple reason that, in the new steady state, the government is paying out more interest as a result of the higher rates.

CHART 2B
The banks' response to higher bill and bond rates is shown in Chart 2B. It is assumed that banks do not instantly or fully respond, by changing the rates at their discretion, to what has happened; this way we can see how and why they are forced to act. If the banks were to leave rates on money unchanged, there would soon be a switch by households out of money into government securities which would reduce banks' defensive assets pari passu. Banks respond to this by putting up interest rates on money until the defensive belt is restored. As banks raise rates on money they must raise loan rates as well if they are to remain profitable.

OTHER SIMULATIONS

This section concludes with some obiter dicta concerning other simulations. If the model is hit with inflation, while nominal interest rates are raised so as to maintain real interest rates unchanged, a large reduction in real demand occurs. Apart from the "tax" which inflation imposes on high powered money (a small part of the story), the postulated rise in nominal rates of interest results in heavy nominal capital losses on bonds and the effect of this on real household income and wealth is compounded by the rise in product prices (A21). Following the recession which inflation causes, the economy slowly recovers to roughly its previous level (by equation 1) so long as the government’s real fiscal stance remains unchanged. But it would be incorrect to interpret this as meaning that the status quo ante is restored, because the onset of inflation caused a loss of wealth selectively to people who held securities of a particular kind and the distribution of wealth will have been permanently changed in their disfavour.

Up to this point I have assumed perfect foresight on the part of households (with respect to their incomes) and of firms (with respect to their sales) since assumptions about expectations formation were not necessary to illustrate the particular points so far at issue. But the model can
be used to show what happens when expectations, whether of households or firms, are falsified. Indeed, it is a central contention of this study that a modern economy cannot function without a banking system which allows loans and money balances to fluctuate because expectations turn out to be wrong.

I have introduced expectations into the simulation model by the crude device of assuming that expected sales and disposable income differ from actual values by random numbers. While there is no pretence that expectations are really formed in this way, this device has the merit that it puts the imaginary banking system to a severe test. The failed expectations of firms give rise to random fluctuations in inventories (and therefore loans) while those of households cause comparably large, but unrelated, fluctuations in money holdings. But the banks can handle all this with no difficulty whatever; and they remain profitable so long as they make appropriate adjustments to the interest rates over which they have control.

The "money multiplier" theory of money creation is still the standard, nearly universal, model used in conventional macroeconomics, for the compelling reason that it enables the notion of an exogenous money supply to be carried through from high powered money to credit money. Goodhart in (1989 pp.130-7) argues that this theory is not so much wrong as empty. Even to the extent that the central bank can influence the total stock of cash by open market operations or by interest rate changes (alternatively if it changes the reserve requirement) it does not follow that banks will thereupon change their lending and therefore the stock of credit money by a multiple of the cash base. Rather than call in loans (in the case of a monetary contraction) which may be difficult or impossible, banks are more likely to respond initially by reducing their bill holdings and then raise the rate of interest on money relative to that on bills, inducing households to alter
the structure of their portfolios so that they hold more money and fewer government securities.

This model in this paper has the banks responding in just this way.

Readers are invited to reproduce the model (which is fully described below and furnished with consistent numbers for all variables and parameters in the appendix) and carry out simulations for themselves; this is perhaps the only way the properties of the model can be fully understood.
THE MODEL SET OUT FORMALLY

This final section gives a formal account of the model. A full list of variables and parameters, with numbers corresponding to a full steady state, is given in the appendix. Capital letters describe stock and flow variables at current prices, while the lower case is used for their deflated counterparts. A bar denotes that the variable is valued at cost, that is, it excludes taxes and profits. Auxiliary equations, formally redundant, are numbered A17a, A17b, etc. Suffixes have the following meanings

** Long run steady state
* Expected
_h Holdings of
_s Supplied
_r Required
_x Exchanged

FIRMS

A1) \( \bar{Y} = \bar{s} + i^* - i_{-1} \)
A2) \( N.\bar{w} = \frac{\bar{Y}}{P_L} \)
A3) \( WB = N.\bar{w} \)
A4) \( UC = \frac{WB}{\bar{Y}} \)

In A1 it is assumed that the level of real output, a key decision made by firms, is equal to expected sales plus the expected change in inventories, with everything valued in common units, namely base year unit wage cost. A2 describes employment (valued at base year wage rates) which is determined as part of the output decision given (exogenous) productivity. A3 describes
the wage bill and A4 gives wages per unit of output valued at constant cost; this is the deflator for inventories.

A5) \( i'' = \sigma_0 + \sigma_1 \bar{S}' - \sigma_2 r_1 \)

A6) \( i' - i_{-1} = \gamma(i'' - i_{-1}) \)

A7) \( I' = i' \cdot UC \)

A5 describes the long run desired level of inventories measured at constant cost, given expected sales and the loan rate of interest. A6 gives the expected change in inventories (at constant cost). A7 gives the expected level of inventories valued at actual cost.

A8) \( s*.px = (1 + \tau) (1 + \beta) (WB - (I' - I_{-1}) + r_1 I_{-1}) \)

A9) \( \bar{s}' = \frac{s'}{\Phi} \)

A8 is the pricing decision, which will partly have determined firms' expectations regarding what they can sell. Ex-tax prices are a mark up on the expected historic costs of producing what (it is expected) will be sold and taxes are levied on the value of ex-tax sales. As \( px \) is an index of market prices equal to 1 in the base period, we need the scaling factor (\( \Phi \)) in A9 to convert expected sales at constant cost to expected sales at constant market prices.

The assumption that prices are determined as a mark up on historic costs is far less arbitrary than might be supposed. As explained in Godley and Cripps (1983 pp. 188-195), A8 can be adapted to yield the following identity which describes the distribution of the national income, period by period, between four categories - taxes, profits, wages, and the creditors of the productive system.

A8a) \( Y = (1 + \tau) (1 + \beta) (1 + k. rc) WB \)

where \( k \) is the inventory/sales ratio (sales as well as inventories being valued at constant cost) and
rc is the loan rate of interest deflated by the rate of cost inflation. The point is that the coefficient β, which describes the mark up on historic cost, also governs the share of profits in the national income. This equation is identical in spirit to that used by Graziani (1989).

A10) \( s = c + g \)

A11) \( \bar{S} = \frac{S}{\psi} \)

A12) \( S = s \cdot px \)

A13) \( i = i' + \bar{s}' - \bar{s} \)

A14) \( I = i \cdot UC \)

A15) \( Y = S + \Delta i \cdot UC \)

A16) \( y = s + \Delta i \)

These seven equations (A10-16) describe realised values for sales, inventory levels and output, variously valued.

A17) \( \Delta f = S - T - Wb + \Delta I - \pi I \cdot I_1 \)

A17a) \( \Delta I = \Delta i \cdot UC + \Delta UC \cdot i_1 \)

A17 describes realised profits of firms - the extractable surplus arising from firms' business operations as can be seen from column 2 of the flow matrix. Although this definition of profits follows logically from all the other accounting relationships, it is not quite the same as that used in the national accounts where it is standard practise to deduct stock appreciation while ignoring interest payments although they are an inevitable cost given that production takes time. In A17 profits include stock appreciation (the second term on the RHS of A17a) but deduct the interest cost of holding inventories. Stock appreciation and the interest cost would equal one another if the interest rate were exactly equal to the rate of cost inflation.
A18) \( L_r = I \)

A18 says that revolving finance in the form of bank loans is required if production is to be financed in advance of sales being made and if profits are to be extracted from the firm and paid over to households.

**HOUSEHOLDS**

A19) \( YDP = Ff + Fb + NB + r_m.M_{-1} + r_b.Bsp_{-1} + B_{-1} + \Delta p_b.B_{-1} \)

A20) \( \Delta V = YDP - C \)

A21) \( C = c.px \)

Equation A19 describes nominal disposable income and A20 relates this to changes in the nominal stock of wealth. The final term in A19 describes the capital gain on perpetuities which occurs when long term interest rates change.

A22) \( \frac{ydp'}{px} = \frac{\Delta px}{px} \cdot v_{-1} \)

A23) \( v = \frac{V}{px} \)

A22a) \( \Delta v' = ydp' - c \)

A22 gives expected real disposable income consistently defined so that it is equal to real consumption plus the expected change in the real stock of wealth as shown in A22a.

A24) \( c = a_1 \cdot ydp' + a_2 \cdot v_{-1} \)

Equation A24 is the consumption function. It follows from A22a that A24 can alternatively be written

A24b) \( \Delta v' = a_2 (a_3 ydp' - v_{-1}) \)

where \( a_3 = \frac{1 - a_1}{a_2} \)
and from this it follows, in turn, that in a full (stationary) steady state, when expectations are fulfilled and there is no change in stock or flow variables, the real stock of wealth is in a well defined ratio to real disposable income, namely

\[ A24c) \quad v = v'' = a_{3}.ydp \]

and hence, in a full steady state, the average propensity to consume is unity.

\[ A25) \quad Vn_h = \lambda_c \cdot C \]

\[ A26) \quad V' = V_{-1} + YDP' - C \]

\[ A27) \quad Vn' = V' - H_{p.h} \]

It is assumed, in A25, that households' holdings of cash are determined entirely by a need for transactions purposes, hence wealth which (it is expected) will be available for investment is given by A27.

\[ A28a) \quad \frac{M_{n.h}^*}{Vn_{-1}'} = \lambda_{10} - \lambda_{12}.rrm - \lambda_{13}.rrb - \lambda_{15}.rr - \lambda_{14}.\Pi + \lambda_{15}.YDP' \frac{Vn_{-1}'}{Vn_{-1}'} \]

\[ A29) \quad \frac{M_{h.}^*}{Vn_{-1}'} = \lambda_{20} + \lambda_{22}.rrm - \lambda_{23}.rrb - \lambda_{25}.rr + \lambda_{24}.\Pi - \lambda_{25}.YDP' \frac{Vn_{-1}'}{Vn_{-1}'} \]

\[ A30) \quad \frac{Bsp_{h}}{Vn_{-1}'} = \lambda_{30} - \lambda_{31}.rrm + \lambda_{32}.rrb - \lambda_{33}.rr + \lambda_{34}.\Pi - \lambda_{35}.YDP' \frac{Vn_{-1}'}{Vn_{-1}'} \]

\[ A31) \quad \frac{B_{h..pb}}{Vn_{-1}'} = \lambda_{40} - \lambda_{41}.rrm - \lambda_{42}.rrb + \lambda_{43}.rr + \lambda_{44}.\Pi - \lambda_{45}.YDP' \frac{Vn_{-1}'}{Vn_{-1}'} \]

\[ A28) \quad M_{n.h}^* = Vn_{-1}' - M_{h.}^* - Bsp_{h} - B_{h..pb} \]

The first four equations (A28a-A31) describe households' aspirations regarding asset allocation. The parameters conform with Tobin's constraints, that is, the sum of the constants is
unity while the sum of the coefficients in every other column is zero. As each one of the asset
demand functions is implied by the other three taken together, one of them, in this case A28a, has
been dropped to make it possible to solve the model; so expected deposit money holdings are
determined by residual as shown in A28.

The interest rates in the asset demand functions are all real rates calculated according to the
Fisher discrete time formula.

\[
\begin{align*}
A32) & \quad r_{rm} = \frac{1 + r_{m}}{1 + \pi} - 1 \\
A33) & \quad r_{rb} = \frac{1 + r_{b}}{1 + \pi} - 1 \\
A34) & \quad r_{r} = \frac{1 + r}{1 + \pi} - 1 \\
A35) & \quad \pi = \frac{\rho_{x}}{\rho_{x-1}} - 1
\end{align*}
\]

Long bonds are assumed to be perpetuities each paying 1$ per period. Hence the price of
bonds is

\[
A36) \quad pb = \frac{1}{r}
\]

\[
A37) \quad M_{n-h} = (V_{n} - M_{h} - B_{sp_{h}} - B_{h}.pb).Z_{1}
\]

\[
A38) \quad Z_{1} = (V_{n} - M_{h} - B_{sp_{h}} - B_{h}.pb).GE.0
\]

\[
A39) \quad M_{h} = M_{h}' \cdot Z_{1} + (V_{n} - R_{sp_{h}} - R_{h}.pb) \cdot LT.0
\]

\[
A40) \quad Z_{2} = (V_{n} - M_{h} - B_{sp_{h}} - B_{h}.pb).LT.0
\]

\[
A41) \quad V_{n} = V - Hpd
\]

In equations A37 to A409 the term GE means “greater than or equal to” while LT means
“less than”. In A38 and A41, Z1 and Z2 take on the value 1 or 0 depending upon whether the
statement on the RHS are true or false. So realised holdings of money in deposit accounts are
given, in A37, by the residual between realised wealth and other assets so long as this expression
yields a positive number; A38 is then a non-negativity constraint. A39 and A40 ensure that money held in the form of time deposits takes on the residual function previously performed by \(Mn_h\) as soon as this falls to zero.

**THE GOVERNMENT AND CENTRAL BANK**

\[ A42) \quad \Delta H = G + rb \cdot B_{s,x} - T - \Delta B_{x} \cdot pb - \Delta B_{s,x} \]

\[ A43) \quad G = g \cdot px \]

\[ A44) \quad T = S \cdot \frac{1}{1+t} \]

A42 and A43 describe the government's budget constraint and its receipts of tax, assumed here to be all indirect.

The government determines short and long term nominal interest rates which is another way of saying that it stands ready to exchange all financial assets on demand at those rates. These passive responses are represented by the following four equations A45-A48.

\[ A45) \quad B_{s,x} = B_{s,p} + B_{sb,h} \]

\[ A46) \quad B_{sb,x} = B_{sb,r} \]

\[ A47) \quad B_{x} = B_{h} \]

\[ A48) \quad H_{b,x} = H - Hpx \]
These three equations give the banks' balance sheet (A49), the banks' appropriation account which implies a definition of their profits (A50) and the fractional reserve requirement which they must observe (A51).

A52) \( \Delta \alpha m = Z3.A1 - Z4.A1 \)

A53) \( Z3 = BR_1 \cdot GE.B1 \)

A54) \( Z4 = BR_1 \cdot LT.B1 \)

A55) \( BR = \frac{Bsb_h}{Mn_x + M_x} \)

It is assumed that banks have a norm (B1) for the ratio of defensive assets (bills) to liabilities (BR). The logical functions in A53 and A54 mean that banks will increase the rate of interest on money at a rate described by A1 whenever BR falls below the norm and reduce it (at the same rate) when it rises above the norm.

A56) \( r11 = (1 + \phi_1) \cdot rm \)

A57) \( r12 = (1 + \phi_2) \cdot rb \)

A58) \( r1 = r11.z5 + r12.z6 \)

A59) \( z5 = r11.GE.rb \)

A60) \( z6 = r11.LT.rb \)

This group of equations (A56-A60) ensures that banks always charge a rate of interest on loans which exceeds that on any of their other assets or liabilities. The normal state is described
by A56, where the rate on loans is a simple mark up on the money rate. However banks are unlikely to let the loan rate fall below the bill rate (as it would then be more profitable to hold bills than make loans). Accordingly A57 makes the loan rate a mark up on the bill rate whenever the mark up on the money rate in A56 is insufficient.

A61) \( M_{n-x} = M_{n-h} \)
A62) \( M_x = M_{h} \)
A63) \( H_{p-x} = H_{p-h} \)
A64) \( L_s = L_r \)
A65) \( Bsp_x = Bsp_h \)

This final group of equations describes the passive response of banks when households wish to hold, or find themselves holding, their assets in various forms. It also describes how, given all the other assumptions of the model, banks supply loan finance when it is needed to finance inventories.

A few additional points remain to be made for the benefit of anyone trying to reproduce the model. First it should be noted that there is an equation both for the banks’ reserve holdings (A51) and also for the quantity of bank reserves exchanged by the government (A48). These two equations should always yield the same number by the rules of accountancy and it is advisable always to check that they do indeed do so, particularly when any change is made to the model. To complete the model as it stands some assumption about expectations must be made even if this amounts to perfect foresight viz. \( YPD^* = YPD \) and \( s^* = s \). It should finally be obvious, particularly given the very simple linear forms attributed to the functions, that the model will not survive very rough treatment. It is easy to find combinations of parameter values which generate
oscillations or outright instability.

When making experiments I have allowed myself a considerable amount of flexibility with regard to adjustment processes and time lags. For instance I have varied the speed with which banks respond to a fall in their bill holdings by raising interest rates (A1 in equation A52) to generate acceptable patterns. Similarly I have introduced lags into the consumption function and into the asset holding functions whenever simultaneous interdependence threatened to generate meaningless oscillations.

CONCLUSION

In this paper I have deployed a fully articulated stock flow model, capable (subject to many simplifying assumptions) of describing the evolution of a whole economy through real time. Credit and money creation are shown to be essential features, given that production takes time and the future is uncertain. Some of the findings are qualitatively different from those in the standard post-Keynesian literature; but the main purpose of the paper has been to extend the insights of the endogenous money school to cover income determination and distribution, as well as asset allocation along Tobinesque lines. A very great deal remains to be done. Fixed investment, equity and capital must be introduced, and with them a motivation for the profit mark-up. Inflation will be endogenised. A “world” version of the model will incorporate international trade, international investment and exchange rate determination. And all this will be brought on with an empirical representation of selected real economies using the same stock flow approach.
This appendix defines the variables and parameters of the model and gives the numbers which have been attributed to each of them in order to obtain an initial steady state. The number of equations exceeds the number of endogenous variables listed below because the variables in the model describe values which are expected, desired, exchanged etc.

**EXOGENOUS VARIABLES**

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<th>Variable</th>
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<td>B1</td>
<td>Desired level for banks’ bill ratio (BR below)</td>
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<tr>
<td>W</td>
<td>Wage rate</td>
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<td>FR</td>
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<td>rb</td>
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<td>r</td>
<td>Rate of interest on bonds</td>
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<td>τ</td>
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**ENDOGENOUS VARIABLES**

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<th>Variable</th>
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<tr>
<td>B</td>
<td>Total bond issue</td>
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<td>BR</td>
<td>Ratio of banks’ bills to liabilities</td>
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<tr>
<td>Bs</td>
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<td>Bills held by banks</td>
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<td>Hp</td>
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<tr>
<td>s</td>
<td>Total sales valued at constant cost</td>
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</tr>
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\[
\begin{align*}
T &= \text{Yield of taxes} & 26.572 \\
UC &= \text{Unit labour cost} & 1.0 \\
V,v &= \text{Wealth} & 107.86 \\
WB &= \text{Wage bill} & 95.978 \\
Y,y &= \text{GDP} & 132.86 \\
\ddot{y} &= \text{GDP valued at constant cost} & 96.978 \\
YDP,ydp &= \text{Personal disposable income} & 107.86
\end{align*}
\]
PARAMETERS

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<td>φ</td>
<td>1.37</td>
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REFERENCES


Lavoie, Marc. 1985. “Credit and Money: the Dynamic Circuit, Overdraft Economics, and Post-


ENDNOTES


2. This obviously contrasts with the neo-classical assumption that firms are all on their production frontiers where price is equal to marginal cost. As Hicks (1989 p.22) put it "There is no need to assume that there is a single optimum output for which the firm is designed; it is better, being more realistic, to think of it as having a regular range of outputs...which it is...fitted to produce [and]...over that range marginal cost is simply running cost per unit of output...which could be considered constant."

3. This is among the most awkward of all the simplifications which have been made. In the real world retained profits are the main source of finance for fixed investment.


5. The "tax rate" here means the share of income taken in taxes. If, as in this study, taxes are all indirect \( \theta = \frac{\tau}{1 + \tau} \) where \( \tau \) is the indirect tax rate on pre-tax sales.

6. To spell it out,
   a) \( y^{*'} = \frac{(g + xx.dg^{*'})}{\theta} \)
   b) \( v^{*'} = a.yd^{*'} \)
   c) \( dg = v - i \)

7. The data will be made available in machine readable form on request as well as the files which create the model (for people who use MODLER software).
CHART 1B SIMULATION 1: EFFECT ON WEALTH & ITS COMPONENTS

TIME (see text)
CHART 1C  SIMULATION 1: EFFECT ON BANKS' BALANCE SHEET

Time (see text)
CHART 2A  SIMULATION 2: EFFECT ON REAL GDP

Time (see text)
CHART 2B  SIMULATION 2: EFFECT ON BANKS' BALANCE SHEETS

- Bills (The Dotted Line)
- LCANS
- Money (Both Kinds Together)

Time (see text)