I would like to thank Anwar Shaikh for his guidance and critical comments. Without holding them responsible in any way for my views, I would also like to thank Wynne Godley, Tom Palley, Dimitri Papadimitriou, Randy Wray, and Ajit Zacharias who provided very helpful comments at various stages of this work.
INTRODUCTION
The aim of this paper is to integrate finance, government spending, and the banking sector into a new model of cyclical growth which is rooted in the classical and Harrodian traditions. This classical growth and cycles (CGC) model is an extension of the framework developed by Shaikh (1989, 1991, 1992, 1996b, 1996c, 1996d, 1997a, 1997c), and also incorporates the flow-of-funds approach of Earley, Parsons, and Thompson (1976) as well as the social accounting matrix (SAM) methodology developed by Godley (1996, 1997, 1999). The CGC model belongs to a general class of linear and nonlinear models that study real-financial interactions (Skott, 1989; Taylor and Connell, 1989; Woodford, 1989; Franke and Semmler, 1989; Shaikh, 1989; Duménil and Lévy, 1989; Palley, 1996, 1997). Its dynamic disequilibrium properties, rooted in a stock-flow framework with endogenous bank credit (Kaldor, 1982; Moore, 1988; Shaikh, 1989; Wray, 1990; Palley, 1996) distinguishes it from orthodox macroeconomic models. A comparison of the CGC model with the state of the art of the heterodox macroeconomic literature (Tobin, 1980; Tobin and Buiter, 1980; Tobin, 1982; Taylor, 1985, 1991, 1997; Godley, 1996, 1997,1999) highlights the following features. As with this literature, the CGC model does not utilize standard neoclassical tools such as intertemporal optimizing behavior, production and utility functions, and the full employment assumption (McCafferty, 1990; Blanchard and Fischer, 1989). Moreover, it follows certain authors in this literature (Godley, 1996, 1997,1998; Tobin, 1980; Tobin and Buiter, 1982; Taylor, 1997) in relating sectoral expenditures to their respective finance requirements along with the corresponding changes in stocks and flows. Bank credit is a crucial type of finance in the CGC model although unlike Godley (1999) and Taylor (1997) the banking sector is not incorporated into the SAM by making any implicit assumption about bank net worth.

With regard to the business cycle the CGC model follows other heterodox approaches by endogenizing business cycles (Dore, 1993). This feature of these models distinguishes them from real business cycle (RBC) models in which cycles are caused by exogenous shocks alone. The short-run nonlinear business cycle debt dynamics in the CGC model is very much in the Minsky tradition and is therefore similar in spirit to the models of Franke and Semmler (1989), Palley (1996, 1997), Flachsel, Franke, and Semmler (1997), and Duménil and Lévy (1989).

However as in Harrod, but in contrast to the above heterodox literature, growth in the
CGC model is a continuous and persistent feature of the system at every point in time (Kregel, 1980). Thus its point of departure is not a given level of output but a continuous rate of growth. The implication is that, at the most fundamental level, the system operates in a growth environment and is internally driven by business investment. This quintessentially classical feature of the model (Eltis, 1993) distinguishes it from the Keynes/Kalecki tradition in which the short run is static and growth takes place only in the long run via exogenous demand-related factors. As discussed below, this dynamic feature of the CGC model is based on its distinction between fixed and circulating capital and the classical/Leontief input-output relationship.

Further, the disequilibrium dynamics of the CGC model rest on its distinction between ex ante plans and expectations and ex post outcomes. In fact both market disequilibria and growth in the CGC model arise from these discrepancies and their feedback signals. These features are generally not explicitly represented in the heterodox literature.

Finally, the CGC model follows the classical assumption that over the long run the economy fluctuates around normal capacity (Winston, 1974; Shaikh, 1992; Lavoie, 1995) which however is consistent with persistent employment as Goodwin (1967) demonstrated in his famous predator-prey model. Models in the Keynes/Kalecki tradition typically assume persistent excess capacity.

In short, the CGC model follows von Neumann and the line of classical thought which includes the Physiocrats and Marx (Chakravarty, 1989; Eltis, 1993, 1998) in demonstrating the dynamic, though turbulent, nature of market economies. In fact endogenous growth, the regulating role of the rate of profit, and the SAM methodology are consistent with the dynamic and “circular flow” view of the system that one finds in Quesnay’s *Tableau Economique*. Moreover, the debt and cyclical dynamics are in the Minsky tradition. It is this synthesis that is derived in the next section.
THE CORE FEATURES OF THE CGC MODEL

First, as with the post-Keynesian tradition, investment spending by firms under conditions of uncertainty (Keynes, 1936; Davidson, 1991) drives capital accumulation; these features of the model endogenize both growth and cycles. It is with regard to the role of investment that models in the classical and post-Keynesian traditions differ from neoclassical ones which are driven by the intertemporal consumption decisions of households.

However, unlike post-Keynesian models, central to the CGC framework is the explicit recognition that circulating and fixed investments have different effects: following Ricardo, von Neumann, and Leontief, the former adds to output whereas the latter adds to capacity. Shaikh demonstrates in a number of papers (Shaikh, 1989, 1991, 1992) that this distinction provides a solution to the famous knife-edge problem.

Second, economic cycles are intrinsic to the model and reflect the fact that business investment decisions are made under uncertainty. For example, the model’s fast adjustment process represents the adjustment between aggregate demand and supply and corresponds to the 3-5 year inventory cycle. The model’s slow adjustment process represents the adjustment between actual and normal capacity utilization and corresponds to the 7-11 year fixed capital cycle (Shaikh, 1989). Each of these cycles corresponds to a different adjustment mechanism.

Unlike financial markets, adjustments to disequilibria in the goods market are relatively slow since production involves real historical time. Actual output at any point in time is based on sales expectations (and planned additions to inventories) of firms in the previous time period when the production process was initiated. The confrontation of actual output with the planned demand by customers may generate an excess demand which would bring about undesired changes in the inventories of firms. But this discrepancy will also entail the revision of production plans (including the demand for inputs) and output in subsequent periods. This

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Note that implicitly the short run in the Keynes/Kalecki tradition is of a shorter time duration than it is in the classical tradition since it ignores disequilibrium adjustments. The point of departure of models in the Keynes/Kalecki tradition is the equality between aggregate demand and supply (Taylor, 1985, 1991; Lavoie, 1995; Palley, 1996). That is, all annual output is the equilibrium level of output in these models and no significance is attached to market disequilibria.
dynamic is at the core of the fast adjustment process.

A reduction of inventories below their desired levels is an immediate response of firms in the face of a positive excess demand. If the latter persists, they will respond in the following period by increasing investments in raw materials and labor power which produces more output. It is this classical input-output relationship which is central to the fast adjustment process and distinguishes it from both orthodox and heterodox macro-models. It is an immediate response of firms and ensures that the economy grows at every point in time\(^2\). Thus the model is a synthesis of Harrod’s warranted growth path and the classical input-output relationship.

This dynamic feature of the short run or fast adjustment process should be contrasted with models of the economy in the Keynes/Kalecki tradition. This literature follows Keynes who disagreed with Harrod’s approach and instead saw “growth [as] a long-period conception” (Kregel, 1980). This was also the viewpoint of Kalecki (1959). As Shaikh points out, these models in the Keynes/Kalecki tradition are static because they begin with a static specification of the short run; the significance of investment in circulating capital, which produces a change in output, is ignored while balance between aggregate demand and supply is assumed so as to define a level rather than a path of output. Thus in these models the short-run constancy of output makes growth a long-run phenomenon.

If the fast process roughly or tendentially equalizes aggregate supply and demand, then it needs to be asked what the effects are of deviations between actual and normal\(^3\) levels of capacity utilization. In the classical approach, this deviation between actual and normal levels of capacity utilization reacts back on the accumulation rate by altering the rate of fixed investment and thereby changes the paths of actual output and capacity, which in turn adjusts the initial

\(^2\) In the CGC model the growth path along the fast adjustment process need not be an equilibrium one, since it can be the result of either stochastically generated cycles or deterministic limit cycles. Stochastically sustained cycles arise from generally nonlinear stable oscillatory systems that are randomly perturbed by a turbulent and uncertain economic environment. Deterministic limit cycles represent local instability that is contained by bounded forces.

\(^3\) (Shaikh, 1991; Winston, 1974) A distinction needs to be made between normal capacity and the engineering capacity. The normal capacity is the economically feasible capacity and is defined as that level which is determined by the normal intensity and length of the working day, the number of shifts, costs etc. It should be distinguished from the engineering capacity, which is the technical upper limit to normal capacity.
deviation and feeds back on accumulation and so on.

Third, the dynamic characteristic of the model implies that all variables are written as shares of some variable such as output. This dynamic specification allows for the continuous variations of variables over time, so that all adjustments take place relative to any trends in these variables. This means that all variables are modeled in terms of growth rates or ratios, rather than levels. Following Goodwin, Shaikh argues that working with levels of variables rather than their ratios (to, say, output) excludes the possibility of endogenous growth\(^4\). The point is that the form in which a model is written determines whether or not it includes the possibility of growth.

From a dynamical perspective, then, there is a difference between a rise in the level of government spending \(G\) and a rise in the share of government spending \(g = G/Y\) where \(Y\) is output. A one-time increase in \(g\) is an acceleration of \(G\) relative to \(Y\) whereas a one-time increase in \(G\) produces a pulse in \(g\) which eventually dies out: each of these fiscal policies has a different long-run effect on the system although the short-run stimulus is identical. Thus in a growth context, the nature of the fiscal policy needs to be specified. Models such as those of Blinder and Solow (1973), Taylor (1985, 1991), Godley (1999), and Tobin (1980) which focus on levels and do not differentiate between the types of fiscal policy.

Fourth, the CGC model has its basis a SAM which relates each sector’s planned expenditures, expected income, and planned external finance requirements to every other sector. These *ex ante* budget restraints are not accounting identities but are behavioral restrictions on each sector’s planned expenditures and expected revenues: they ensure that every sector’s plans, expectations, and borrowing from other sectors are financially consistent (Clower, 1965).

Sectoral budget restraints are aggregated into an economy-wide budget restraint which, when written in an appropriate form, shows the relationship between net injections of purchasing power in the form of additions to the money supply and leakages of money as desired reserves or buffer stocks (Goodhart, 1984; Laidler, 1990, 1993).

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\(^4\) For example, if \(x = X/Y\) then the steady state value \(dx/dt = 0\) of a differential equation in \(x\) implies that either \(X\) and \(Y\) are growing at the same rates or that they are at some levels. On the other hand, the solution \(dX/dt = 0\) of a differential equation in \(X\) implies that \(X\) is not growing and is at the same level.
Money is injected into the economy via bank credit and fiscal policy. Neither of these sources of money need correspond to the public’s desire to hold money (currency and various types of deposits) as reserves or buffer stocks (Goodhart, 1984, 1989; Laidler, 1990, 1993). It is the discrepancy between the two which works its way through the different markets\(^5\). Thus, as with the commodity market to which it is linked by bank loans and government policy, the money market disequilibria will also adjust over the course of the fast adjustment process. The fast adjustment process is described by the dual disequilibria relationship which relates goods market disequilibria to the imbalance between the money supply and agents’ desired money reserves or buffer stocks (Goodhart, 1984; Laidler, 1990, 1993).

The desired increase in money holdings (\(L_{M_d}\)) refers to the holdings of money as a “buffer stock” (Rabin, 1979, 1993; Coghlan, 1981; Yeager, 1986; Goodhart, 1984, 1989; Dow and Dow, 1989; Laidler, 1990, 1993). The rationale for this notion is that in a complex market economy, supply and demand do not necessarily mesh instantaneously so that agents are likely to incur costs in finding a buyer for assets when money is required. Under these conditions of uncertainty each individual agent might be expected to hold some portion of his or her wealth in the form of money as a “temporary abode of purchasing power,” to use Friedman’s expression (cited in Laidler, 1990, p. 25).

Baumol (1952), Tobin (1956), Miller and Orr (1968), Orr (1970) and Akerlof (1979, 1980) were the ones who initially conceptualized reserve money holdings as flexible inventories. Drawing on this literature, Goodhart (1984) argues that in the face of uncertainty, both goods and financial assets are held as buffers to absorb unforeseen shocks. When these inventories rise or fall to some desired limit an adjustment process gets set in motion. For example, if at the end of a period a company accumulates undesired inventories, it will carry out certain readjustments in subsequent periods to either add to depleted stocks or run down unwanted ones. A similar argument can be made for desired money reserves. Goodhart concludes that for this reason shifts in desired money balances relative to desired levels should enter into expenditure functions.

The sources-and-uses of funds approach formulated by Earley, Parsons and Thompson

\(^5\) In a neoclassical model, the interest rate would adjust rapidly to clear the money market.
(1976) and its extended version by Shaikh (1989) include such a variable, thereby ensuring that fluctuations in money reserves affect expenditures. Thus, fluctuations in money reserves act as an additional factor in the monetary transmission mechanism discussion. It should also be noted that the buffer stock demand for money is essentially a *stock* demand for money and is expected to rise with liquidity preference, a point which follows from Keynes and is discussed by some post-Keynesian authors such as Coghlan (1981), Goodhart (1984) and Dow and Dow (1989).

From this it follows that any deviation of the economy’s stock of money from desired levels will lead to some dynamic adjustment processes that take a while to work themselves out in the system. Hence the notion of disequilibrium money (Goodhart, 1984).

.....the windfall gains and losses which the agent experiences from time to time might well manifest themselves in unexpected variations in cash holdings. A discrepancy between actual and long-run target money holdings can just as well arise from this source as from changes in the arguments of the agent’s long-run demand-for-money function, and there is no reason to believe that the response to such a discrepancy depends in any way upon what it generates. *Given that a discrepancy exists, the agent will attempt to move towards a long-run target demand for money by altering the current rate of flow of expenditures on goods, services, and asset accumulation.* That is to say, for the individual agent, a discrepancy between actual and desired cash balances will set in motion a real balance effect (Laidler, 1990, p. 27, emphasis added).

During the time that there is monetary disequilibrium, aggregate demand will be different from aggregate supply. However, in both the ‘Keynesian’ neoclassical synthesis and new classical economics such disequilibria are annulled instantaneously. Interest rate flexibility in the neoclassical synthesis case and wage and price flexibility in the new classical case ensure continuous market-clearing. Laidler argues that in these two theoretical approaches market disequilibria are corrected in *meta* time whereas for the buffer-stock theorist disequilibrium adjustments work themselves out in *real* time. Laidler concludes, “Money is very easy for the individual agent to get rid of, but very difficult for the economy as a whole to get rid of, if it is being pumped through credit creation” (Goodhart, 1984, p. 257).

Fifth, in the spirit of post-Keynesian models, bank credit in the CGC model fills the gap endogenously whenever planned investment exceeds available savings. However, this does not
This takes place via a fixed savings propensity which is a standard assumption in macromodels. Of course, the fundamental role of bank credit in accumulation remains as long as the savings propensity is constant. Marx, however, was to demonstrate in the schemes of reproduction that accumulation can take place via pure internal finance without bank credit. In such a situation higher planned investment can take place when capitalists reduce their consumption spending, thereby releasing the additional savings needed to finance the investment (Bleaney, 1976; Shaikh, 1989).

It should be pointed out that savings, notably business retained earnings, play a crucial role in both the fast and slow adjustment processes. While investment can exceed savings\(^6\) in the fast adjustment process, the very fact that the accumulation of finance charges slows down investment implies that business profits play a central role in regulating the investment rate. This is even more evident with the warranted growth path which is determined by the social savings rate \(s^* = s + (t - g)\) where \(s\) and \((t - g)\) are the private and public savings rates respectively. The reliance of accumulation on savings is not surprising since in the classical tradition it is the total mass of surplus value that firms have which determines investment.

In the CGC model two different mechanisms determine profitability in the fast and slow adjustment processes respectively. Over the course of the business cycle profits are determined by demand because capacity utilization is endogenous. Along the warranted path, for any given capital stock, the mass of profits is regulated by the normal rate of profit which itself is determined by income distribution and technology (Sraffa, 1960).

To summarize, along the warranted path of output capacity utilization fluctuates around the normal (potential) level, the actual rate of profit fluctuates around the normal (potential) profit rate and growth is internally generated from the reinvestment of profits even when there is no technological change.

However, if normal capacity growth is not attainable because of the knife-edge problem

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\(^6\) This takes place via a fixed savings propensity which is a standard assumption in macromodels. Of course, the fundamental role of bank credit in accumulation remains as long as the savings propensity is constant. Marx, however, was to demonstrate in the schemes of reproduction that accumulation can take place via pure internal finance without bank credit. In such a situation higher planned investment can take place when capitalists reduce their consumption spending, thereby releasing the additional savings needed to finance the investment (Bleaney, 1976; Shaikh, 1989).
the central role of profitability would have to be replaced by other regulating factors such as population growth, waves of exogenously-given technological change, government spending, expectations etc. The bulk of the growth theory literature uses some combination of these factors to explain growth because of the apparent inability of the system to sustain balanced growth.

One common solution to the knife-edge problem has been to assume that the actual growth path of the economy is the warranted path. The analysis then shifts to the properties of this path or the relationship between it and the labor utilization rate, i.e. to the relationship between the warranted path and the natural growth rate which is determined by population growth and technological change. The Swan-Solow models (Sen, 1970), the ceiling-floor growth-cycle model of Hicks (1950) and Goodwin’s (1967) non-linear growth-cycle model all fall in this broad approach.

Another common response to the knife-edge problem is to take growth as given, so that the focus of this line of research is on the cyclical fluctuations around this exogenous trend. The Lucas Rational Expectations models and the Nordhaus Political Business Cycles models all fall into this category (Mullineaux, 1984), as do the non-linear cyclical models of Kaldor (1940), Hicks (1950), and Goodwin (1951). The various cyclical models of Kalecki also fall in this category (Kalecki, 1971; Steindl, 1981).

Finally, multiplier-accelerator models comprise the third response to this problem (Shaikh, 1992). In these models, certain parameter ranges yield damped oscillations around a stationary path while other parameter ranges yield growth that is asymptotic to a non-warranted path. However these models do not yield warranted growth (Shaikh, 1992).

To conclude, the growth literature can be subdivided into the following three categories. The first group consists of short-run models. Of these, the static models assume short-run equality between aggregate demand and supply so that I = S is the point of departure (Kalecki, 1971; Steindl, 1979; Kregel, 1980; Taylor, 1985, 1991; Lavoie, 1995). In these models growth of output takes place via exogenous factors. On the other hand, the series of models developed by Shaikh (1989, 1991, 1992) are dynamic and in the short run do not assume that I and S are equal. The possibility of short-run aggregate excess demand is a crucial aspect of Shaikh’s approach since this is precisely the feature that makes growth a persistent feature of the system.
The second group assumes that over the medium-run capacity utilization $u$ is approximately at the normal level $u_n$ and that the economy’s actual growth equals the warranted growth. Harrod’s famous warranted path model (Sen, 1970; Hacche, 1979) and the various models by Shaikh (1989; 1991; 1992) fall in this category. Other authors such as Robinson (1956, 1962) and Solow (1956) discuss the warranted growth path, but their concern was also with the convergence between it and the natural growth rate (Sen, 1970).

This is the research agenda of the third group of models. Their goal is to identify the particular long-run conditions that make the warranted growth rate $g_w$ converge to the natural growth rate $g_n$ (Sen, 1970). This equilibrium growth rate is what Hacche (1979) calls the *steady-state growth rate*. If $s$ is the savings propensity, $\beta$ the capital/output ratio, $n$ the rate of population growth and $m$ the rate of technological change then this convergence implies that $s/\beta = g_w = n + m = g_n$. The goal of this literature is to identify the particular circumstances that make the savings propensity, the capital/output ratio or the rate of technological change adapt to ensure the above equality. Non-neoclassical models in this category are those by Kaldor (1955-56, 1957), Kaldor and Mirrlees (1961-62), Kalecki (1954), Pasinetti (1962, 1965) and Robinson (1956, 1962). The basic neoclassical models are those by Solow (1956) and Swan (1956) and the literature that follows (see Sen, 1970 and Hacche, 1979). The CGC model investigates the interaction between the short run and the medium run as well as the convergence to the warranted path. The question of the convergence between $g_w$ and $g_n$ is beyond the scope of the present work.

**THE FORMAL MODEL**

As the CGC model is an extension of Shaikh’s growth and cycles framework, we begin this section by representing the Shaikh (1989) model as an *ex ante* SAM. (The expression $L$ against a variable X represents the *desired increment* of that variable, $LX = X - X_{-1}$. It should be contrasted with $L$ which is the *actual increment* of that variable).

<table>
<thead>
<tr>
<th></th>
<th>Households</th>
<th>Firms</th>
<th>Ex Ante Gaps</th>
</tr>
</thead>
</table>

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7 The superscripts “p” and “e” stand for plans and expectations respectively.
It should be emphasized that budget restraints are behavioral financial constraints faced by each sector and are not accounting identities; thus planned expenditures are always determined by expected income inflows.

Table 1. *Ex Ante* Social Accounting Matrix of a Pure Private Sector Economy

<table>
<thead>
<tr>
<th>Consumption Of domestic goods</th>
<th>- Cₚ</th>
<th>+ C*</th>
<th>+ C* - Cₚ</th>
</tr>
</thead>
<tbody>
<tr>
<td>Investment of domestic goods</td>
<td>- Iₚ</td>
<td>+ I*</td>
<td>+ I* - Iₚ</td>
</tr>
<tr>
<td>Subtotal</td>
<td></td>
<td>(C* + I* + G*) - (Cₚ + Iₚ)</td>
<td></td>
</tr>
<tr>
<td>Wages</td>
<td>+ w.(N*)</td>
<td>- w.(Nₚ)</td>
<td>+ w.(N* - Nₚ)</td>
</tr>
<tr>
<td>Dividends (* = dividend yield)</td>
<td>+ *(EQₚ)</td>
<td>- *(EQₚ)</td>
<td>+ *(EQₚ) - *(EQₚ)</td>
</tr>
<tr>
<td>Subtotal</td>
<td></td>
<td>- [(Yₚ - Yₕ) + w.(N* - Nₚ) + *(EQₚ) - *(EQₚ)]</td>
<td></td>
</tr>
<tr>
<td>Interest Flows on Loans</td>
<td>- iₑ.(Lₑₚ)</td>
<td>- iₑ.(Lₑₚ)</td>
<td></td>
</tr>
<tr>
<td>Ch. In Equity</td>
<td>- (LEQₚ)</td>
<td>+ (LEQₚ)</td>
<td>+ (LEQₚ) - (LEQₚ)</td>
</tr>
<tr>
<td>Ch. In Bank Loans</td>
<td></td>
<td>+ LLₚ</td>
<td>+ LLₚ</td>
</tr>
</tbody>
</table>

This simple model of a pure private sector economy relates the budget restraints of the business and household sectors to each other so that most uses of funds are coherently related to their sources; the sole exceptions are the flows related to credit which are not traced to their source because the banking sector is not introduced explicitly into this model. The model deliberately ignores an explicit treatment of the banking industry and assumes that the interest rate on loans is determined exogenously via some appropriate policies. In contrast to the Foley (1987) model, Shaikh’s purpose is to demonstrate that business cycles can be produced without any variations in the interest rate.

Appendix 1 shows how the aggregate budget restraint from the *ex ante* SAM can be used to derive an expression for excess demand, which is the key operational variable in the fast adjustment process in Shaikh’s model. Excess demand, which is defined as the difference between total demand and total supply, captures unfulfilled expectations. Any discrepancy between supply and demand will be reflected in undesired changes in final goods inventories.

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8 It should be emphasized that budget restraints are behavioral financial constraints faced by each sector and are not accounting identities; thus planned expenditures are always determined by expected income inflows.
This difference between \textit{ex ante} and \textit{ex post} was explicitly recognized by Buiter (1980) who, however, restricted it to planned and expected dividends.

\begin{align*}
\text{UCINV}_t &:\quad 1. \quad E_t = D_t - Q_t = - UCINV_t \\
&\text{Thus, in the event that excess demand is positive final goods inventories will fall below their desired levels and the undesired change will be a negative one.}
\end{align*}

From the aggregate budget restraint,

\begin{align*}
2. \quad \{- E + w.(N^e - N_d) + \{*(EQ_{t-1})^e - *(EQ_{t-1})^p}\} - i_t(L_{dt-1}) + LL_d &= 0 \\
&\text{Assuming zero expectational errors,}
\end{align*}

\begin{align*}
3. \quad E = I_d + S_e = LL_d - i_t(L_{dt-1}) = L_{dt} - L_{dt-1} - i_t(L_{dt-1}) = L_{dt} - (1 + i_t)L_{dt-1} = L_{dt} - F_t \\
&\text{where } F_t = \text{finance charges on debt = principal due + interest due. Equation 3 parallels the equation linking excess demand and the net injection of credit in the Shaikh (1989) model. The difference between the two equations is that in the latter } B_t \text{ is a flow (therefore } B_t/LL_d \text{ ) and the finance charges } F_t \text{ are in terms of the flow of the period’s debt: } F_t = (1 + I)B_{t-1}. \text{ In equation 2 } F_t \text{ is in terms of the stock of the previous period’s debt: } F_t = (1 + i)L_{dt-1}. \text{ These differences aside, the economic meaning of these equations is the same. That is, they both show that the excess of planned investment over available saving is fueled by the net injection of bank credit so that the money supply adjusts endogenously to accommodate the needs of accumulation.}
\end{align*}

In the SAM in Table 1 the assumption that \textit{ex ante} sources and uses of funds of each sector are internally consistent can be imposed by the requirement that columns sum to zero. On the other hand, rows add to zero when all cross-sectoral expectations are exactly correct. This is the situation when the wage payments planned by firms (-W^p_f) exactly equal the wages expected by households (+W^e_h) or the consumption planned by households (-C^p_h) exactly equals the consumption demand expected by firms (+C^e_f) (Shaikh, 1997d). However, in general in an \textit{ex ante} mode such exact equalities need not hold since the wages \textit{planned} by firms (-W^p_f) need not match the wages \textit{expected} by households (+W^e_h)^9. From the standpoint of those disequilibria modeled by Shaikh, the central significance of the \textit{ex ante} gaps is that they allow for a discrepancy to arise between the output that results from firms’ sales expectations and the planned demand for output by customers.

The above SAM incorporates a fundamental feature of the methodology applied by

\begin{flushright}
\textbf{9} This difference between \textit{ex ante} and \textit{ex post} was explicitly recognized by Buiter (1980) who, however, restricted it to planned and expected dividends.
\end{flushright}
Godley (1996, 1997, 1999) and Shaikh (1989, 1990, 1997a). In the way that was pioneered by Richard Stone and Wynne Godley, such an approach relates stocks and flows of money in a coherent fashion so that the model of the economy has no "black holes". This methodology imposes an analytical discipline to the way that the flows are represented.

Godley’s SAM is cast in terms of *ex post* accounting identities although, as with the CGC model, his economic model also distinguishes between *ex ante* and *ex post*. His *ex post* SAM is a description of what actually happened, so that the funds received by one sector in a transaction exactly equal the funds paid out by another. As with the CGC model, funds received or paid out and all financial balances have as their counterpart changes in assets and liabilities. In Godley’s model all column sums and row sums add to zero, respectively.

For some other analytical purposes it is also important to consider *ex ante* variables in the SAM. This allows sectoral budget restraints to be represented in the SAM. While *ex post* such a SAM has to also respect accounting identities, its *ex ante* feature allows it to show how each sector’s planned expenditures are related to its expected income. This implies that the funds received by one sector may not necessarily equal those paid out by another. Hence in the *ex ante* SAM, all column sums add to zero but the row sums do not necessarily do so. Thus the significance of the *ex ante* gaps.

This distinction in the treatment of the SAM has an important implication. As derived below, a central result of the *ex ante* SAM is the so-called dual disequilibria relationship linking excess demand in the goods market to monetary disequilibria which arise when money stocks from injections (from the banking sector and the government in a closed economy) do not coincide with agents’ desired money holdings (determined by liquidity preference). On the other hand, Godley (1998) rejects the possibility of any distinction between the money supply and the money which agents either desire to or actually do hold. In other words, the methodological difference between the treatment of the SAM in the CGC model and Godley’s model has far-reaching macroeconomic implications.

The extended *ex ante* SAM shown in Table 2 is a further extension of Shaikh (1989) with four sectors: household, firms, commercial banks, and the government (all variables are
I am grateful to Wynne Godley and Anwar Shaikh for a number of helpful comments with regard to this SAM.

The aggregate column sum of each sector (i.e. the sum of its current and capital accounts) is its budget restraint. Alternatively, as in Godley (1999), the distinction between the current and capital accounts reveals that for each sector the gap between its income and expenditure equals its net asset accumulation.

<table>
<thead>
<tr>
<th>Households</th>
<th>Firms</th>
<th>Banks</th>
<th>Govt.</th>
<th>Ex Ante Gaps</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>Consumption Of Domestic goods</strong></td>
<td>- $C_d^p$</td>
<td>+ $C_e$</td>
<td></td>
<td></td>
</tr>
<tr>
<td><strong>Gross Fixed &amp; Materials Investment</strong></td>
<td>+ $I_{lm}^e$</td>
<td>- ($I_{lm}^d)_p</td>
<td>- ($I_d^p)_{bk}$</td>
<td>+ $I_{lm}^e - (I_{lm}^d)_p$</td>
</tr>
<tr>
<td><strong>Inventory Investment</strong></td>
<td>+ $I_v^e$</td>
<td>- ($I_v)_d^p</td>
<td></td>
<td>+ $I_v^e - (I_v)_d^p$</td>
</tr>
<tr>
<td><strong>Govt. Expenditures on Domestic Goods</strong></td>
<td>+ $G^e$</td>
<td></td>
<td></td>
<td>- $G_d^p$ + $G_e - G_d^p$</td>
</tr>
<tr>
<td><strong>[Output]</strong></td>
<td>[Y^e]</td>
<td></td>
<td></td>
<td>+ $T^e$ - $T^p$</td>
</tr>
<tr>
<td><strong>Subtotal</strong></td>
<td></td>
<td></td>
<td></td>
<td>+ $w^p(N_{t+1})$ - $w^p(N_{t+1})_{T^p}$</td>
</tr>
<tr>
<td><strong>Taxes</strong></td>
<td>- $T_h^p$</td>
<td>- $T_f^p$</td>
<td>- $T_{bk}^p$</td>
<td>+ $T^e$ - $T^p$</td>
</tr>
<tr>
<td><strong>Wages</strong></td>
<td>+$w^p(N_{t+1})$</td>
<td>-$w^p(N_{t+1})_{T^p}$</td>
<td>-$w^p(N_{t+1})_{bk}$</td>
<td>+$w^p(N_{t+1})$ - $w^p(N_{t+1})_{T^p}$</td>
</tr>
</tbody>
</table>
| **Int. Flows on Deposits, Bonds and Loans** | + $i_d^p(D_{t+1})_{bk}$ | + $i_d^p(D_{t+1})$ | - $i_d^p(D_{t+1})_{bk}$ | + $i_d^p(D_{t+1})$
| | + $i_h^p(BG_{t+1})_{bk}$ | + $i_h^p(BG_{t+1})$ | - $i_h^p(BG_{t+1})_{bk}$ | + $i_h^p(BG_{t+1})$
| | - $i_h^p(L_{t+1})_{bk}$ | - $i_h^p(L_{t+1})$ | - $i_h^p(L_{t+1})_{bk}$ | - $i_h^p(L_{t+1})$
| | + $i_h^p(BP_{t+1})_{lk}$ | + $i_h^p(BP_{t+1})$ | - $i_h^p(BP_{t+1})_{bk}$ | - $i_h^p(BP_{t+1})$
| | + $i_d^p(BR_{t+1})$ | + $i_d^p(BR_{t+1})$ | - $i_d^p(BR_{t+1})_{bk}$ | - $i_d^p(BR_{t+1})$

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I am grateful to Wynne Godley and Anwar Shaikh for a number of helpful comments with regard to this SAM.
<table>
<thead>
<tr>
<th>[Profit]</th>
<th>[P₁]</th>
<th>[P₅ₙ]</th>
<th>0</th>
<th>0</th>
<th>0</th>
<th>0</th>
</tr>
</thead>
<tbody>
<tr>
<td>Retained Earnings</td>
<td>-(REₜ)</td>
<td>+ (REₜ)</td>
<td>-(RE₅ₙ)</td>
<td>+ (RE₅ₙ)</td>
<td>0</td>
<td>0</td>
</tr>
<tr>
<td>Dividends (* = dividend yield)</td>
<td>+*(EQₜ₋₁)</td>
<td>-<em>ᵦ</em>(EQₜ₋₁)</td>
<td>-<em>ᵦ</em>(EQₜ₋₁)</td>
<td>+*(EQₜ₋₁)</td>
<td>- *(EQₜ₋₁)</td>
<td><em>ᵦ</em>(EQₜ₋₁)</td>
</tr>
<tr>
<td>Ch. In High Powered Money (net of fiscal and monetary policies)</td>
<td>-(LCuₜ₊)ₜ</td>
<td>-(LCuₜ₊)ₜ</td>
<td>-LRₜ₊</td>
<td>+LHₜ</td>
<td>+LHₜ -(LCuₜ₊)ₜ +LRₜ</td>
<td>0</td>
</tr>
<tr>
<td>Ch. In Borrowed Reserves (Discount Window Loans)</td>
<td></td>
<td></td>
<td>+LBRₑ</td>
<td>-LBRₜ</td>
<td>+LBRₑ - LBRₜ</td>
<td>0</td>
</tr>
<tr>
<td>Subtotal (for sources and uses of high powered money)</td>
<td></td>
<td></td>
<td></td>
<td>(LHₜ - LBRₑ) - LCuₜ₊ - (LRₜ - LBRₑ) = LHₜ - (LCuₜ₊ + LRₜ) + (LBRₑ - LBRₜ)</td>
<td>0</td>
<td>0</td>
</tr>
<tr>
<td>Ch. In Deposits</td>
<td>-(LDₜ₊)ₜ</td>
<td>-(LDₜ₊)ₜ</td>
<td>+ (LDₑ)ₗₕ</td>
<td>+ LDₑ - LDₜ₊</td>
<td>0</td>
<td>0</td>
</tr>
<tr>
<td>Ch. In Equities</td>
<td>-(LEₜ₊)ₜ</td>
<td>+ (LEₑ)ₗ</td>
<td>+ (LEₑ)ₗₕ</td>
<td>+ LEₑ - LEₜ₊</td>
<td>0</td>
<td>0</td>
</tr>
<tr>
<td>Ch. In Private Bonds</td>
<td>-(LBPₜ₊)ₜ</td>
<td>+ (LBPₑ)ₗ</td>
<td>-(LBPₑ)ₗₕ</td>
<td>+ LBPₑ - LBPₜ₊</td>
<td>0</td>
<td>0</td>
</tr>
<tr>
<td>Net Ch. In Gov. Bonds (net of fiscal and monetary policies)</td>
<td>-(LBₜ₊)ₜ</td>
<td>-(LBₑ)ₗ</td>
<td>-(LBₑ)ₗₕ</td>
<td>+ LBₑ</td>
<td>+ LBₑ - LBₜ₊</td>
<td>0</td>
</tr>
<tr>
<td>Ch. In Bank Loans</td>
<td>+(LLₜ₊)ₜ</td>
<td>+(LLₑ)ₗ</td>
<td>- LLₑ</td>
<td>+ LLₑ - LLₜ₊</td>
<td>0</td>
<td>0</td>
</tr>
<tr>
<td>Column Sums</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>0</td>
</tr>
</tbody>
</table>

**Table 2. Ex Ante Social Accounting Matrix with the Government Sector**

Notes: (1) Expected output is gross of depreciation and corresponds to the BEA’s gross product originating. (2) İ = İₑₑ + İₑ and İₑ = (İₑₑ)ₑₑ + ( İₑₑ)ₑₑ + ( İₑₑ)ₑₑ where İₑₑ = fixed and materials investment
by non-bank firms, $I_v = \text{inventory investment by non-bank firms}$, and $(I_d^p)_{bk} = \text{fixed investment by banks}$. (3) The government sector includes both the Treasury and the monetary authority. (4) The treatment of retained earnings in the current and capital accounts follows Godley (1996). (5) In the firm budget restraint the expected inventory sales term $I_v^e$ appears on the sources-of-funds side while the planned inventory investment appears $(I_v^p)_{bk}$ on the uses-of-funds side. This distinction allows one to derive the standard definition of profits (Godley and Cripps, 1983, p. 186; Shaikh, 1991). However, one cannot make an economic distinction between these two variables\(^{11}\). (6) Aggregate savings $= S_h + S_f + (S_{bk} + S_g) = S + S_g$ where $S_h = \text{household savings}$, $S_f = \text{business retained earnings}$, $S_{bk} = \text{savings of banks-as-businesses}$ and $S_g = \text{government savings}$. (7) If the interest and dividend rates are known in advance then the \textit{ex ante} gaps for these two entrees are equal to zero. Financial flows correspond to the stocks of assets bought at the end of the previous period. (8) If labor is assumed to be hired before production begins then wage flows correspond to the total quantity of labored that was hired before production starts. (9) For notational simplicity, $LBG / p_b (LB_G)$: given the bond price $p_b$, a change in the value of the stock of bonds between two periods $= p_b (LB_G) + ) p_b (B_G)$ in which the first term stands for the flow in bond transactions and the second one represents the capital gain when the bond price changes (the same notation applies to equity).

With regard to the firm sector, the sum total of the current and capital accounts gives the aggregate budget restraint of this sector so that aggregate sources of funds - aggregate uses of funds $= 0$. Further, the current account column also sums to zero since clearly current planned expenditures have to be restrained by current income expectations. It therefore follows that the capital account column has to necessarily equal zero also.

For households, firms, and the government the columns sum to zero since total uses - total income $= \text{total external funds for each of these sectors}$. These external funds are bank loans for households and firms and bonds plus high powered money for government. But for banks, caution needs to be exercised in writing an \textit{ex ante} budget restraint since from the endogenous money perspective the supply of credit is not determined by such a restraint. That is, this particular source of funds is not restrained in any direct technological sense by the other inputs into the banking firm. Godley (1996,1999) and Taylor (1997) introduce the banking sector into the SAM by implicitly assuming that bank net worth is either constant or zero since only then will ) (assets) = ) (liabilities)\(^{12}\). One advantage of this is that it converts the balance

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\(^{11}\) I am grateful to Ajit Zacharias for pointing this out to me.

\(^{12}\) In Godley (1998), for example, since every other column sums horizontally to zero the bank column must also do so, leading to the constant or zero net worth assumption. One could in fact argue that some special assumption such as this is inevitable in an \textit{ex post} matrix. In such a
sheet identity assets = liabilities + net worth into a constraint. The problem with this assumption is that at best it is a special case but is not consistent with the banking models of a number of post-Keynesian authors who treat the banking firm as a profit-making entity (Rousseas, 1985; Moore, 1988; Wray, 1990; Palley, 1996). Moreover the zero net worth assumption reduces banks to mere passive entities, a view that is inconsistent with the literature that has discussed the aggressive nature of bank lending (Minsky, 1982a; Darity and Horn, 1988; Wray, 1990).

To deal with the problem of the banking sector it is proposed for analytical purposes that it has a dual identity: banks-as-businesses and banks-as-banks. The former role describes the normal activities of the banking firm which implies that the sum total of its sources and uses of funds has to equal zero, as with non-banking firms. Thus in this role the bank can be represented by a budget restraint:

\[
\text{Sources} = \text{Uses}
\]

4. \[i_L(L_{t-1}) + i_b(BG_{t-1})_{bk} + i_b(BP_{t-1})_{bk} - *p(EQ_{t-1})_{bk} - i_d(D_{t-1}) - i_w(BR_{t-1})] + (EQ^p)_{bk}
\]

\[= T^p_{bk} + (I^b)_{bk} + w^p(N_{t-1})_{bk}\]

where \(i_L(L_{t-1})\) = interest receipts on bank loans, \(i_b(BG_{t-1})_{bk} + i_b(BP_{t-1})_{bk}\) = interest receipts on government and private bonds held by banks, \(*p(EQ_{t-1})_{bk}\) = dividend payment on bank equity at the rate \(*\), \(i_d(D_{t-1})\) = interest payments on deposits, interest payments on borrowed reserves, \((EQ^p)_{bk}\) = bank equity, \(T^p_{bk}\) = tax payments, \((I^b)_{bk}\) = investment, \(w^p(N_{t-1})_{bk}\) = wage payments at rate \(w^p\) for \(N\) workers. The term in brackets groups together the income earned by banks net of all interest and dividends paid.

The role of banks-as-banks has to do with those kinds of activities that specifically distinguish banks from other sectors. These activities pertain to the supply of deposits and loans, borrowing from the discount window and maintaining an adequate level of reserves. From the endogenous money framework, the provision of loans is not related in some mechanical fashion to reserves (as in the exogenous money approach) but is determined by banks’ desired balance sheet liquidity (Wray, 1990; Pollin, 1991; Palley, 1996). One would
therefore not expect the variables corresponding to banks-as-banks to be part of a budget restraint since they are in the final instance determined by other factors such as the desired balance sheet liquidity, the pace of accumulation etc. Our next step is to show that these variables are related to one another via a particular relationship which is derived by taking into account not only the *forms* in which money is kept (currency plus a broad range of deposits) but also the *sources* of money\(^{13}\).

The change in money supply

5. \(\text{LM}_s' \ \text{LL}^\text{H}'\%\text{LH}'\)

where \(\text{LH}'\) = exogenous component of the money supply, \(\text{BR} = \) borrowed reserves (discount window loans), \(\text{LH}' \ \text{LC}^\text{R}, \text{L}^\text{H}'\) aggregate bank credit to public and private sectors. In other words, the exogenous increase in the money supply equals the aggregate injection of base *net* of discount window loans. This exogenous component is the net resultant of fiscal and monetary policies and corresponds to the net purchase or sale of government bonds by state institutions (comprising the Treasury and the central bank). It turns out that the expression for the aggregate money stock \(\text{M}_s' \ \text{L}^\text{H}'\%\text{H}^\text{H}')\) allows us to introduce a relationship which constitutes the *differentia specifica* of the banking sector.

The change in aggregate loans extended by banks is

6. \(\text{LL}' \ \text{LL}^\text{H}'\%\text{LBG}_{bk} \%\text{LB}_{bk}')\)

in which the first term on the right is new bank credit to the private sector and the second term in parentheses is the purchase of government and corporate bonds out of commercial bank excess reserves. Let the form in which money exists be \(\text{M}_s' / \text{C + D} = \) currency + non-interest and interest bearing deposits. Then in a closed economy, the aggregate change in money supply is given by

7. \(\text{LM}_s' \ \text{LC}^\text{D}' \ \text{LL}'\%\text{LBG}_{bk} \%\text{LB}_{bk}') \ \text{LL}'\%\text{LB}_{bk}' \ \text{LL}'\%\text{LH}'\%\text{LBG}_{bk} \%\text{LB}_{bk}').\)

This equation identifies the sources of money supply in a closed economy. The two bond

\(^{13}\) For notational simplicity we will ignore all superscripts and subscripts pertaining to plans and expectations in deriving the important relationship.
purchases out of commercial bank excess reserves essentially involve the monetization of those reserves since these purchases increase the money stock. However, this monetization does not entail any increase in aggregate base. For simplicity, it will be assumed that all bonds purchased by banks are done so through excess reserves. Therefore,

\[ LM_s = LC + LL + \text{other reserves} \]

Writing the money supply in terms of its sources enables us to derive an expression that captures a particular relationship between loans, deposits and reserves. Since \( LM_s \) is \( \%LH \) it follows that:

\[ LC = LL + \text{other reserves} \]

In other words, in terms of plans and expectations,

\[ LD = LL + \text{other reserves} \]

This is a crucial result that relates the key operational variables of banks-as-banks to each other. The term \( LBR_e \) is the expected increase in bank reserves through discount window borrowing from the central bank, i.e. it is the expected sale of this particular asset by banks to the central bank. No assumptions about bank net worth were made in deriving this result. Brunner and Meltzer (1990) derive a similar result but by assuming that bank net worth equals zero.

Assume first that both the current and the capital account are merged into one column. Eliminating the relationship represented by equation 11 from this column would leave the budget restraint of banks-as-businesses which was derived above (equation 4). The combination of this budget restraint and equation 11 would ensure that the aggregate column sum of the banking sector equals zero. But, as with non-bank firms, we also know that current planned expenditures are restrained by expected income inflows. It therefore follows logically that if both the aggregate column (obtained by merging the current and capital account) and the
current account column sum to zero, then the capital account will also sum to zero.

The current accounts of the firm and banking sector allow us to derive expressions for the expected profits of these two sectors (Shaikh, 1991; Godley, 1998):

**Firms:**

12. \[ P_f^c \epsilon (C^c \% \Delta_{fm}^c \% G^c \% \Delta_v^c) \& w^p(N_{t&d})_f \& T_f^p \& INT_f^p \]

**Banks:**

13. \[ P_{bk}^c \epsilon INT_{bk}^c \& w^p(N_{t&d})_{bk} \& T_{bk}^p \]

where INT_f^p is the net interest paid by firms and INT_{bk}^c is the net interest inflow into the banking sector which constitutes its revenue.

Table 3 shows the balance sheets for the economy, which are the stock counterparts of the flow matrix in Table 2 (NW represents the net worth of each sector, \( K_f \) is the stock of capital, INV is the stock of finished goods inventories, and DG is total government liability).

<table>
<thead>
<tr>
<th></th>
<th>Households</th>
<th>Firm</th>
<th>Bank</th>
<th>Govt</th>
<th>Row Sum</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>Capital Stock</strong></td>
<td></td>
<td>+K_f</td>
<td></td>
<td></td>
<td>K_f</td>
</tr>
<tr>
<td><strong>Inventories</strong></td>
<td></td>
<td>+INV</td>
<td></td>
<td></td>
<td>INV</td>
</tr>
<tr>
<td><strong>Cash</strong></td>
<td>+C_u_h</td>
<td>+C_u_f</td>
<td>+R</td>
<td>-H</td>
<td>0</td>
</tr>
<tr>
<td><strong>Deposits</strong></td>
<td>+D_h</td>
<td>+D_f</td>
<td>-D_{bk}</td>
<td></td>
<td>0</td>
</tr>
<tr>
<td><strong>Govt. Bonds</strong></td>
<td>+B_G_h</td>
<td>+B_G_f</td>
<td>+B_G_{bk}</td>
<td>-B_G</td>
<td>0</td>
</tr>
<tr>
<td><strong>Pvt. Bonds</strong></td>
<td>+B_P_h</td>
<td>-B_P_f</td>
<td>-B_P_{bk}</td>
<td></td>
<td>0</td>
</tr>
<tr>
<td><strong>Equity</strong></td>
<td>+E_Q_h</td>
<td>-E_Q_f</td>
<td>-E_Q_{bk}</td>
<td></td>
<td>0</td>
</tr>
<tr>
<td><strong>Loans</strong></td>
<td>-L_h</td>
<td>-L_f</td>
<td>+L_{bk}</td>
<td></td>
<td>0</td>
</tr>
<tr>
<td><strong>Column Sum</strong></td>
<td>NW_h</td>
<td>NW_f</td>
<td>NW_{bk}</td>
<td>DG</td>
<td>K_f+INV</td>
</tr>
</tbody>
</table>

Table 3. Balance Sheets of the Economy

We next turn to fiscal and monetary policies. The Treasury has basically three sources which it can use to finance its deficit (Ritter and Silber, 1991). The impact on high-powered money and the money supply depends on who buys the government bonds: (a) if either the non-bank public or banks with zero excess reserves buy the bonds then both the money supply and the high-powered money remain unchanged; (b) if banks with excess reserves buy the bonds, the money supply expands though there is no change in high-powered money and (c) if the central
bank buys the bonds, there is an expansion of both the money supply and high-powered money. Note that the purchase of bonds by the central bank is “the modern-day equivalent of printing money to finance a deficit,”(Ritter and Silber, 1991, p. 262).

For the sake of completion we should also mention what the effects on the money supply and high powered money would be because of the monetary policies of the central bank. The latter alters the quantity of high powered money through its open market operations and discount window loans. Some of these actions also affect the aggregate money supply. Thus the purchase of bonds from the nonbank public injects high powered money and expands the money supply. The purchase of bonds from commercial banks increases high powered money (by expanding bank reserves) and not necessarily the money supply. Finally, bank reserves can expand from discount window loans though again this may have no effect on the money supply. Note that from a neoclassical perspective, any expansion of bank reserves automatically leads to an expansion of money via the money multiplier (Mishkin, 1995).

The government budget restraint is given by:

\[ G & T' \ LM_G \%LBG \]

or (suppressing subscripts and superscripts)

15.  \[ G & T' \ LM_G \%LBG \]

where

16.  \[ LM_G \%LBG \]

is the monetized portion of the budget deficit and consists of the money “printed” to finance the

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14 In the U.S. the Fed does not directly buy securities from the Treasury. Instead, the Federal Reserve banks act as the Treasury’s fiscal agent and hold auctions at which they bring newly issued bonds to the market. Securities dealers buy the newly issued Treasury bonds. In pursuit of its monetary policies the Fed could decide to buy back some of these securities from the dealers by issuing money (Ritter and Silber, 1991).

15 However, under existing institutions in the U.S., it is the prerogative of the Fed in deciding how much of the Treasury debt it should monetize. This relative independence of the Fed in pursuing monetary policy was established after the Treasury-Federal Accord of 1951.
budget deficit $H^{L \& L}$ and that which originates from banks’ purchase of government bonds out of excess reserves. The portion of $LM_G$ which originates from central bank borrowing directly injects new high powered money and expands the money supply. This is called “government money creation” (Burdekin and Langdana, 1992, p. 3). Note that $\$\$\$ is a variable that is inclusive of the fiscal and monetary policies of the Treasury and the central bank respectively. Thus the above equation is the net injection of money and bonds from the joint interaction of fiscal and monetary policies. Finally, $G$ is defined as government spending gross of net interest payments.

$H^{L \& L}$ part of the deficit that is financed through central bank borrowing (“printed” money) and $L^{L \& L}$ aggregate bank credit = $L + (BP_d)_{bk} + (BG_d)_{bk}$ where $L + (BP_d)_{bk} = DB$ is bank credit to the private sector and $(BG_d)_{bk} = DG$ is bank credit to the government (purchased out of bank excess reserves). Since $M_G^{L \& L}$ $H^{L \& L}$ = money created to finance the budget deficit, $L^{L \& L}$ $L_h^{\%} (BP_d)_{bk} (BG_d)_{bk}$ = aggregate bank credit, and $L + (BP_d)_{bk} = DB$ = aggregate bank credit to businesses it follows that the aggregate stock of money $M_s$ can equivalently be written as

\[
M_s^{L \& L} H^{L \& L} [L_h^{\%} (BP_d)_{bk} (BG_d)_{bk}]^{\%} D_{G}^{L \& L} M_s^{L \& L} D_B^{L \& L}
\]

That is, the planned money stock is determined by the planned borrowing of the business and household sectors and the stock of money that the government plans to print.

From Appendix 1, the expression for aggregate excess demand $E$ is derived:

18. $E = (I^c - S^p) + (G^p - T^e) = (I_c^{p} + I_f^{p} + I_v^{p} - S^p) + (G^p - T^e)$

where $I_c^{p}, I_f^{p}$, and $I_v^{p}$ are investments in circulating capital, fixed capital, and finished goods inventories respectively.

Appendix 1 also shows how the ex ante SAM can be used to derive an aggregate budget restraint for the economy and thereby relate excess demand to its sources of finance. We repeat here just the principal results. Assuming zero expectational errors (at least in the short run)

19. $E = [LH_p - (LCu_d^{p} + LL_d^{p})] + (LB_B^{e} - LBR_d^{e}) + (LD^{e} - LD_d^{p}) + (LL_d^{p} - LL^{e}) = LH_p + LL_d^{p} - (LCu_d^{p} + LD_d^{p}) - LR_d^{p} + (LD^{e} - LL^{e}) + (LB_B^{e} - LBR_d^{e})$

But since $L D^{e} \& L L^{e} \& L R_d^{e} % L B R^{e} % (L B G_d^{p})_{bk} % (L B P_d^{p})_{bk}$

20. $E = (LH_p - LBR_d^{e}) + LL_d^{p} + (LB G_d^{p})_{bk} + (LBP_d^{p})_{bk} - (LCu_d^{p} + LD_d^{p})$
Thus,

\[ E = (I_d - S^p) + (G_d - T^c) = (LH^p - LBR_d) + [LL_d^p + (LBG_d^p)_{bk} + (LBP_d^p)_{bk}] - (LCu_d^p + LD_d^p) = LM_d^p - LM_d^p \]

This equation shows that the private sector and the government budget deficits are financed by the injection of bank credit, the injection of bank excess reserves, through the “printing” of money (i.e. the sale of bonds to the Fed) and via the running down of the money reserves of the non-bank private sector. This last source of finance was discussed by Earley, Parsons and Thompson (1976) and is also a line of argument that can also be found in the classical and post-Keynesian (Sawyer, 1985) traditions.

Now, following the endogenous money approach, banks obtain the credit that they demand (Palley, 1996). Further, let us assume that the government actually prints the total quantity of money that it desires to add to the economy’s money stock in order to finance the deficit. Then if \( \Delta \) represents the actual change in a stock it follows that \( LM_d = LD_b + LM_G = (M_t^p - M_{s-1}) = ) M_t^p. \) Then

\[ E = LM_t^p - LM_d^p = ) M_t^p - LM_d^p = (M_t^p - M_{s-1}) - (M_d^p - M_{s-1}) = M_t^p - M_d^p \]

Thus, suppressing the superscripts,

\[ E = M_t - M_d \]

Or in terms of levels of output

\[ e_m = M_t \& M_d \]

Equation 22 in fact represents the dual disequilibria relationship that is at the core of the CGC model’s fast adjustment process. It says basically that if the actual amount of the money injected into the economy exceeds the amount of money that firms desire to hold then excess money supply will fuel spending in the market for goods and services.

The dual disequilibria relationship arises as a residual from the flow matrix because all columns sum to zero and all intersectoral row transactions are explicitly linked to each other, i.e. it arises because there are no “black holes” in the SAM. If, with the exception of aggregate demand and supply, all these transactions sum to zero then a non-zero excess demand in the market for goods and services will necessarily imply an imbalance between money supply and demand; only when \( E = 0 \) as an average over the course of a business cycle will the condition \( M_t = M_d \) be satisfied. On the other hand, the SAM in Godley (1999) is written in an \textit{ex post}
mode in which aggregate demand and supply are equal; thus the condition $M_s - M_d$ is satisfied continuously.

We will now turn to a formalization of the CGC model. Given the complexity of the system of nonlinear differential equations, we will make the simplifying assumption that firms are the only private sector entities that borrow from banks. This is not to imply that household debt is unimportant. However, following the classical tradition, this assumption makes firms the driving force of accumulation and therefore places them at the core of the CGC model.\(^{16}\)

We turn to the financing needs of businesses. From the aggregate budget restraint $E = M_s - M_d$, so that

\[ E' = \Delta M_s - \Delta M_d \]

From equations 15 and 18,

\[ (I - S) + (G - T) = LD_B + LM_G - LM_d \]

Therefore

\[ LD_B = (I - S) + (G - T) - (LM_G - LM_d) \]

But given equation 15

\[ LD_B' = (I[S] & (LM_G \% LM_d))' \]

Therefore,

\[ LD_B' = I[S & (LM_d \% LBG)] \]

The economic meaning of this equation is as follows. Aggregate total savings $S$ is the sum of business retained earnings and household savings. A portion of aggregate savings is set aside to add to money reserves ($LM_d$) and another portion ($LBG$) to purchase government bonds. It is

\(^{16}\) Neoclassical economists would disagree with this since households constitute the point of departure of neoclassical macromodels (McCafferty, 1990).
the remaining part of S which is available to finance investment. The excess of planned investment over the actually available savings determines the gap that must be filled by businesses through net new borrowing $LD^*_B$.\footnote{Note that negative borrowing is essentially the reduction of existing bank debt.}

Abstracting from household credit the aggregate stock of money $M$, as discussed above is

\begin{equation}
M_s = H \frac{dL}{dL} M_G \frac{dD}{dD} B
\end{equation}

In terms of ratios to output

\begin{equation}
m_s = m_G \frac{dD}{dD} B
\end{equation}

As discussed earlier, the demand for money is a demand for buffer stocks (Coghlan, 1981; Laidler, 1991). This stock of money reserves is stored in the form of cash and non-interest- and interest-bearing deposits (e.g. $M_2$ which would be inclusive of very liquid interest-bearing demand deposits).

\begin{equation}
m_d = m_d(i) \quad \frac{M_{d_d}}{M} < 0
\end{equation}

where $i$ is some average bond rate of interest. Now a rise in this interest rate leads to a compositional change in the money stock as the private sector increases the proportion of interest-bearing deposits relative to cash and non-interest-bearing deposits. But there will also be an increase in the demand for bonds and a fall in the demand for money, as asset demand functions of the Tobin type show (Godley, 1999).

As in Keynes (Chick, 1983, p. 214), the interest rate is the variable which equalizes the available money supply with the desire to hold money in liquid form:

\begin{equation}
i = \frac{\epsilon_i(m_s \epsilon m_d)}{j > 0}
\end{equation}

Together, equations 23 and 30 imply that whenever the supply of money exceeds the stock demand for money determined by liquidity preference, agents will spend the excess money
in both the market for goods and services as well as financial markets. The former expenditure will lead to a positive excess demand whereas the latter one will increase the demand for bonds which will raise bond prices and therefore lower the interest rate.

Since an excess money supply implies that e rises and i falls, equations 23, 29, and 30 jointly imply that

$$m_{d0}^{-1} \left( \xi \left( \frac{m}{m_0} \right)^{e} m_0 \right) \left( \frac{m}{m_0} \right)^i \frac{m_0}{\xi \frac{m}{m_0}} \left( \frac{m}{m_0} \right) \left( \frac{m}{m_0} \right)^e \left( \frac{m}{m_0} \right)^i > 0$$

The reaction coefficient $m_{d0}$ is a measure of the responsiveness of money demand relative to the demand for bonds when the interest rate changes: the lower (higher) is this coefficient the less (more) responsive will the change in money demand (and thus the change in the demand for bonds) be to a given change in the rate of interest.

Internal finance available to the firm at time $t$ is $X_t$ and is given by

$$X_t = \text{(realized profit at time } t) - \text{(debt service at time } t+1) = (P+E)_t - F_{t+1}$$
$$= (P+E)_t - (1+i)D_{B(t+1)}$$

where $P$ is the mass of profits. Accumulation of circulating capital investment is assumed to be proportional to the excess of firms’ internally available finance at time $t$ over potential profits at the beginning of time period $t$ (Shaikh, 1990):  

$$33. \quad \left( \frac{I}{Y} \right)_{e(1\%)} \left( \frac{I}{Y} \right)_{i(1\%)} \left( \frac{G}{Y} \right)_{e(1\%)} \left( \frac{G}{Y} \right)_{i(1\%)} \frac{(P+E)_t - (1+i)D_{B(t+1)}}{Y_t} \left( \frac{E_t - (1+i)D_{B(t+1)}}{Y_t} \right)$$

where $h > 0$. Therefore, in continuous terms

$$34. \quad a_{c}^{+1} h [e \xi (1\%) d_B \xi (1\%) \frac{D_{B}}{Y}]$$

This equation shows that the rate of change of $a_c$ is positively related to $e$ and negatively related to $d_B$ and the interest rate $i$. Models in the Keynes/Kalecki tradition do not have a growth function such as equation 34.

---

18 Note that $(1+i)D_{B(t+1)} = (1+i) D_{B(t+1)} + (1+i)D_B$, where the first term on the right hand side represents the interest payments on net new debt incurred in the time period between $t$ and $t+1$ and the second term is the interest payment on the stock of debt incurred at time $t$.

19 The prime represents the time derivative.
The significance of this accumulation reaction function is that it roots the “animal spirits” of firms to their net cash flow and balance sheet liquidity. Thus if any given rate of accumulation produces a level of internal finance above potential output, firms will increase their accumulation rate for the next period. Conversely, if internal finance falls below potential profits the accumulation rate will fall. In other words, the “animal spirits” of firms are related to their financial strength.

This mechanism linking business investment to expectations and financial liquidity is similar to the model of Flaschel, Franke, and Semmler (1997) as well as the business cycle model of Wolfson (1994) who draws on Minsky, Wojnilower, and others. Wolfson refers to a number of business variables such as the interest coverage ratio, the debt/asset ratio, and the liquidity ratio all of which can be used to gauge the financial strength of firms.

Equation 34 has one reaction coefficient and therefore puts the stimulus to accumulate (excess demand) on the same footing, so to speak, as the factor which slows it down (debt). For the purpose of investigating the opposing effects of excess demand and financial charges the following more general form of the accumulation reaction function is used:

\[ a_c \cdot h_1 e^{h_2 [(1 \text{ %})d_B \cdot (1 \text{ %})D_B]} \]

The expansion in output is related to investment in circulating capital via the input-output coefficient \( \mu \). Thus the growth rate of output is related to the share of circulating capital \( a_c \) by

\[ \frac{Y}{Y} \cdot \mu a_c \]

The equation system 34 or 35 and 36 are the key ones describing the short-run dynamics of the model and constitute the *classical* features of the model since they relate the expansion of output to investment in circulating capital. The feedback links between excess demand, debt, and investment in circulating capital which they describe are absent in the fiscal policy literature (Blinder and Solow, 1973; Tobin and Buiter, 1976; Nguyen and Turnovsky, 1983; Tobin, 1980; Tobin and Buiter, 1980; Godley, 1999; Taylor, 1985, 1991, 1997).
Given the government budget restraint, equation 15, and the business budget restraint, equation 27, we get

37. \( \text{LD}_B \cdot E \text{&} (\text{LM}_G \text{&} \text{LM}_d) \)

where \( \text{MG} \) = money creation from budget deficit. But \( \text{LD}_B = \text{DB}_t - (1+i)\text{DB}_{t-1} \) so that

38. \( \text{DB}_t \cdot (1\%)\text{DB}_{t(d)} \%E \text{&} (\text{LM}_G \text{&} \text{LM}_d) \)

In continuous terms

39. \( \text{DB} \cdot \text{DB} \cdot (1\%)\text{DB} \cdot \%E \text{&} (\text{LM}_G \text{&} \text{LM}_d) \)

Dividing through by \( Y \)

40. \( \frac{\text{DB}}{Y} \cdot \%\text{DB} \cdot (1\%)\text{DB} \cdot \%E \text{&} \left(\frac{\text{MG}}{Y}\text{&} \text{MD}_{d}\right) \)

Making substitutions of the form \( \text{DB} \cdot \%E \text{&} \left(\frac{Y}{Y}\right) \) and simplifying we get

41. \( \text{DB} \cdot \text{DB} \cdot \%\text{DB} \cdot \%E \text{&} \left(\text{MG} \text{&} \text{MD}_{d}\right) \frac{Y}{Y} \)

Written as shares of output equation 18 becomes

42. \( e = (a_c + a_r + a_v - s) + (g - t) \)

We will assume that the desired inventory stock/sales ratio and thus the inventory/output ratio, \( v = \text{INV}/Y \), is a constant\(^{20} \). Given a constant input-output coefficient \( \mu \) it follows that \( \text{LINV} = vLY = v\mu I_c = I_v \) (Shaikh, 1991). Thus

43. \( a_v \cdot \mu v a_c \cdot Db \)

where \( D = v \mu \).

Therefore using equations 36, 42, and 43 we get

\(^{20} \) Since sales are on average equal to output over the course of the cycle.
In the fast adjustment process a is taken as a constant.

\[
\frac{\mu a_c}{Y} \mu \frac{(e \Delta a_i \% \& \Delta(g \& d))}{(1+\mu v)} \mu \frac{(e \% d)}{(1+\mu v)}
\]

where \(d = s - a - (g - t)\) and \(s - (g - t) = s^*\) is the social savings rate (Shaikh, 1992). Let \(P = \frac{\mu}{1+\mu v}\) so that

\[
\frac{\mu a_c}{Y} P(e \% d)
\]

If in the short run \(d\) is a parameter then

\[
e^{j'} a_c
\]

Using equations 35, 44, 45 and the substitution,

\[
\frac{D_B^j}{Y} \frac{\mu a_c}{Y} \frac{\mu w}{(1+\mu v)}
\]

we get the following equation

\[
e^{j'} h_1 e^{\Delta h_2} \Delta b \Delta h_2 \Delta b \Delta h_2 \Delta b \Delta P \Delta h_2 \Delta b \Delta P \Delta d \Delta h_2 \Delta (1+\mu v)\Delta d
\]

Equations 30 and 31 jointly imply that

\[
i^{j'} e^{\Delta m_0} \Delta k m_0
\]

so that

\[
i^{j'} \Delta k m_0
\]

Substituting equation 49 into equation 47 we get the following equation for excess demand

\[
e^{j'} h_1 e^{\Delta h_2} \Delta b \Delta h_2 \Delta b \Delta h_2 \Delta b \Delta P \Delta h_2 \Delta b \Delta P \Delta e^{\Delta k m_0} \Delta k m_0 \Delta h_2 \Delta b \Delta P
\]

Using equations 41, 44, and 49 we get following equation for business debt:

\[
d_B^j \Delta e^{\Delta m_0} \Delta h_2 \Delta h_2 \Delta b \Delta h_2 \Delta b \Delta h_2 \Delta b \Delta P \Delta e^{\Delta k m_0} \Delta k m_0 \Delta h_2 \Delta b \Delta P \Delta d \Delta h_2 \Delta b \Delta P
\]

Using the continuous form of equation 16 we get

\[
M^i_G \frac{Y}{m_g} \frac{Y}{m_g} \Delta m_g \Delta (G \& T) \Delta \Delta (g \& t)
\]

Using equation 44, the equation for government money injection becomes

\[
m_g^i \Delta (g \& t) \Delta m_g \Delta P(e \% d)
\]

\[21\] In the fast adjustment process \(a_i\) is taken as a constant.
With regard to $\$, we will adopt the following policy function that

$$54. \ $'(\i) \ \frac{d\$}{di} > 0$$

That is, the government lowers the degree of bond financing and increases money financing when the interest rate rises. From equation 30

$$55. \ i' (m_g \& m_d) \& \$e$$

so that a rise in $e$ lowers $i$ and conversely. Thus $\$ can be related to $e$ via the following equation

$$56. \ \$' (\i) \& \$e \ \to 0$$

Equations 31 and 56 jointly imply that

$$57. \ $' (m_g \& m_d) \& \$e \ \to 0$$

Substituting equation 57 into 53 we get

$$58. \ m_g' \ & \ m_e (g \& t) \ & \ m_g (e \%i)$$

To summarize, the cyclical dynamics (fast adjustment process) of the model are determined by the following 4 x 4 nonlinear differential equation system:

$$50. \ e' \ & \ i \ & \ m_g \ & \ m_d \ \to 0$$

$$51. \ d' \ & \ m_g \ & \ m_d \ \to 0$$

$$58. \ m_g' \ & \ m_e (g \& t) \ & \ m_g (e \%i)$$

$$31. \ m_d' \ & \ m_e \$$

The steady state of this system corresponds to $e = 0$, $d = 0$, $m_g = 0$, and $m_d = 0$; that is aggregate demand equals aggregate supply and accumulation is internally financed. Appendix 2 investigates the stability properties of this system.

The slow adjustment process of the model is based on the degree of capacity utilization and investment in fixed capital $a_f$:

$$59. \ \frac{a_f'}{a_f} \ & \ 6(u \& 1) \ \to 0$$

In other words, investment in fixed capital increases when the capacity utilization rate is above the normal rate. Equations 50, 51, 58, and 31 capture the interaction process between the
We finally turn to some key simulations that highlight the role of the excess demand variable as well as some of the classical features of the model. Figure 1 shows that a rise in the budget deficit raises excess demand and business debt. Note that the cyclical comovements of excess demand and debt are persistent macroeconomic features (Shaikh, 1989) since they reflect the investment decisions of firms operating in an uncertain environment. Figure 1 shows the additional stimulus that is given to the system when the budget deficit rises.
Figure 2 shows the warranted path of growth with fluctuation of capacity utilization around the normal level. Note that this growth path corresponds to a given level of the budget deficit. In other words, growth of output is not predicated on the growth of exogenous demand $g = G/Y$ but is rather internally generated by profitability.

Finally, Figure 2 underscores the crucial role of the rate of profit. Curve A corresponds to a higher rate of profit than Curve B. Since
the rate of profit is ultimately determined by income distribution and technology in the classical tradition, any factor which influences these will also alter the growth rate of output and employment. Following the classical tradition the rate of profit in the CGC model is a key variable that drives investment spending. Thus, in contrast to the Keynes/Kalecki tradition, the path of accumulation of the system is fundamentally regulated by the rate of profit. From a policy standpoint, increases in productivity growth relative to wage growth, a rise in the turnover time of capital, and other measures to increase long-run profitability will have the effect of raising the warranted growth rate.

CONCLUSION

The classical theory of effective demand that underpins the CGC model is the framework developed by Shaikh (1989). Shaikh’s classical model is distinct from the two major traditions in economics, which assume either that aggregate supply generates its own demand (Say’s Law and neoclassical models) or that the system is demand-constrained (models in the Keynes/Kalecki and stagnationist traditions). In fact this model follows the classical tradition in which both aggregate demand and supply are regulated by more fundamental factors, notably the rate of profit (Kenway, 1980; Foley, 1983). The turbulent and dynamical system ensures that aggregate demand and supply fluctuate around an endogenously generated growth path (Bleaney, 1976; Shaikh, 1978; Garegnani, 1979).

Models in the stagnationist and post-Keynesian traditions (Taylor, 1985, 1991; Lavoie, 1995; Palley, 1996) are essentially static in the classical and Harrodian sense (Kregel, 1980) since they begin with a given short-run level of output which is established from the savings-investment balance, \( I + G = S + T \). Thus, implicitly in these models investment in circulating capital is zero and growth becomes a long-period phenomenon caused by exogenous factors. These factors include the degree of monopoly and technological change (Kalecki, 1965; Taylor, 1985, 1991), the structure of labor markets, demand-determining policies or expectations (Palley, 1996). Authors in other theoretical traditions also rely on exogenous factors to explain growth. For example, the model of Foley (1985) depends on the exogenous growth of money while Goodwin (1986) bases his growth and cycles model on population growth and technical
The fundamental equation of finance $e = (a_c + a_v + a_f - s) + (g - t) = m_s - m_d$ is the key
dynamical relationship that describes the CGC model. It links the two sources of demand in a
closed economy to their finance requirements. Excess demand is fueled by the excess of
monetary injection $m_s$ over monetary leakage $m_d$ and is responsible for both the cyclical
dynamics and the growth path of the system. In other words, the condition $e > 0$ does not
correspond to a level of but rather to a path of output.

The fundamental equation of finance and the interest rate mechanism jointly imply that
an excess money supply fuels spending in both the commodity and bond markets; it raises the
demand for goods and services as well as that for bonds so that the interest rate falls. In this
vital respect the model is a synthesis of the Keynesian and monetarist approaches. In the
former, an excess money supply spills over into an increased demand for bonds only whereas for
the latter it exclusively fuels a higher demand for goods and services (Chick, 1985).

On the other hand, it would be fair to say that the debt dynamics that are central to the
fast adjustment process in the CGC model constitute a feature that it shares with a variety of
models in the Keynes/Kalecki tradition, some of which focus on the impact of debt on
consumption (Palley, 1996, 1997) and others on business investment (Franke and Semmler,
1989).

However, an important distinction between the CGC model and the heterodox literature
is with regard to the role of circulating capital investment in the former. This is the feature that
makes it a dynamic model. Aggregate excess demand leads to an expansion of output which
takes place via the accumulation reaction function 34 (or 35) and the classical input-output
relation 36. This is a process that is fueled by bank credit which attenuates the expansion as
business debt rises.

Thus in this model the proportion of profit that is devoted to the expansion of output
responds positively to excess demand and negatively to debt. This dynamic produces business
cycles around a growth path. In the medium run, the proportion of output that is used to
increase capacity (via fixed capital investment) rises if capacity utilization is above normal and
falls if capacity utilization is below normal capacity. In other words, the model generates two
cycles, one of which represents imbalances between aggregate demand and supply and the other
represents discrepancies between actual and normal capacity utilization.

The model’s consistent stock-flow accounting with “no black holes” (Godley, 1999) along with its internally-generated growth process locates it in the broad tradition of Quesnay’s *Tableau Economique*, Marx’s reproduction schema, the von Neumann growth model, and Harrod’s warranted growth path analysis. The endogenous business cycles and debt dynamics are in the tradition of Kalecki and Minsky.

While the rate of profit remains the ultimate regulator of the path of accumulation, the question is what impacts do fiscal policy and foreign trade have on both growth and cycles? In the above simulations, the impact of fiscal policy on the warranted path has been deliberately side-stepped as it depends on the composition of government spending. These issues along with policy implications are formally investigated in Moudud (1999a, 1999b).

**APPENDIX 1**

**Ex Ante Social Accounting Matrix in a Closed Economy with the Government Sector**

1. Note that the row sum of the last column (ex ante gaps) will not be zero unless the row sum of every other column is zero. For households, firms, and government the row sum = 0 implies total uses - total income = total external funds (bank loans for HH and firms and bonds plus high powered money for government). But for banks, this does not hold directly. The separately derived aggregate constraint $M = L'' + H''$ must be added in here.

\[ A1.1 \]

\[ \frac{LM_s}{LCu} + LD = LL + (LH - LBR) + LBG_{bk} + LBP_{bk} \]

\[ = LL + LH'' + LBG_{bk} + LBP_{bk} \]

The change in money supply $LM_s' \% LL''$, where $LH'' \% LBR, BR = borrowed reserves (discount window loans)$, and $LH' \% LCu \% L_{R^{22}}$. The change in aggregate loans extended by banks is $LL'' \% LBG_{bk} \% LBP_{bk}$ in which the first term on the right is new bank credit to the private sector (which directly creates new money as in the endogenous money literature) and the second and third terms are respectively the purchase of government and

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22 Brunner and Meltzer (1990) derive a similar relationship by assuming that bank net worth equals zero.
private bonds out of bank excess reserves. For simplicity, it will be assumed that only all government bonds purchased by banks are done so through excess reserves. Note that \( M_s \) comprises demand and time deposits, i.e. it is “broad money”. Thus

\[
A1.2 \quad LD' \ LL \%LBG_{bk} \%LP_{bk} \%LR \& LBR
\]

or

\[
A1.3 \quad LD' \ [LL \%LBG_{d} \%LP_{d} \%LR_{d} \& LBR]
\]

This is a crucial result that describes the behavior of banks in their role of \textit{banks-as-banks} as discussed below. The term \( LBR_e \) is the expected increase in bank reserves through discount window borrowing from the central bank, i.e. it is expected sale of this particular asset by banks to the central bank.

In operational terms one can analytically distinguish \textit{banks-as-banks} from \textit{banks-as-businesses}. The former role has to do with the those kinds of activities that specifically distinguish banks from other sectors. These activities pertain to the supply of deposits and loans, borrowing from the discount window and maintaining an adequate level of reserves. From the endogenous money framework, the provision of loans is not related in some mechanical fashion to reserves (as in the exogenous money approach) but is determined by banks’ desired balance sheet liquidity. One would therefore not expect the variables pertaining to \textit{banks-as-banks} to be part of a budget restraint since they are in the final instance determined by other factors (desired balance sheet liquidity, the pace of accumulation etc.). However these variables are related to one another via the equation (A1.3) which is derived by taking into account not only the \textit{forms} in which money is kept (currency plus a broad range of deposits) but also the \textit{sources} of money.

All the other variables in the banking sector column describe the normal activities of \textit{banks-as-businesses} which means that the sum total of these sources and uses of funds has to equal zero, as with non-banking firms. One would thereby obtain the result that the sum total of the entire banking sector column is equal to zero.
2. This section deals with the aggregate *ex ante* budget restraint. We first begin by deriving an expression for aggregate savings. For firms, if $Y^s$ = output gross of depreciation, $DIV^p$ = planned dividend payment and $RE^p$ = planned retained earnings then

**Firms**

\[ A1.4 \quad Y^s \& w^p (N_{t+1}) \& T^p_f \& INT^p_f \quad P^e \quad DIV^p_f \% RE^p_f \]

**Banks**

\[ A1.5 \quad INT^e \& w^p (N_{t+1}) \& T^p_{bk} \quad P^e_{bk} \quad DIV^p_{bk} \% RE^p_{bk} \]

**Households**

\[ A1.6 \quad DIV^e \% w^p N_{t+1} \& C^e \& T^p_h \& INT^p_h \quad S^p_h \]

Adding these three equations

\[ A1.7 \quad [Y^s \& w^p (N_{t+1}) \& T^p_f \& INT^p_f] \% [INT^e \& w^p (N_{t+1}) \& T^p_{bk}] \% [DIV^e \% w^p (N_{t+1}) \& C^e \& T^p_h \& INT^p_h] \quad (DIV^p \& DIV^e) \% S^p \]

Let $T^p = T^p_h + T^p_f + T^p_{bk}$, $S = S^p_h + RE^p_f + RE^p_{bk} = S^p_h + RE^p$. Rewriting equation A1.7,

\[ A1.8 \quad Y^s \% w^p N_{t+1} \& C^e \& T^p_h \& INT^p_h \quad (DIV^p \& DIV^e) \% S^p \]

Let

\[ A1.9 \quad E' \quad Y^s \& Y^e \quad (C^e \& d^e \% G^d) \& (C^e \% d^e \% G^c) \]

So that

\[ A1.10 \quad (C^e \% d^e \% G^c) \& (C^e \% d^e \% G^d)' \& E \]

Adding $(T^p - T^p) + (w^p N_{t+1} - w^p N_{t+1})$ to A1.10

\[ A1.11 \quad (C^e \% d^e \% G^c) \& (C^e \% d^e \% G^d)' \& (T^e \& T^p) \% (w^p N_{t+1} \& w^p N_{t+1}) \]

Therefore

\[ A1.12 \quad Y^s \& C^e \& (I^e \% G^d) \& (T^e \& T^p) \% (w^p N_{t+1} \& w^p N_{t+1}) \]

\[ A1.13 \quad Y^s \& C^e \& T^p \% (w^p N_{t+1} \& w^p N_{t+1}) \]
\[ & E & T c \% (T c & T p) \% d w \% N_{t, d} \% w \% N_{t, d} \% d \% G_{d} \% c \& \% \% p \] 

Substituting equation A1.13 into equation A1.8

A1.14 \[ & E & T c \% (T c & T p) \% d w \% N_{t, d} \% w \% N_{t, d} \% d \% G_{d} \% c \& \% \% p \] 

A1.15 \[ E + (T^c - T^p) + (w^cN - w^pN) = (DIV^p - DIV^c) - [(L_d - S^p) + (G_d - T^c)] \]

A1.16 \[ E + (T^p - T^c) + (w^cN - w^pN) + (DIV^p - DIV^c) = (I_d - S^p) + (G_d - T^c) \]

We will assume that the wage rate is set in advance before production is initiated so that \( w^c = w^p = w \). Further, if interest and dividend rates are known in advance then \( INT^c = INT_t^p + INT_h^p \) and \( DIV^p = DIV^c \) then

A1.17 \[ E = (I_d - S^p) + (G_d - T^c) \]

From the SAM

A1.18 \[ \{ E + (T^c - T^p) + (w^cN - w^pN) + [\% \% (EQ_{t, c}) - \% \% (EQ_{t, p})] + [i^c (BG_{t, c}) - i^p (BG_{t, p})] + [i^c (D_{t, c}) - i^p (D_{t, p})] + [i^c (L_{t, c}) - i^p (L_{t, p})] + [i^w (BL_{t, c})] \]

- \( i^w (BL_{t, c}) + [L^p (LC_{u, d} + LR_d)] + (LB R^e - LB R_d) + (L D^e - LD_d^p) + (LEQ_p) \)

- \( LEQ_p^c \) + \( (LB G^c - LB G_d^p) + (LB P^e - LB P_d^p) + (L L_d^p - L L^c) = 0 \)

If dividend and interest rates are known in advance (e.g. \( *^c = *^p = * \) and \( i_b^e = i_b^p = i_b \), \( LB P^e - LB P_d^p = 0 \) and \( LEQ_p^c = LEQ_p^p \) = 0 the above equation reduces to

A1.19 \[ E = [L H^p - (LC_{u, d} + LR_d)] + (LB R^e - LB R_d) + (L D^e - LD_d^p) + (L L_d^p - LL^c) = L H^p + L L_d^p - (LC_{u, d} + LD_d^p) - LR_d + (L D^e - LL^c) + (LB R^e - LB R_d) \]

But since \( LD^c \& \% LL \& \% R_d \% \& BL \& \% (LB G_d^p) \) then

A1.20 \[ E = (L H^p - LB R_d) + LL_d^p + (LB G_d^p) - (LC_{u, d} + LD_d^p) \]

A1.21 \[ \Rightarrow E = (L H^p - LB R_d) + LL_d^p + (LB G_d^p) - (LC_{u, d} + LD_d^p) = LM_d - LM_d^p \]

As explained in the text, if the actual increase in the money stock = the desired increase in the money stock so that \( \) \( M_s = LM_s \) then

A1.22 \[ E = LM_s^p - LM_d^p = ) M_s^p - LM_d^p = (M_s^p - M_{s+1}) - (M_d^p - M_{d+1}) = M_s^p - M_d^p \]

Note that the term \( LH^p \& \% BR_d^c \& \% LH^p \) is the net injection of high-powered money into the system originating from the government budget restraint (fiscal policy) and from open market operations of the central bank (monetary policy). It expand the money supply directly. The part that originates in the government budget restraint is also the exogenous component of the
money supply and is called “government money creation” (Burdekin and Langdana, 1992, p.3). The total change in high-powered money is \( LH^{p1} LH^{p2} \% LBR_d \).

Using equations A1.17 and A1.21 we get the following relationship, which relates the private and public deficits to the excess of money injection relative to money leakage:

\[
A1.23 \quad (I_d - S^p) + (G_d - T^p) = (LH^p - LBR_d) + [LL_d^p + (LBG_d^p)_{bk}] - (LC_d^p + LD_d^p)
\]

Simplifying, ignoring all superscripts and subscripts, and remembering equation A1.22

\[
A1.24 \quad (I - S) + (G - T) = M_s - M_d
\]

GLOSSARY OF TERMS IN THE SOCIAL ACCOUNTING MATRIX

C: private consumption, \( I_{fm} \): fixed and materials investment, \( I_v \): inventory investment, S: savings, G: government expenditure, T: tax revenue, N: quantity of labor, EQ: equity, Y: output, \( i_d \), \( i_b \), \( i_{dw} \) and \( i_L \): deposit, bond, discount window and bank loan rates of interest respectively, \(* \): dividend rate, w: wage rate, D: deposits, BG: government bonds, BP: private bonds, L: bank loans, BR: borrowed reserves (discount window loans), H: high-powered money, Cu: currency, R: bank reserves, RE = retained earnings

Variables with the subscripts h, f, bk, and g refer to those pertaining to households, firms, banks, and the government respectively.

**Endogenous Variables (Fast Adjustment Process):**

e: excess demand; \( d_h \): business debt; \( a_c \): circulating investment; \( a_i \): inventory investment; Y: output; i: interest rate; \( m_d \): money demand; u: capacity utilization

**Exogenous Variables (Fast Adjustment Process):**

s: savings rate; g - t: budget deficit; \( a_f \): fixed investment; \( \mu \): input-output coefficient; v: inventory/sales ratio; \( m_g \): money created from the deficit

**Endogenous Variables (Slow Adjustment Process):**

\( a_f \): fixed investment
Exogenous Variables (Slow Adjustment Process):

With the exception of \(a_c\), all the other variables that are exogenous to the fast adjustment process are exogenous for the slow adjustment process also. Finally, since actual output fluctuates around potential output capacity utilization fluctuates around the normal level and therefore is exogenously determined.

Parameters:

These are listed in Appendix 2 with their assigned values.

APPENDIX 2

Stability of the Fast Adjustment Process

The steady state values of the system of equations 51, 52, 59, and 32 are \(e = 0\), \(d = 0\), \(m_G = 0\), and \(m_d = 0\). Evaluated at these values, the Jacobian of the system is

\[
A2.1
\]

\[
J = 
\begin{bmatrix}
  h_1 & -h_2 - \chi d & 0 & 0 \\
  1+ \chi d & -\chi d & -\chi d & \chi d \\
  0 & 0 & -\chi d & -\gamma \\
  m_0 & 0 & 0 & 0 
\end{bmatrix}
\]

The determinant of this system is given by

\[
\text{Det } J = (-m_0)(-h_1)(h_2 + h_3d^2)(Pd)(J + Pd^2) = (m_0)(h_1 + h_2d^2)(Pd)(J + Pd)
\]

The terms in the first and third parentheses are unambiguously positive\(^{23}\). Now a necessary condition for stability is that the determinant of this system should have the sign \((-1)^n\); that is, it

---

\(^{23}\) The variable \(d = s - (g - t) - a_t > 0\). This condition follows from \(e = a_c + a_f + a_v - s + (g - t)\). If \(e = 0\) over the course of a short-run cycle then \(s - (g - t) - a_t = d = a_c + a_v\). Thus since \(a_c + a_v > 0\) it follows that \(d > 0\).
has to be positive since \( n = 4 \). This stability condition ultimately hinges on the signs of the terms in the second and fourth parentheses. If

1. \( J( + Pd) > 0 \) then \( (h_2 + h_2dP) = h_2(1 + dP) > 0 \) so that \( h_2 > 0 \)
2. \( J( + Pd) < 0 \) then \( h_2(1 + dP) < 0 \) so that \( h_2 < 0 \)

Condition (2) is clearly meaningless economically since the accumulation of circulating investment is assumed to be proportional to the excess of firms’ internally available finance at time \( t \) over potential profits at the beginning of time period \( t \) (see equations 32 and 33). Thus the necessary condition for stability implies that only condition (1) can be true, i.e. \( h_2 > 0 \). 

In order to study the system the following parameter values were used: \( h_1 = 0.0075, \mu = 0.55, \nu = 0.25, P = \mu/(1 + \mu \nu) = 0.52, m_0 = 0.04, s = 0.12, a = 0.08, d = s - a - (g - t) = 0.04 - (g - t), g - t = 0.01, \$, = 0.2, J = \$/m_0 = 5, j = 0.06, k = j/m_0 = 1.5. \)

The reaction coefficient \( h_2 \) in equation 35 was varied to investigate the stabilizing effect of finance charges. Simulations show that for a given value of \( h_1 \) (as well as given values of the other parameters) if \( h_2 \) is decreased the system explodes once the latter reaction coefficient crosses some value \( h_2^* \). That is, for \( h_2 < h_2^* \) the retarding effect of debt is too weak and the system becomes unstable.

When \( h_2 \) is zero all eigenvalues are real, of which two are negative, one is positive and the remaining one is zero. Thus the system is locally unstable. For \( 0 < h_2 < h_2^* \) all eigenvalues are complex, of which one pair has positive real components and the other one has negative real parts. Beyond the critical value \( h_2 = h_2^* \) (roughly equal to 0.001 for the above parameter values), all four complex eigenvalues have negative real parts and the system becomes stable.

That is, at this critical point corresponding to \( h_2 = h_2^* \), the system undergoes a Hopf bifurcation since the following conditions are satisfied (Gandolfo, 1995, p. 477):

a. \[ [\text{Re}(\Theta)]_{b_2^*} h_2' \]

b. \[ \frac{d[\text{Re}(\Theta)]}{dh_2} b_2^* h_2' \]

The economic intuition to explain this pattern is fairly obvious once it is remembered that a progressive rise in \( h_2 \) relative to \( h_1 \) tends to increase the stability of the model. Note that for higher values of the budget deficit, the value of \( h_2^* \) has to be higher. This is because the higher

---

\(^{24}\) Of course, there is also the possibility that \( J( + Pd) = 0 \). But in this case the necessary stability condition would be violated whatever is the sign of \( h_2(1 + dP) \).
deficit implies a greater value of e so that a stronger negative feedback effect of debt would be required in order to stabilize the system.

Together, the above stability analysis shows that while \( h_2 > 0 \) is a necessary condition for stability, the necessary and sufficient requirement for stability is that \( h_2 \leq h_2^{*} \).

An implication of this analysis is that, given \( h_1 \) and \( h_2 \), an excessive fall or rise in the social saving rate \( s^{*} = s - (g - t) \) could push the system into instability. In the former case, a fall in \( s^{*} \) brings about an expansionary scenario which stimulates e via a greater degree of monetary injection; if this stimulus is too strong relative to the anchoring effect of debt the system becomes unstable. Similarly in the contractionary scenario characterized by a fall in \( s^{*} \) e falls too much and cannot be brought back to the short-run growth path.

Figure 3 depicts the phase diagram for \( h_2 = 0.001 \) and shows the existence of damped cycles.
The above equation system can be reduced to a 3 x 3 system by setting $s_0 = 0$ in equation 56 so as to eliminate $m_g$ from the above system, i.e. by considering a pure bond-financed budget deficit. Nothing fundamental to the model is changed by this procedure since in the case that $s_0 = 0$ in equation 56 only a lower degree of stimulus is provided when the deficit increases. The advantage in reducing the model to a 3 x 3 system is that the modified Routh-Hurwitz conditions can be used to analyze model stability (Flaschel, Franke and Semmler, 1997, p. 139).

The steady state values of the model still remain $e = 0$, $d_B = 0$, and $m_d = 0$. The Jacobian matrix $J$ evaluated at equilibrium is

$$J = \begin{bmatrix} h_1 & -h_2 - \chi d & 0 \\ 1 + \chi d & -\chi d & \chi d \\ m_0 & 0 & 0 \end{bmatrix}$$

**Necessary and sufficient** conditions for all eigenvalues to have negative real parts are

(a) $A_1 = -\text{trace } J > 0$ (b) $A_2 = J_1 + J_2 + J_3 > 0$

(c) $A_3 = -\det J > 0$ (d) $B = A_1 A_2 - A_3 > 0$

where $J_i$ is the i-th principal minor. The Jacobian yields the following results:

$A_1 = \text{Pd} - h_1$, $A_2 = -h_1 \text{Pd} + h_2 + 2 h_2 \text{Pd} + h_2 (\text{Pd})^2$, $A_3 = m_0 h_2 \text{Pd} + m_0 h_2 (\text{Pd})^2$, $B = (\text{Pd} - h_1)(-h_1 \text{Pd} + h_2 + 2 h_2 \text{Pd} + h_2 (\text{Pd})^2) - m_0 h_2 \text{Pd} - m_0 h_2 (\text{Pd})^2$
From condition (c)

\[ A_3 = m_0 h_1 P_d + m_0 h_2 (P_d)^2 > 0 \]

\[ [m_0 h_2 P_d ][1 + P_d] > 0 \]

Since both \( P \) and \( d \) are numbers which are less than one, it follows that \([1 + P_d] > 0\) so that \([m_0 h_2 P_d] > 0\). Since we know that \( m_0 > 0, d = s - (g - t) - a_t > 0 \) (see footnote 22) and \( P > 0 \) (positive input-output coefficient and inventory/sales ratio), it follows that \( h_2 > 0 \) so that a positive reaction coefficient on the debt term in the accumulation function is sufficient to stabilize the system.

Finally, in order to establish the range of \( h_2 \) relative to \( h_1 \) for the system to be stable we consider condition (d):

\[
(P_d - h_1)(- h_1 P_d + h_2 + 2 h_2 P_d + h_2 P^2 d^2) - m_0 h_2 P_d - m_0 h_2 P^2 d^2 > 0
\]

Since stability requires that \( A_1 = P_d - h_1 > 0 \) and given that \( A_1 = m_0 h_2 P_d + m_0 h_2 (P_d)^2 > 0 \), it follows that

\[
(- h_1 P_d + h_2 + 2 h_2 P_d + h_2 P^2 d^2) > 0
\]

Thus,

\[
h_2 > \frac{h_1 P_d}{1 \% P_d \% P^2 d^2}
\]

Or

\[
h_2 > h_1 \frac{P_d}{(1 \% P_d)^2} \quad \text{where} \quad \frac{P_d}{(1 \% P_d)^2} < 1
\]

Note that in the 4 x 4 system, the additional “pumping” effect of \( m_z \) implies that the value of \( h_2 \) has to be higher so as to stabilize the system.

**STABILITY OF THE SLOW ADJUSTMENT PROCESS**

These conditions are the same as in Shaikh (1989) and will not be repeated here.
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