Kaleckian Models of Growth in a Stock-Flow Monetary Framework: A Neo-Kaldorian Model

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This paper integrates a stock-flow monetary accounting framework, as proposed by Godley and Cripps (1983) and Godley (1993, 1996, 1999), with Kaleckian models of growth, as proposed by Rowthorn (1981), Dutt (1990) and Lavoie (1995). Our stock-flow accounting is related to the social accounting matrices (SAM) originally developed by Richard Stone in Cambridge, with double-entry bookkeeping used to organise national income and flow of funds concepts. We present a consistent set of sectoral and national balance sheets where every financial asset has a counterpart liability; and budget constraints for each sector describe how the balance between flows of expenditure, factor income and transfers generate counterpart changes in stocks of assets and liabilities. These accounts are comprehensive in the sense that everything comes from somewhere and everything goes somewhere, or to put it more formally, all stocks and flows can be fitted into matrices in which columns and rows all sum to zero. Without this armature, accounting errors may pass unnoticed and unacceptable implications may be ignored.

The paper demonstrates the usefulness of this framework when deploying a macroeconomic model, however simple. The approach was used by Godley (op. cit.) to describe an economy which tended towards a stationary steady state, with no secular growth. In this paper, the same methodology is used to analyse a growing economy.

A useful starting point for our study is the so-called neo-Pasinetti model proposed by Kaldor (1966). In Kaldor's model, the budget constraint of the firm plays an important role in determining the macroeconomic rate of profit, for a given rate of accumulation. In addition, through his "valuation ratio", which is very similar to what later became known as Tobin's \( q \)-ratio, Kaldor provides a link between the wealth of households and the financial value of the firms on the one hand, and the replacement value of tangible capital assets on the other.

One drawback to Kaldor's 1966 "neo-Pasinetti" model, as Davidson (1968) was quick to point out, is that it does not describe a monetary economy, for Kaldor assumed that households hold their entire wealth in the form of equities and hold no money deposits. This assumption gave rise to the bizarre conclusion that households' propensity to save has no impact on the steady-state macroeconomic profit rate, a conclusion which gave the model its name.\(^{1}\) To take money into account, Davidson proposed the concept of a "marginal propensity to buy placements out of household savings" (1972, p. 272; cf. 1968, p. 263), while Skott (1981) set
out explicit stock-flow norms linking the two components of wealth (money and equities) to the consumption decision. The Skott model in its various incarnations (1988, 1989) is closest to the model used here, since Skott uses explicit budget constraints with money/credit stocks for both firms and households.

Our model extends Kaldor's 1966 model by assuming that firms obtain finance by borrowing from banks as well as by issuing equities. It includes an account of households' portfolio behaviour à la Tobin (1969) where the proportion of wealth held in the form of money balances and equities depends on their relative rates of return. It also includes an investment function, which makes the rate of growth of the economy largely endogenous. The model is Kaleckian because, in contrast both with Cambridge models of growth à la Robinson and Kaldor, and also with classical models of growth (Shaikh 1989, Duménil and Lévy 1999, Moudud 1999), rates of utilization in the long period are not constrained to their normal or standard levels. Our model may be said to be neo-Kaldorian because it includes many features, such as markup pricing, endogenous growth and flexible rates of utilization, as well as endogenous credit money and exogenous interest rates, that Kaldor (1982, 1985) emphasized towards the end of his career.

The first section of the paper presents our social accounting matrices while the second section gives the behavioural equations of the model. The third section describes experiments in which we explore the impact of changes in the propensity to consume, liquidity preference, the rate of interest, the rate at which securities are issued, the retention ratio and the real wage on variables such as the rate of accumulation, the rate of profit, the rate of capacity utilization, Tobin's $q$ ratio, and the debt ratio of firms.

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**The social accounting framework**

We have made many drastic simplifications in the service of transparency. Our postulated economy has neither a foreign sector nor a government, while banks have zero net worth. Firms issue no bonds, only equities, and hold no money balances, implying that whenever firms sell goods, they use any proceeds in excess of outlays to reduce their loans. No loans are made to households and there is no inflation.

The balance sheet matrix of this economy is presented in Table 1 while Table 2 gives the flow matrix which describes transactions between the three sectors of the economy and which distinguishes, in the case of firms and banks, between current and capital transactions. Symbols with plus signs describe sources of funds, while negative signs indicate uses of funds. The financial balance of each sector -- the gap between its income and expenditure reading each column vertically -- is always equal to the total of its transactions in financial assets, so every column represents a budget constraint. The subscripts $s$ and $d$ have been added to relevant variables (denoting, very roughly speaking, "supply" and "demand"), the purpose of which is to emphasise that each variable must make behavioural sense wherever it appears. The inclusion of these subscripts in no way qualifies the obvious fact that each row of the flow matrix must sum to zero; but we shall be at pains to make explicit the means by which this equivalence comes about. The watertight accounting of the model implies that the value of any one variable is logically implied by all the other variables taken together. It also implies that any one of the columns in Table 2 is logically implied by the sum of the other four.

In writing out our system of equations, each endogenous variable will only appear once on the LHS, facilitating the counting of equations and unknowns and making it easier for the reader...
to reconstruct the whole model in his or her mind. When a variable does appear on the LHS for a second time -- therefore in an equation which is logically implied by other equations -- that equation will be numbered with the suffixes A, B, etc..

Take the first column of Table 2. The regular income of households, $Y_{hr}$, is defined as the sum of all the positive terms of that column, wages $W_s$, distributed dividends $F_D$, and interest received on money deposits $r_m M_{d(-1)}$, where $r_m$ is the rate of interest on money deposits, and $M_{d(-1)}$ is the stock of money deposits held at the end of the previous period.

$$Y_{hr} \equiv W_s + F_D + r_m M_{d(-1)}$$

From the first column of Table 1, we know that the wealth, $V$, of households is equal to the sum of money holdings plus the value of equity holdings:

$$(2A) \quad V \equiv M_d + e_d p_e$$

where $e_d$ is the number of equities and $p_e$ is the price of equities. We can rewrite (2A) as:

$$\Delta M_d \equiv V - \Delta [e_d p_e]$$

where $\Delta$ is a first difference operator.

The second term on the RHS of equation (2) can be written:

$$(2) \quad \Delta [e_d p_e] \equiv (e_d p_e) - (e_{d(-1)} p_{e(-1)}) \equiv \Delta e_d p_e + \Delta p_e e_{d(-1)}$$

which says that the change in the value of the stock of equities is equal to the value of transactions in equities ($\Delta e_d p_e$) plus capital gains on equities held at the beginning of the period ($\Delta p_e e_{d(-1)}$). The change in wealth, using column 1 of Table 2 again, can be written as:

$$(3) \quad \Delta V \equiv Y_{hr} - C_d + G$$

where $C_d$ is consumption and $G$, the value of the capital gains that accrue to households in the period, is given by:

$$(4) \quad G \equiv \Delta p_e e_{d(-1)}$$

Rearranging equation (3) allows us to retrieve the Haig-Simons definition of income, $Y_{hs}$, according to which income is the sum of consumption and the increase in wealth.

$$(3A) \quad Y_{hs} \equiv C_d + \Delta V \equiv Y_{hr} + G$$

The current account of the firm sector, shown in column 2 of Table 2, yields the well-known identity between national product and national income.
(1A) $C_s + I_s \equiv W_d + F_T$

where $I_s$ is investment and $F_T$ is total profits. This equation, since it is logically implied by the other four columns of Table 2, was dropped when we came to solve the model.

Total profits $F_T$ are made up of distributed dividends $F_D$, retained earnings $F_U$, and interest payments on bank loans $r \cdot L_{d(-1)}$, where $r$ is the rate of interest on loans $L_{d(-1)}$ outstanding at the end of the previous period:

$$F_U \equiv F_T - F_D - r \cdot L_{d(-1)}$$

The capital account of the firm sector is given in column 3 of Table 2, which shows the financial constraint of firms:

$$\Delta L_d \equiv I_d - F_U - \Delta e \cdot p_e$$

Equation (6) says that investment $I_d$ must be financed by some combination of retained earnings, sale of new equities and additional borrowing from banks. This is the budget constraint of firms that was introduced by Kaldor (1966).

Our banking system is the simplest possible one. There is no government sector, so a fortiori there is no government debt, no high powered money, and no currency. This is a pure Wicksellian credit economy, where all money takes the form of bank deposits. As an added simplification, banks do not make profits, so the rate of interest on money deposits and the rate of interest on loans are identical. With these assumptions, the banks' balance sheet is given by:

$$M_s = L_s$$

while its appropriation account implies:

$$r_m = r_l$$

2. Behavioural relationships

Firms

Firms have four categories of decision to take. They must decide what the markup on costs is going to be (see Coutts, Godley and Nordhaus 1978, and Lavoie 1992, ch. 3). In the present model, it is assumed that prices are set as a mark-up on unit direct costs which consist entirely of wages. We have a simple markup rule:

$$p = (1 + \mu)w / \mu$$

with $p$ the price level, $w$ the nominal wage rate, the markup, and where $\mu$ is output per unit of labour such that:

$$N_d = \frac{S}{\mu}$$
where $N_d$ is the demand for labour and output, $S$, is:

\[(11) \quad S \equiv C_s + I_s\]

We shall assume that the parameters in the above equations are all constant, implying constant unit costs and constant returns to scale. The wage rate is also assumed to be exogenous (and constant), and the markup stays the same regardless of the degree of capacity utilization. These are very strong assumptions made in order to bring a limited range of problems into sharp focus; it will be not be difficult to amend them in a later model. We also define units in such a way that the price level is equal to unity, so that there is no difference between nominal and real values.

Under these assumptions the main purpose of the pricing decision is to determine the share of income between profits and wages. For instance, since the total wage bill is $W_d \equiv w.N_d$, while the total wage income of households is $W_s \equiv w.N_s$, and since there is assumed to be an infinitely elastic supply of labour,

\[(12) \quad N_s = N_d\]

total profits are given by:

\[(13) \quad F_T = \rho/(1 + \rho)S\]

Entrepreneurs must next decide how much to produce. It is assumed that firms adapt supply to demand within each period. This implies that sales are always equal to output, and hence aggregate supply $S$ is exactly equal to aggregate demand, given by the sum of consumption $C_d$ and investment $I_d$. We thus have the first of our two equilibrium conditions, where equilibrium is achieved by a quantity adjustment (an instantaneous one), as is always the case in standard Keynesian or Kaleckian models:

\[(14) \quad S = C_s + I_s = C_d + I_d\]

The third kind of decision made by firms concerns the quantity of capital goods that should be ordered and added to the existing stock of capital $K$ -- their investment. Because we have a growth model, the investment function is defined in growth rates. We shall identify the determinants of the rate of accumulation of capital $g$, such that:

\[I_d = \Delta K = g.K\]

Investment functions are controversial. In Kaldor (1966) there was no investment function, the growth rate being exogenous. In Joan Robinson (1956) there was an investment function, where the rate of capital accumulation depends on the expected profit rate. Some authors believe that it is more appropriate to take the rate of capacity utilization and the normal rate of profit (rather than the realized one) as the determinants of the investment function (Bhaduri and Marglin 1990, Kurz 1990). These models usually assume away debt and money. Obviously, in a monetary model, the interest rate and the leverage ratio should play a role. The possibilities are endless.\[5\]
We have decided to use the investment function recently tested empirically by Ndikumana (1999). His model is inspired by the empirical work of Fazzari and Mott (1986-87), which they present as a Kalecki-Steindl-Keynes-Minsky investment function. In the Ndikumana model, there are four variables which explain the rate of accumulation: the ratio of cash flow to capital, the ratio of interest payments to capital, Tobin's $q$ ratio, and the rate of growth of sales. We shall use the first three of these and replace the fourth by the rate of capacity utilization, which was one of the variables implicitly used by Fazzari and Mott. Before setting out the investment function, we make the following five definitions:

The rate of capacity utilization $u$, which is the ratio of output to full-capacity output $S_{fc}$:

\[ (16) \, u \equiv \frac{S}{S_{fc}} \]

where the capital to full capacity ratio $\sigma$ is defined as a constant:

\[ (17) \, S_{fc} \equiv \frac{K}{\sigma} \]

Tobin's $q$ ratio, which is the financial value of the firm divided by the replacement value of its capital:

\[ (18) \, q \equiv \frac{V}{K} \equiv \left( \frac{L}{s} + e \cdot p \cdot e \right)/K ; \]

The leverage ratio $l$, which is the debt to capital ratio of the firms:

\[ (19) \, l \equiv \frac{L_d}{K} ; \]

The rate of cash flow $r_{cf}$, which is the ratio of retained earnings to capital:

\[ (20) \, r_{cf} \equiv \frac{F_U}{K} \]

The investment function, or more precisely the rate of capital accumulation $g$, is given by equation (21), with $\gamma_0$ comprising exogenous investment ("animal spirits") and all other $\gamma$'s being (positive) parameters. The parameters are all assumed to take effect after one period, on the assumption that investment goods must be ordered and that they take time to be produced and installed, and that entrepreneurs make their orders at the beginning of the period, when they have imperfect knowledge concerning the current period.

\[ (21) \, g = \gamma_0 + \gamma_1 r_{cf} (-1) - \gamma_2 r (-1) - \gamma_3 q (-1) + \gamma_4 u (-1) \]

In this model, as in the model tested by Ndikumana (1999), interest payments have two negative effects; they enter the investment function twice, once directly, but also indirectly by reducing cash flow and therefore the ability to finance investment internally. The direct effect of high interest payment commitments is to reduce the creditworthiness of firms and increase the probability of insolvency which may cause firms to slow down their expansion projects; this is because entrepreneurs will be more prudent, to ensure that they stay in business (Crotty 1996, p. 350); and banks will be more reluctant to provide loans to firms with high debt commitments.

Tobin's $q$-ratio is not usually incorporated into heterodox growth models with financial
variables. For instance, it is not present in the models of Taylor and O'Connell (1985) and Franke and Semmler (1989), although these models do have some mainstream features such as a fixed money supply. The valuation ratio, however, is to be found in the investment functions of Rimmer (1993) and of Gatti, Gallegati and Gardini (1990). The latter refer to their investment function as a Keynes-Davidson-Minsky theory of investment determination, citing Davidson (1972) and Minsky (1975). Thus, it is clear that various Post Keynesians have considered the introduction of the valuation ratio (the \( q \)-ratio) as a determinant of investment, although Kaldor himself did not believe that such a ratio would have much effect on investment.

Introducing the valuation ratio may reduce the rate of accumulation decided by entrepreneurs whenever households show little desire to save or to hold their wealth in the form of equities. As pointed out by Moore (1973, p. 543), such an effect "leads back to the neoclassical conclusions of the control of the rate of accumulation by saver preferences, albeit through a quite different mechanism. A reward to property must be paid ... to induce wealth owners to hold voluntarily, and not to spend on current consumption, the wealth accumulation that results from business investment". We shall see that some of the usual conclusions of Keynesian or Kaleckian models can indeed be overturned, depending on the values taken by the reaction parameters, when the valuation ratio is included as a determinant of the investment function.

There is nothing in the model to force the \( q \)-ratio towards unity. We could have written the investment function by saying that capital accumulation is a function \( \gamma_3 \) of the difference \( q - 1 \). But this is like subtracting \( \gamma_3 \) from the constant in the investment function; it does not imply \( q \) converges to unity in steady state growth. For this to happen, we would need to claim that the change in the rate of accumulation is a function of the difference \( q - 1 \). Formally, we would need to write the difference equation: \( \Delta g = \gamma(q - 1) \), so that \( g \) becomes a constant when \( q = 1 \). In stationary neoclassical models, this result is achieved by assuming that \( I = I(q - 1) \), as in Sargent (1979, p. 10).

One may wonder where expectations enter the investment function, since (nearly) all the determinants of investment are one-period lagged variables. For instance, in the investment functions of Taylor and O'Connell (1985) and Franke and Semmler (1989), the rate of accumulation depends on the current rate of profit augmented by a premium which represents expectations of future rates of profit relative to the current one. As a first step these authors assume the premium to be an unexplained constant; in elaborations of the model, the premium is an inverse function of the debt ratio. In other words, it is assumed that expected future rates of profit decline when debt ratios rise. We have a similar mechanism by virtue of the term \( \gamma_2 r_{-1} \), on the grounds that an increase in debt commitments will slow down accumulation. In addition, a change in the exogenous term in the investment function, \( \gamma_0 \), can represent a change in expectations regarding future profitability or future sales relative to current conditions.

Finally, we consider the fourth category of decision which firms must take. Once the investment decision has been taken, firms must decide how it will be financed. Which variable ought to be considered as the residual one? Franke and Semmler (1991: 336), for instance, take equity financing as a residual. However, they note that the recent literature on credit and financial constraints may suggest rather that "debt financing should become the residual term to close the gap between investment and equity finance", and this is exactly what will be done here. Firms borrow from the banks whatever amount is needed once they
have used up their retained earnings and the proceeds from new equity issues. As Godley (1996, p. 4), suggests, bank loans "provide residual buffer finance". This has already been given a formal representation in equation (6) which gave the budget constraint of firms: \[ \Delta L_d = l_d - F_U - \Delta e_s \cdot \rho_e. \]

We propose two behavioural equations, one determining the split between distributed dividends and retained earnings, and the other determining the amount of new equities to be issued. Distributed dividends are a fraction \((1 - s_f)\) of profits realized in the previous period (net of interest payments). Again, a lag is introduced on the ground that firms distribute dividends each period on the basis of the profits earned the previous period, having imperfect knowledge of soon-to-be realized profits. It is assumed however that these distributed dividends are upscaled by a factor which depends on the past rate of accumulation, to take into account of the fact that the economy is continuously growing.

\[ (22) F_D = (1 - s_f) \cdot (F_T (-1) - r_l (-1) \cdot L_s (-2)) \cdot (1 + g (-1)) \]

This formulation of the dividend decision, though without the lags, can be found in Kaldor's 1966 model \((F_D = (1 - s_f) \cdot F_T)\). Similarly, Kaldor assumes that firms finance a percentage \(x\) of the investment expenditures, regardless of the price of equities or of the value taken by the valuation ratio.\(^{10}\) This is clearly an oversimplification, but we shall adopt it as an approximation, so that:

\[ \Delta e_s \cdot \rho_e = x \cdot l \]

With the above two equations, and remembering that Kaldor assumes away bank debt, Kaldor (and Wood (1975)) arrives at the following determination of the overall rate of profit:

\[ r = g \cdot (1 - x) / s_f \]

where \(r = F_T / K\) is the overall rate of profit and where \(g\) is the exogenous rate of accumulation. This equation is the source of Kaldor's (1966) surprising belief that the rate of household saving has no impact on the rate of profit, for a given rate of growth. By contrast, when there is bank debt and money, the budget constraint (omitting time lags) is telling us that:

\[ (L_d / K) = g = s_f \cdot (F_T - r_l l_d / K) + x \cdot l_d / K + (\Delta L_d / L_d) (L_d / K) \]

In the steady-state case where bank debts are growing at the same rate as the capital stock, i.e., when \(\Delta L_d / L_d = g\), the equilibrium value of the rate of profit is given by a variant of Kaldor's equation:

\[ r = g \cdot (1 - x - l) / s_f + r_l \cdot l \]

Thus, in steady-state growth, the rate of profit is positively related to the rate of accumulation \(g\) and to the rate of interest on bank loans \(r_l\).\(^{11}\) The problem here, however, is that the debt ratio of firms, \(l\), can be considered as a parameter, given by history, only in the short period. In the long period, the debt ratio is among the endogenous variables, to be determined by the model and dependent, among other things, on the rate of household saving.
and the growth rate of the economy, so that the above expression is hardly informative.\textsuperscript{(12)} Simulations will allow us to observe the actual relationship between the rate of profit, the rate of growth and the debt ratio.\textsuperscript{(13)}

**BANKS**

Banks make loans on demand and obviously they accept and exchange deposits as well as pay and receive interest.

\begin{equation}
L_s = L_d
\end{equation}

The equality between loan demand and loan supply should be interpreted as representing the equality between the effective demand for loans and the supply of loans.\textsuperscript{(14)} All credit-worthy demands for loans are granted in this system. In the present model, when debt commitments increase, the symptoms of the crumbling credit-worthiness of firms, accompanied by a shift in the effective demand for loans (and possibly in the notional demand for loans), appear as a downward shift of the investment function (21), under the negative impact of the term representing debt commitments.

It would have been possible to make the rate of interest on loans a positive function of the debt ratio of firms, introducing a kind of Kaleckian effect of increasing risk, but this would have simply compounded the negative effect of high leverage ratios on investment.

**HOUSEHOLDS**

Households must decide how much they wish to consume and save, thereby determining how much wealth they will accumulate. They must also decide the proportions of their wealth they wish to hold in the form of money and equities. We have already discussed, in the first section, the budget constraint that households face when taking these decisions. Here we focus on behaviour.

Using a modified version of the Haig-Simons definition of income, consumption is held to depend on expected regular household income and on capital gains which occurred in the previous period. When they make their spending decisions, households still do not know exactly what their income is going to be.\textsuperscript{(15)} The consumption equation is then:

\begin{equation}
C_d = a_1 Y_{hr}^* + (a_1/\alpha) G_{(-1)}
\end{equation}

with \(0 < a_1 < 1, \alpha > 1\), and:

\begin{equation}
Y_{hr}^* = (1 + g_y_{(-1)}) Y_{hr_{(-1)}}
\end{equation}

\begin{equation}
g_y = \Delta Y_{hr} / Y_{hr_{(-1)}}
\end{equation}

where the * symbol represents expected values.

Expected regular household income is assumed to depend on the realized regular household income of the previous period, and on the rate of growth, \(g_y\), of regular household income.
the previous period. The implication of such a consumption function is that unexpected income increases are not spent in the current period; rather they are saved, much in line with the disequilibrium hypothesis put forth by Marglin (1984, ch. 17) and other non-orthodox authors. This unexpected saving is held entirely in the form of additional money deposits since the allocation of wealth to equities has already been decided on the basis of expected income. Thus actual money balances are a residual -- they constitute an essential flexible element of the system (Godley 2000, p. 18; Lavoie 1984, p. 789).

Our consumption function is nearly the same as that suggested by Kaldor (1966: 318) in a footnote to his neo-Pasinetti article, where there is a single savings propensity for the household sector applying equally to wages, dividends and capital gains. Here the propensity to consume applies uniformly to wages, dividends and interest income. It is doubtful whether, in a world of uncertainty, households would treat accrued capital gains, i.e., non-realized capital gains, on the same footing as regular income. Indeed, some empirical studies have found no relationship between consumption and contemporaneous capital gains. However, "studies that have included lagged measures of capital gains have often found a significant impact" (Baker, 1997, p. 67). As result, we have assumed that only lagged capital gains enter the consumption function, and that a smaller propensity to consume applies to these gains.

It would have been possible to introduce a third element in the consumption function, namely the stock of wealth accumulated previously, \( V(-1) \), with a certain propensity to consume out of it, say \( a_2 \), an addition akin to the mainstream models of consumption (the life-cycle and the permanent income hypotheses). In models dealing with stationary steady states without growth, such an addition is a necessary requirement, because if the \( a_1 \) coefficient is less than one wealth must be rising relative to income, without limit (Godley 1999, p. 396). However, in a growth model wealth is continuously growing, and hence the standard Keynesian consumption function, with \( a_1 < 1 \) and \( a_2 = 0 \), is adequate. In a growing economy, equation (25), where consumption only depends on flows of regular or accrued income, still makes it possible to incorporate the theory of credit, money and asset allocation into that of income determination in a coherent way. We shall therefore stick with the Kaldorian consumption function for the time being.\(^{16}\)

Coming to households' portfolio choice, we follow the methodology developed by Godley (1999), and inspired by Tobin (1969).\(^{17}\) It is assumed that households wish to hold a certain proportion \( \lambda_0 \) of their expected wealth \( V^* \) in the form of equities (and hence a proportion \( 1 - \lambda_0 \) in the form of money deposits), but that this proportion is modulated by the relative rates of return on bank deposits and equities, and by the transactions demand for money (related to expected household income). The two asset demand functions are thus:

\[
(28) \quad (p_e . e_d)*/ V^* = \lambda_0 - \lambda_1 r_m + \lambda_2 r_{e(-1)} - \lambda_3 (Y_{hr} / V^*)
\]

\[
(28A) \quad M_*/V^* = (1 - \lambda_0) + \lambda_1 r_m - \lambda_2 r_{e(-1)} + \lambda_3 (Y_{hr} / V^*)
\]

where the \( \lambda \)s are parameters, where the * symbol again represents expected values, and where \( r_{e(-1)} \) is the rate of return obtained on equities in the previous period. The rate of return on equities of the current period is defined as the ratio of dividends received plus capital gains over the value of the stock of held equities in the previous period.
\[(29) \ r_e \equiv (F_D + G)/(p_e(-1) \cdot e_d(-1))\]

The two asset demand functions are homogeneous in wealth, i.e., the proportions of the two assets being held does not vary in the long run with the absolute size of wealth although, by virtue of the final term in each function, there is a transactions demand for money which can make a temporary difference. The two asset functions sum to one because households are assumed to make consistent plans, symmetric to the adding-up condition of equation (2A). Portfolio plans, under the adding-up assumption, are thus:

\[(30) \ M_d^* \equiv (V^* - (e_d \cdot p_e))^* \]

Equation (30) implies that one of the two asset demand functions must be dropped for the model to solve. And this is indeed what is done in the simulations; equation (28A) describing the money demand function has been dropped and replaced by (30).

Expected regular household income was defined by equation (26). Expected capital gains are assumed to depend on past capital gains and the rate of accumulation of capital in the previous period, so that:

\[(31) \ G^* = (1 + g(-1)) \cdot (G(-1)) \]

On the other hand, for households to have consistent plans, the expected level of wealth must be in line with its expected budget constraint. The realized budget constraint of households was already defined by equation (3). The following equation is its equivalent, within the realm of expectations:

\[(32) \ V^* \equiv V(-1) + Y_{hr}^* + G^* \cdot C_d \]

When expectations and plans are fulfilled, the ratios targeted in equations (28) and (28A) will be exactly realized. In this case, the only element of flexibility resides in the price of equities \(p_e\), since all the other elements, including \(e\) --the number of equities-- are predetermined. The price of equities will rise until the targeted ratio is attained since there cannot be any discrepancy between the number of shares which have been issued and the number of shares that households hold. In other words, there has to be a price-clearing mechanism in the equity market, such that:

\[(33) \ e_d = e_s \]

What happens when expectations about regular income are mistaken? As pointed out above, an extra element of flexibility resides in the amount of money balances held by households. On the basis of their expectations, regardless of whether they are realized or not, households invest in the stock market in such a way that:

\[(34) \ p_e \cdot e_d = (p_e \cdot e_d)^* \]

System-wide implications

We now have the same number of equation as unknowns, including equations in both the "demand" (equation 2) and the "supply" of money (equation 7). So the whole model is now
closed and there is therefore neither a need nor a place for an equilibrium condition such as:

\[(7A) \ M_s = M_d \]

However, from the balance sheets of Table 1 we know that the equality between the money deposits households find themselves holding and the money deposits supplied by banks -- which are equal to the loans they have made -- must invariably hold. Indeed this property of the model provides a way in which its accounting logic can, in practise, be tested. Having solved the model, we can check the accounting, using the simulations, to verify that the numbers do indeed generate \( M_s = M_d \). It is only when an accounting error has been committed, that the equality given by (7A) will not be realized. With the accounting right, the equality must hold. And in the present model, the equality holds with no need for any asset price or interest rate adjustment.

If household income, and hence household wealth, turn out to be different from expected levels, the adjustment factor is the amount of money left with households, \( M_d \), compared with \( M_d^* \). For instance, suppose that actual household income is higher than its expected level: \( Y_{hs} > Y_{hs}^* \). As a result, because consumption does not depend on actual current income, there will be a corresponding gap between the actual and expected change in wealth: \( \Delta V > \Delta V^* \). As a consequence, the amount of money held by households will be higher than what they expected to hold by exactly the amount that income has been underestimated. Formally, we have:

\[(19) \]

\[(2B) \ M_d = M_d^* + (Y_{hs} - Y_{hs}^*) \]

Equation (2B) shows that the planned demand for money can be different from the realized one. In other words, we know that it is possible to have: \( M_s > M_d^* \). But this has no bearing on whether or not an excess supply of money can arise. This inequality is due to mistaken expectations; it has no causal significance of its own. In particular, it cannot be said that the excess money supply, defined here as \( M_s - M_d^* \), can be a cause of an excess demand on the goods market, or of an excess demand on the equities market (which would push down financial rates of return).

Is it for a moment surprising that the stock of money people fetch up with, whether or not they have made wrong predictions, is identically the same amount as the loans which firms find that they have incurred -- although this follows from a distinct set of decisions. Our model is so simple that it reveals with unusual clarity why this must be so. Kaldor’s (1982) intuition -- that there can never be an excess supply of money -- is vindicated.

Kaldor’s assertion has often been called into question. Some authors have noted that, because money deposits are created as a result of loans being granted to firms, money demand could exceed money supply. Coghlan (1978, p. 17) for instance, says that: “If we accept that advances can be largely exogenous ....then the possibility must exist that bank deposits can grow beyond the desires of money holders”. That claim is wrong, however. As shown here, and as explained informally by Lavoie (1999), such a misunderstanding arises as a result of ignoring the overall constraints imposed by double-entry financial bookkeeping.

Finally, it should be pointed out that the seeds of our generalization of Kaldor’s 1966 model
to a monetary economy can already be found in Joan Robinson's works (1956, 1971). (21) Robinson endorsed Kaldor's neo-Pasinetti theorem, with the proviso that "the banking system is assumed to be generating a sufficient increase in the quantity of money to offset liquidity preference" (1971, p. 123). She had argued earlier that "banks must allow the total of bank deposits to increase with the total of wealth", and that banks must "lend to entrepreneurs (directly or by taking up second-hand bonds), the difference between rentier saving and rentier lending" (Robinson 1956, p. 277). (22)

EXPERIMENTS

The model presented above was solved numerically and subjected to a series of simulation experiments. First we assigned values to the various parameters using reasonable stylized facts. Then we solved the model, and found a steady state solution through a process of successive approximation. Having found a steady-state, we conducted experiments by modifying one of the exogenous variables or one of the economically-significant parameters of the model at a time. The advantage of this approach is that it is always possible to find out exactly why the model generates the results it does. The disadvantage is that we can only analyse local stability: we do not know if there are other equilibria, or if these other equilibria are stable. What we do show, is that over a reasonable range of parameter values, including obviously the values which we chose, the model does yield a stable solution.

We quickly discovered that the model could be run on the basis of two regimes. In the first regime, the investment function reacts less to a change in the valuation ratio -- Tobin's q -ratio -- than it does to a change in the rate of utilization. In the second regime, the coefficient of the q -ratio in the investment function is larger than that of the rate of utilization (γ_3 > γ_4). The two regimes yield a large number of identical results, but when these results differ, the results of the first regime seem more intuitively acceptable than those of the second regime. For this reason, we shall call the first regime a normal regime, while the second regime will be known as the puzzling regime. The first regime also seems to be more in line with the empirical results of Ndikumana (1999) and Franke and Semmler (1996), who find very small values for the coefficient of the q -ratio in their investment functions, that is, their empirical results are more in line with the investment coefficients underlying the normal regime.

CHANGES IN THE PROPENSITY TO CONSUME

Let us first consider changes in the propensity to consume. We shall spend more space on this issue, because it is a particularly touchy one, as indicated in the previous section. The paradox of savings -- a higher propensity to consume or a lower propensity to save leads to faster growth -- is a crucial component of the Keynesian/Kaleckian school, in contrast to the classical/ Marxian models of growth or to the neoclassical models of endogenous growth, where the opposite occurs. Here, whether the paradox of savings occurs or not depends on the value taken by the coefficient of the q -ratio in the investment function.

In the normal regime the paradox of savings holds. An increase in the propensity to consume leads to an increase in the rate of accumulation, both in the short period and in the long period, despite the fall in the q - ratio.

The logic of this result is the following. The increase in the propensity to consume leads to higher rates of utilization and higher rates of profit, both of which encourage entrepreneurs to increase the rate of accumulation. The higher profits of entrepreneurs allow them to
reduce their dependence on debt and reduce the leverage ratio $l$. All these effects are shown in Figure 1(a), where, as in all following figures, the various series are expressed as a ratio of the steady-state base case.

On the other hand, the initial fall in savings is accompanied by a falling demand for equities, which initially slows down the rate of increase in the price of equities, and hence reduces the $q$-ratio and the rate of return on equities $r_e$ (see Figure 1(b)). The initial fall in $r_e$ increases the demand for money as a share of wealth. However, as profits and capital keep on growing, the rate of return on equities recovers, and hence, in the new steady-state the money to wealth ratio is lower than in the previous steady-state (Figure 1(c)). Because entrepreneurs hardly react to the fall in the $q$-ratio, accumulation keeps going strongly: its steady-state rate is higher than that of the initial steady-state, but it is lower than the previously achieved peak (Figure 1(a)). The paradox of savings holds in this regime.

In the puzzling regime, the paradox of savings does not hold. The faster rate of accumulation initially encountered is followed by a floundering rate, due to the strong negative effect of the falling $q$-ratio on the investment function. The turnaround in the investment sector also leads to a turnaround in the rate of utilization of capacity. All this leads to a new steady-state rate of accumulation which is lower than the rate existing just before the propensity to consume was increased (see Figure 1(d)). Thus in the puzzling regime, although the economy follows Keynesian or Kaleckian behaviour in the short-period, long period results are in line with those obtained in classical models or in neoclassical models of endogenous growth: the higher propensity to consume is associated with a slower rate of accumulation in the steady state. In the puzzling regime, by refusing to save, households have the ability over the long period to undo the short-period investment decisions of entrepreneurs (Moore 1973). On the basis of the puzzling regime, it would thus be right to say, as Duménil and Lévy (1999) claim, that one can be a Keynesian in the short period, but that one must hold classical views in the long period.

**CHANGES IN THE INTEREST RATE ON LOANS AND DEPOSITS**

The key difference between the behaviour of the normal and the puzzling regimes is the impact of a change in the (real) interest rate on loans (and deposits). Recall that an increase in the interest rate has two effects on effective demand. On the one hand, as is shown in mainstream IS/LM models, an increase in the rate of interest has a negative impact on investment. But on the other hand, an increase in interest rates has a favourable effect on consumption demand and hence on the rate of capacity utilization, since more income is now being distributed to households. This effect is underlined in the models of stationary steady states presented by Godley (1999), where a higher interest rate leads to a higher stationary level of output. The positive effect on effective demand, for a given level of investment, is also present in Skott (1988), in a model which is closely related to the present one.

In our model, with the chosen parameters, the negative investment effect is initially strongest in both regimes. In the normal regime the negative effect of the higher debt commitments carries over to the long period (Figure 2(a)). However, in the puzzling regime, despite the heavier debt commitments due both to the higher rate of interest and the higher leverage ratio $l$, an increase in interest rates eventually drives up the steady-state rate of accumulation to a level which exceeds the growth rate associated with the lower rate of interest (Figure 2(b)) -- a rather surprising and counter-intuitive result. This counter-intuitive result justifies the name puzzling which we have attributed to this second regime.
In both regimes, despite an initial downward move, the steady-state rate of utilization ends up higher than its starting value (see Figure 2(a)). In addition, the $q$-ratio is quickly pushed upwards (see Figure 2(b)), as more disposable income allows households to spend more on equities. This effect has particularly strong repercussions on capital accumulation in the second regime, which explains why the increase in the rate of interest drives up the steady-state rate of growth.

It may be also noted that in the normal regime the higher lending rates of interest are associated in the long period with lower rates of return on equities, while in the puzzling regime there is a positive long-period link between lending rates of interest and rates of return on equities.

**CHANGES IN THE PROPENSITY TO HOLD EQUITIES**

The other experiments show little difference of behaviour between the first and second regimes. For instance, in both regimes, a shift in liquidity preference, out of money deposits and into equities, symbolized by an increase in the $\lambda_0$ parameter of the portfolio equations, leads to an increase in the short and the long period rate of accumulation. The view of liquidity preference in the present model is consistent with that offered by Mott (1985-86, p. 230), according to whom "liquidity preference is a theory of the desire to hold short- versus long-term assets". Here money deposits are the short-term asset, while equities are the long-term one.

Our experiments give considerable support to the Post Keynesian belief that liquidity preference, defined in a broad way, does matter in a monetary economy. The favourable impact of lower liquidity preference can be observed independently of any change in the confidence or animal spirits of entrepreneurs or their bankers (as proxied by the $\lambda_0$ coefficient in the investment equation, or by the level of the real rate of interest). Our model allows us to identify the mechanisms by which pure liquidity effects can affect the real economy.

The favourable impact of the increasing desire of households to hold equities instead of money can be attributed to two standard effects. On the one hand, the increase in the stock-demand for equities pulls up the price of equities and creates capital gains (Figure 3(a)). These gains are then partly consumed, thus raising the rate of capacity utilization, and hence, in the next period, it shifts up the investment function. On the other hand, the increase in the demand for equities pushes up the $q$-ratio, the increase in which also contributes to shift up the investment function. All these effects are accompanied by a lower money to wealth ratio and a lower debt ratio, which also contributes to the faster accumulation rate of the economy (all these effects are shown in Figure 3(b)).

There is a feedback loop which operates as a result of the initial increase in the desire of households to hold securities; there is an acceleration in the rate of growth of the economy and the rate of utilization rises. All this drives up the rate of return on securities $r_e$, thus reinforcing the desire of households to reduce their money deposits relative to their overall wealth.

Mott (1985-86, p. 231) asserts that "liquidity preference is governed primarily by the profitability of business". In all of our experiments, the steady-state values of the rate of accumulation and the rate of return on equities moved in the same direction. Since the demand
for equities depends on the rate of return on equity, we may say that there is indeed a link between the good performance of the economy and the preference of households for long-term assets. (24)

CHANGES IN REAL WAGES

A typical Kaleckian effect is also to be found in the present model. Assume that there is a decrease in the markup $\rho$, which, ceteris paribus, implies that there is an increase in the real wage of workers, relative to their productivity, $(w/p)/\mu$. (25) This means that the share of wages is now higher, while that of profits is lower. In standard Kaleckian growth models, an increase in the real wage leads to an increase in the long-period rate of accumulation and in the long-period rate of capacity utilization (Rowthorn 1981, Dutt 1990, Lavoie 1995). The same result is obtained here.

The increase in real wages leads to an increase in consumer demand, because firms will now be distributing more income to households, while retaining less. As a consequence, the rate of capacity utilization is pushed upwards. Note that the increase in capacity utilization will only be felt one period later since consumption depends on expected regular household income, rather than on realized regular income.

Initially, in the short period, despite the increase in the rate of utilization, the rate of profit of businesses falls, because of the lower markup. This short-period result is in contrast with the result achieved in time-continuous Kaleckian models, because in these models everything is simultaneous, so that firms react immediately to the higher rate of utilization by speeding up their rate of accumulation, generating higher rates of profit in the process.

In the present model, by contrast, the rate of capital accumulation set by firms depends on the variables of the previous period, and as a result the increase in the rate of utilization induced by rising real wages has no immediate effect on accumulation. In later periods, however, the rate of accumulation starts recovering from the lower rate of profit initially induced by the lower markup. Over time the faster accumulation helps to improve profitability. In the long period, the rate of accumulation is much higher with higher real wages, whatever the regime of the model. In the normal regime, the more likely one, the rate of profit does not totally recover. (26) This last result, as pointed out above, is in contrast with the time-continuous Kaleckian models of growth. In addition, the lower markup set by firms leads to a higher debt ratio, a not-so-obvious result. All these effects are shown in Figure 4.

CHANGES IN PARAMETERS CONTROLLED BY THE FIRMS

When discussing the behaviour of firms, it was assumed that firms had the ability to set the number of equities that they wished to issue each period -- a rule was given according to which firms financed $x\%$ of their investment by issuing new shares -- and that firms chose a retention ratio on profits (net of interest payments). What happens when firms decide to change these percentages? (27)

First consider the case when the $x$ ratio is increased. Firms issue more securities. This leads to an initial fall in the rate of growth of equity prices, and hence to a fall in the $q$-ratio. This fall induces a capital loss and hence a slowdown in consumption demand growth. This slowdown leads to a fall in the rate of utilization and hence in the cash flow of firms. The fall in these two determinants of the rate of accumulation, as well as the fall in
its third determinant -- the $q$ -ratio -- lead to a permanent slowdown in the rate of accumulation, as shown in Figure 5(a). The only positive effect of issuing more securities is that the debt ratio is reduced, but this appears to be a second-order effect.

If the model correctly describes the behaviour of a true economy, the reluctance of companies to issue equities may appear to be well founded. Larger issues of equities have detrimental effects on a monetary economy, leading to a fall in the growth rate, the rate of profit, and the rate of return on equities.

Let us now consider the case of an increase in the retention ratio of firms. This increase has two contradictory effects on effective demand. On the one hand, it automatically increases the cash flow which is available to firms to finance their investments, thus pushing up the investment function. In addition, firms have to borrow less, and hence can reduce their debt ratios. On the other hand, households are left with less regular income, and hence the rate of growth of consumption demand slows down. With the chosen parameters, the positive effects on the rate of accumulation initially overwhelm the negative ones, but over the long period an increase in the retention ratio does have a negative effect on the rate of growth of the economy. All these effects are shown in Figure 5(b). In the steady state, there is also a negative impact on the overall rate of profit and the rate of return on equities.

OTHER CONSIDERATIONS

Within the Post Keynesian tradition, the works of Minsky (1975) have attracted a considerable amount of attention. His financial fragility hypothesis, within the context of the present model, can be represented with the help of a proxy, the leverage or debt ratio $l$, which is the ratio of debt to capital (valued at production prices). Minsky has argued at length that he expects the debt ratio of firms to rise in booms, as firms would attempt to take advantage of the more relaxed borrowing standards set by banks. During recessions, the opposite would occur, as firms would reduce the relative weight of their debt. Thus, according to Minsky and some of his followers, there is a procyclical movement between economic activity and debt or leverage ratios.

Lavoie and Seccareccia (2000) have argued that Minsky makes an error of composition, by confusing intentions and realizations. It may be that, during the boom, all entrepreneurs would like to increase leverage and take advantage of the more relaxed borrowing standards. Whether they actually succeed in doing so is an entirely different story, since the actual increase in debt will depend on realized profits. It may well be that realized debt ratios fall during the boom, when entrepreneurs would like them to rise, and that realized debt ratios increase during a slowdown, just when entrepreneurs would prefer to see them fall. This is what Steindl (1952) calls the paradox of debt.

The empirical work conducted on this issue has not been entirely conclusive. The consensus among specialists used to be that debt ratios were essentially counter-cyclical (Wolfson 1994), thus favouring Minsky's financial fragility hypothesis, but the more recent economic cycles seem to have produced pro-cyclical debt ratios. Table 3 summarizes the qualitative relationship between the steady state rate of accumulation and the steady state debt ratio. Clearly these relations depend on the parameter that was modified. However, the analysis of the previous subsection has shown that when entrepreneurs reduce the retention ratio $s_f$ and the share issuing ratio $x$, this leads to a long-period increase in the debt ratio of firms. Thus, if entrepreneurs attempt to take advantage of the more relaxed financial standards by reducing those two financial ratios, they will indeed be successful in increasing their
long-period debt ratios. In that case, Minsky is clearly correct.

However, as argued by Kalecki, the long run is simply a succession of short runs. In other words, economies are most of the time in a traverse from one (hypothetical) steady state to another, as Hicks would have it. What is the relationship between the rate of accumulation and the debt ratio during the transition between two steady states? Table 4 allows us to give some kind of answer to this question. The table was compiled by examining the graphs of various simulations. The first line gives the short-run relationship between the growth rate of capital and the debt ratio right after the parameter change has been inflicted on the economy. In most cases, the rate of accumulation and the debt ratio move in opposite ways.

The second line of Table 4 yields the short-run relationship between the rate of accumulation and the debt ratio at the end of the transition, when the economy is actually converging towards its new steady state. In all cases, whatever the source of the disturbance, the relationship is a positive one. Whenever the convergence process is actually occurring, the rate of accumulation and the debt ratio are moving in the same direction. For instance, when there is overshooting, the deceleration of the rate of accumulation towards its steady state position will always be accompanied by a decrease in the debt ratio.\(^{(28)}\)

The above shows that the relationship between economic activity and debt ratios in a demand-led monetary economy is highly complex. With one exception, there are no simple rules, and one cannot assert that debt ratios should move one way or another in a boom. This of course explains why it is so difficult to determine whether debt ratios are pro- or counter-cyclical. However, we know for sure that the mechanism of convergence towards a steady state growth equilibrium requires that the rate of accumulation and the debt ratio move in the same direction at the end of the transition path. We also know that, if entrepreneurs decrease the retention ratio and the share issuing ratio in an attempt to take advantage of the more relaxed financial standards in a boom period, the new steady-state debt ratio will indeed be higher.

**CONCLUSION**

Post Keynesian economics, as reported by Chick (1995), is sometimes accused of lacking coherence, formalism, and logic. The method proposed here is designed to show that it is possible to pursue heterodox economics, with alternative foundations, which are more solid than those of the mainstream. The stock-flow monetary accounting framework provides such an alternative foundation which is based essentially on two principles. First, the accounting must be right. All stocks and all flows must have counterparts somewhere in the rest of the economy. The watertight stock flow accounting imposes system constraints which have qualitative implications. This is not just a matter of logical coherence; it also feeds into the intrinsic dynamics of the model.

Secondly, we need only assume, in contrast to neoclassical theory, a very limited amount of rationality on the part of economic agents. Agents act on the basis of their budget constraints.\(^{(29)}\) Otherwise, the essential rationality principle is that of adjustment. Agents react to what they perceive as disequilibria, or to the disequilibria that they take note of, by making successive corrections. There is no need to assume optimization, perfect information, rational expectations, or generalized price-clearing mechanisms.

Another feature of the present analysis is the simulation method. With simulations, a full model can be articulated and its properties ascertained and understood, without the need to
resort to reduced forms. The simulation method enables one to penetrate, with one's understanding, dynamic models of far greater complexity than can be handled by analytic means. Indeed, even practitioners of multi-dimensional stability analysis resort to simulations to figure out how their models behave (see for instance Flaschel et al. 1997). Non-linearities can be easily introduced. For instance, we can program behaviour to change whenever a variable exceeds or drops below some threshold level, as in the model of Godley (1999). In that model, the steady state was stationary. It is quite possible, however to superpose the present model to that previous model, to obtain a growth model with highly complex but coherent features. These would include a government sector, a detailed banking sector, and consumption and production that occurs in real time, with inventories, and with output supply not being generally equal to output demand.

Although narration and verbal explanation are in order -- indeed essential -- we are suggesting a method which has much rigour and demonstrability. In our methodology we can justify every point by reference to a precise system of relationships. If others disagree, they can be challenged to say precisely what simplification or parameter is inappropriate. Every relationship can be changed, and one can find out whether the change makes any difference to the results. This method ought to be helpful to resolve some controversial issues.

The present model has provided two examples. First, our framework has allowed us to discuss the relationship between the supply of credit and the demand for money. We have shown how and why an excess supply of money can never arise. Second, we have been able to assess the relationship between the rate of growth of firms and their debt ratios, a relationship much discussed in the work of Minsky. We discovered that the relationship between those two variables is not the same depending on the stage of the transition process, and depending on whether a comparison of steady states is being made, thus showing that the behaviour of debt ratios is a complex matter, even in simple growth models.

Table 1: Balance sheets

<table>
<thead>
<tr>
<th></th>
<th>Households</th>
<th>Firms</th>
<th>Banks</th>
<th>( \Sigma )</th>
</tr>
</thead>
<tbody>
<tr>
<td>Money</td>
<td>+( M_d )</td>
<td></td>
<td>-( M_s )</td>
<td>0</td>
</tr>
<tr>
<td>Equities</td>
<td>+( e_d \cdot p_e )</td>
<td>-( e_s \cdot p_e )</td>
<td>( K )</td>
<td>( +K )</td>
</tr>
<tr>
<td>Capital</td>
<td></td>
<td>+( K )</td>
<td>( +K )</td>
<td>( +K )</td>
</tr>
<tr>
<td>Loans</td>
<td></td>
<td>-( L_d )</td>
<td>( +L_s )</td>
<td>0</td>
</tr>
<tr>
<td>( \Sigma ) (net worth)</td>
<td>+( V )</td>
<td>( K - (L_d + e_s \cdot p_e) )</td>
<td>0</td>
<td>( +K )</td>
</tr>
</tbody>
</table>

Table 2: Transactions matrix
<table>
<thead>
<tr>
<th></th>
<th>Households</th>
<th>Firms</th>
<th>Banks</th>
<th>Σ</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>Current</td>
<td>Capital</td>
<td>Current</td>
<td>Capital</td>
</tr>
<tr>
<td>Consumption</td>
<td>- C_d</td>
<td>+C_s</td>
<td>0</td>
<td>0</td>
</tr>
<tr>
<td>Investment</td>
<td>+I_s</td>
<td>-I_d</td>
<td>0</td>
<td>0</td>
</tr>
<tr>
<td>Wages</td>
<td>+W_s</td>
<td>- W_d</td>
<td>0</td>
<td>0</td>
</tr>
<tr>
<td>Profits</td>
<td>+F_D</td>
<td>-(F_U + F_D)</td>
<td>+F_U</td>
<td>0</td>
</tr>
<tr>
<td>Interest on loans</td>
<td>- r_l .L_d(-1)</td>
<td>+r_l .L_s(-1)</td>
<td>0</td>
<td>0</td>
</tr>
<tr>
<td>Interest on deposits</td>
<td>+ r_m .M_d(-1)</td>
<td>- r_m .M_s(-1)</td>
<td>0</td>
<td>0</td>
</tr>
<tr>
<td>Δ in loans</td>
<td>0</td>
<td>+ΔL_d</td>
<td>- ΔL_s</td>
<td>0</td>
</tr>
<tr>
<td>Δ in money</td>
<td>- ΔM_d</td>
<td>0</td>
<td>+ΔM_s</td>
<td>0</td>
</tr>
<tr>
<td>Issue of equities</td>
<td>- Δe_d .p_e</td>
<td>Δ_e_s .p_e</td>
<td>0</td>
<td>0</td>
</tr>
<tr>
<td>Σ</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>0</td>
</tr>
</tbody>
</table>

Table 3:
Relative evolution of the rate of accumulation and the debt ratio, measured in new steady-state positions, following a change in one of the parameters.

<table>
<thead>
<tr>
<th>change in</th>
<th>a_l</th>
<th>r_l</th>
<th>λ_0</th>
<th>p</th>
<th>x</th>
<th>s_f</th>
<th>γ_0</th>
</tr>
</thead>
<tbody>
<tr>
<td>fg / dl normal</td>
<td>-</td>
<td>-</td>
<td>-</td>
<td>+</td>
<td>+</td>
<td>+</td>
<td>-</td>
</tr>
<tr>
<td>fg / dl puzzling</td>
<td>+</td>
<td>+</td>
<td>-</td>
<td>+</td>
<td>+</td>
<td>+</td>
<td>-</td>
</tr>
</tbody>
</table>

Table 4:
Relative evolution of the rate of accumulation and the debt ratio, during the transition, following a change in one of the parameters.

<table>
<thead>
<tr>
<th>change in</th>
<th>a_l</th>
<th>r_l</th>
<th>λ_0</th>
<th>p</th>
<th>x</th>
<th>s_f</th>
<th>γ_0</th>
</tr>
</thead>
<tbody>
<tr>
<td>fg / dl right after shock</td>
<td>-</td>
<td>-</td>
<td>-</td>
<td>-</td>
<td>+</td>
<td>-</td>
<td>-</td>
</tr>
<tr>
<td>fg / dl near the steady-state</td>
<td>+</td>
<td>+</td>
<td>+</td>
<td>+</td>
<td>+</td>
<td>+</td>
<td>+</td>
</tr>
</tbody>
</table>
REFERENCES


Figure 1(a): Higher propensity to consume, normal regime
Figure 1(b): Higher propensity to consume

- rate of return on equities
- rate of growth of equity prices
Figure 1(c): Higher propensity to consume

money to wealth ratio
Figure 1(d): Higher propensity to consume, puzzling regime
Figure 2(a): Higher interest rate, normal regime

utilization rate

rate of accumulation

Figure 3(a): Stronger preference for equities
Figure 3(b): Stronger preference for equities
Figure 4: Higher wage share

- Utilization rate
- Debt ratio
- Rate of accumulation
- Profit rate

Figure 5(a): Larger issues of equities

- Rate of accumulation
- Q-ratio
- Growth rate of equity prices

1. The rate of profit in a Golden Age equilibrium ... will then be independent of the "personal" savings propensities.... In this way, it is similar to the Pasinetti theorem.... It will hold in any steady growth state, and not only in a "long-run" Golden Age (Kaldor 1966, p. 318).

2. As in other Kaleckian models, it will be assumed that parameters are such that the rate of capacity utilization does not exceed unity.


4. See Palley (1996) for an analysis of household debt. Of course it would be possible within the present model to suppose that households borrow to speculate on the stock market.

5. For instance, the investment function proposed by Dutt (1995) includes the cash flow ratio, the debt ratio and the rate of utilization.

6. The suggested investment function is also supported by the empirical work of Semmler and Franke (1996).

7. "[The article] 'Money, Portfolio Balance, Capital Accumulation and Economic Growth', written in 1965 ... presented an alternative approach to money and capital accumulation more in tune with Keynes's General Theory and Treatise on Money. This alternative to Tobin's
1965 accumulation analysis involved utilizing the forward market price for capitals (that is, the market price of existing real capital relative to the cost of producing real capital) as the relevant 'invisible hand' ratio directing the entrepreneurial determination of the rate of investment or disinvestment in real capital. This ratio, is of course, the equivalent of the famous q-ratio that Tobin was to discover in 1968". (Davidson 1992, p. 111).

8. "The stock exchange value of a company can fall to say one half of the value of the assets employed in the business. But this does not change the decision as to whether it is worthwhile to undertake some investment or not; the implicit rate of return would only become relevant to the firm's decisions if the normal method of financing investment were to be the issue of ordinary shares for cash -- which in fact plays a very small role. Most of the profits come from ploughed back profits, in which case the expected internal rate of return is relevant and not the implicit rate of return". (N. Kaldor, in a letter to one of the authors, November 9, 1983).

9. It is also what Flaschel, Franke and Semmler (1997, p. 357) end up doing themselves.

10. Alternative formulations would have been possible. For instance, Marris (1972) and Skott (1988) assume that the stock of issued securities grows at a constant rate \( g_S \). That rate could also be assumed to be higher when the valuation ratio exceeds unity.

11. Here, because there is no price inflation, all growth rates are in real terms: the rate of interest is the real rate of interest.

12. Since the propensity to save of households has an impact on the debt ratio \( l \), it also has an impact on the rate of profit, even if there is no change in the rate of growth. Thus, as guessed by Davidson (1968, 1972), introducing money into Kaldor's neo-Pasinetti model does change the main feature which gave it its name!

13. Computing the steady-state value of the debt ratio \( l \) yields an extraordinarily complicated equation, even in such a simple model.

14. The expression "effective" demand for loans, to denote the demand from credit-worthy customers, is utilized by both Lavoie (1992, p. 177) and Wolfson (1996, p. 466).

15. This is one of the crucial aspects which distinguish the present model from that of Skott (1988).

16. It should be pointed out, however, that Kaldor was fully aware that wealthy households could consume without ever having to declare any taxable income. Even if a portion of realized capital gains were to become part of taxable income, these wealthy families could dodge taxation altogether by borrowing their way into consumption, getting loans for consumption purposes, secured on the basis of their large assets, thus slowly depleting their net assets. This is why Kaldor wished to have an expenditure tax replacing the income tax.


18. This assumption can be found in Godley (1996, p. 18): "It is assumed that mistaken expectations about disposable income turn up as differences in holdings of [money deposits] compared with what was targeted".
19. Equation (2B) is the result of combining equations (I) and (II), which, given equations (34) and (3A), arise from the following subtractions:

\[(3) \ V = V_{-1} + Y_{hr} + G - C_d \]
\[(2A) \ V = M_d + e_{d} \cdot p_e \]
\[(32) \ V^* = V_{-1} + Y_{hr}^* + G^* - C_d \]
\[(30) \ V^* = M_d^* + (e_{d} \cdot p_e)^* \]
\[(I) \ V - V^* = Y_{ss} - Y_{hr}^* \]
\[(II) \ V - V^* = M_d - M_d^* \]

20. By contrast, Godley (1999) shows how, in a world with a more sophisticated banking system, the path of loans and deposits can diverge. But the question of the equality between the demand for and the supply of money is an entirely different issue.

21. See Rochon (2000, ch.4) for an overview of Robinson’s unjustly neglected analysis of endogenous credit money.

22. The reader will see some similarity with Davidson’s (1972, p. 335) analysis of growth when the so-called excess flow-demand for securities is negative. See also Dalziel (1999-2000), for a symmetrical analysis when the excess flow-demand for securities is positive.

23. Figure 1(b) shows substantial cyclical fluctuations in the stock market, which are due to the mechanical way in which portfolio decisions are taken. Still the variables do converge to their steady-state values.

24. From the budget constraint of firms, and from the definition of the rate of return on equities, it can be shown that, in the steady-state, \[ r_e = \{ r - i \ln + g \cdot (q - 1) \}/(q - 1). \]

25. In the simulations of the model, the markup \( p \) is reduced while the nominal wage rate \( w \) is simultaneously increased, to keep output prices constant at \( p = 1 \).

26. In the puzzling regime, however, the steady-state rate of profit with higher real wages is much higher than that with low real wages.

27. It turns out that the regime of the model does not matter.

28. Steve Keen tells us that he obtains the same results with his own simulations, based on non-linear models.

29. Other authors, mainly heterodox ones, have made use of balance sheets, to secure appropriate accounting foundations, and of Tobin’s adding-up constraint, to achieve portfolio equilibrium, for instance Franke and Semmler (1989, 1991). But while the stock matrix is given a great deal of attention, the flow matrix is sometimes left out, specially when dealing with the banking sector.