Money, Growth, Distribution
and Prices
in a Simple Sraffian Economy

by

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Abstract

It is well known that relative prices are determined by the profit rate in a Sraffa model. This paper follows Sraffa’s suggestion about the determination of the profit rate: it closes the Sraffian model with a monetary sector. It is shown that the Central Bank, through its control of money supply growth, influences the profit rate and thus the long run distribution of income and relative prices. Furthermore, it is shown that in a Ricardo-Von Neumann-Lewis world, the Central Bank controls the long run growth rate of the economy.

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I. Introduction

Neoclassical theory locates the determination of the profit rate in the interaction of technology, tastes and the population growth rate. A rival explanation is offered by the Classical approach to growth theory, as developed by Ricardo, Von-Neumann (1946) and Lewis (1954). The real wage is viewed as determined by sociological and political forces: labor is supplied elastically at the exogenously given real wage. From the wage-profit relation this determines the profit rate.

Sraffa (1960) provides yet another explanation. Sraffa poses the question: what determines relative prices in a production economy? Rejecting the neoclassical notion of supply and demand, he demonstrates that relative prices are determined once either the profit rate or the real wage rate is fixed. He suggests (1960, p.33) that the profit rate be viewed as determined outside the system of production; in particular, that it may be viewed as controlled by the Central Bank.

His ‘suggestion’ has sparked considerable interest in a ‘monetary theory of the rate of profits’. The term was coined by Panico (1984, chapter 6) who presents a model to demonstrate a connection between ‘liquidity parameters’, exogenously controlled by the government, and the profit rate. However, there is no description of the determination of output, growth and accumulation in this model. Further, it is not clear what determines the money wage (taken as exogenous in the model), and the model cannot allow for fixing the real wage rate. The lack of a sufficiently explicit and complete theoretical model leaves, as Foley (1989) points out, “many of the central questions about the possibility of closing the Sraffian model with a monetary sector unanswered”.

It is the purpose of this paper to close the Sraffian model with a monetary sector. Sraffa’s (1920, 1922, 1932) approach to the monetary sector and Central Bank behavior had a richer institutional component than that developed in this paper. However, by necessity formalisations of the kind presented in this paper must leave out institutional detail. I shall construct a formal model of the Sraffian monetary theory of profits: a framework in which a) relative prices are determined by the profit rate, b) the profit rate is determined outside that production system, namely in the market for assets, and c) the Central Bank through its control of the supply of
money may determine the profit rate. The principal conclusion is that Central Bank policy, through its influence on the profit rate, influences the long run distribution of income and relative prices in this Sraffian world.

I then explore the effect of Central Bank policy in a Classical world. I show that while the real wage (and so the profit rate) is unaffected by monetary factors (since by assumption it is exogenously fixed in the Classical system), the long run growth rate of the economy is controlled by the Central Bank. In this model, then, the Central Bank does not influence the distribution of income but does control the growth of employment of the labor force, the flow composition of output and the capital intensity of the economy.

The paper is organized as follows. Section 2, integrates a continuous-time, two-class, simple Sraffian production economy with an assets market. The production side is a two sector economy which produces capital (basic) goods and consumption (non-basic) goods using Sraffian technology. There are two assets: money, printed and distributed by the Central Bank, and the stock of capital. It is shown that the model is underdetermined. Sections 3 and 4, respectively, consider two labor market closures, full employment and an exogenously fixed real wage; and study the comparative statics of the models. The real side follows Sraffa (1960); the treatment of assets markets and portfolio choice draws heavily on the ideas of Tobin (1965, 1967) and Foley-Sidrauski (1970, 1971). In distinguishing models by different closures - in particular, by different assumptions on labor market equilibrium - the paper follows Marglin (1984A, 1984B). Section 5 concludes.

II. A Monetarv Sraffian Economy

II.A. The Production Economy

The model presented here is a simple version of a non-joint production economy studied by Sraffa (1960, chapters 1-5).

The economy produces two goods, consumption - nonbasic - goods, $C$, and investment - basic - goods, $I$. The goods are produced by profit maximising firms. There is no joint production, and the technology is unchanging and representable by the following constant returns to scale production functions:
\[ C = \min (Lc/a1, Kc/b1) \]
\[ I = \min (Li/a2, Ki/b2); \]

\[ \text{al, a2, bl, b2 are the technical coefficients. K and L denote capital and employed labor respectively. Capital is a basic good: a good used in the production of all other goods. It is the stockpile of the investment good. There is no depreciation of capital and perfect mobility of labor and capital between the two sectors. For expositional simplicity I focus on the case where consumption goods are more labor intensive than investment goods: } D = a1 b2 - a2 b1 > 0. \]

\[ \text{The case where } D < 0 - \text{where investment goods are more labor intensive than consumption goods} - \text{can be analyzed analogously (see fns. 8 and 10).} \]

From equation (1), the supply of the goods is given by

\[ C = (b2 L - a2 K)/D; \]
\[ I = (a1 K - b1 L)/D; \]

Equations (2) and (3) together imply that the capital-employed labor ratio of the economy, \( k = K/L \), is an increasing function of the rate of growth of the economy. Dividing through by \( K \) in (3) and rearranging yields:

\[ k(g) = b1/[a1 - gD]; \quad k'(g) > 0; \]

where \( g = I/K \) is the rate of growth of the economy.

Given the money wage rate, normal prices for each good are determined by the technical conditions and the rate of profit:

\[ p_c = a1 w + b1 r p_i \]
\[ p_i = a2 w + b2 r p_i \]

where \( r \) and \( w \) denote the profit rate and the money wage rate, respectively. Given the competitive tendency towards a uniform rate of profits and uniform rates of remuneration to labor, the relative price of capital goods depends only on the profit rate; and an
increase in the profit rate would increase the price of the good which employs capital more intensively - namely the investment good.

\[ p = P_i / P_C = a_2 / (a_1 - Dr) ; \quad p'(r) > 0. \]  

(7)

Take the consumption good to be the numeraire, \( P_C = 1 \), equations (5) and (6) can be solved for the real wage \( w / P_C \) as a function of the profit rate and the technology. The equation captures the standard negative relationship between the real wage and the profit rate:

\[ w / P_C = [1 - b_2 r] / [a_1 (1 - b_2 r) + a_2 b_1 r] \]
\[ = [1 - b_2 r] / [a_1 - rD]; \quad (w / P_C)'(r) < 0 \]  

(8)

II.B. Assets Markets

There are two social classes, workers and capitalists, and a Central Bank. All workers are alike, they own no property and must take service with those who can provide means of production for them to operate. Workers supply labor and spend all of their income on non-basics.

The Central Bank creates non-interest bearing Bank debt, which is called money, at a rate \( \theta = M / M \), where \( M \) is the total outstanding debt of the Central Bank (the nominal stock of money). The Central Bank deficit, \( \dot{M} \), takes the form of transfer payments to the capitalists, and it is assumed that only capitalists hold money (workers may be paid in money but spend it instantaneously on non-basics).

There are two assets in this economy: real money and the stock of capital. Capitalists consider the stock of real money and the stock of capital as their real wealth: \( W = pK + Mp_m \); where \( W \) is the stock of real wealth and \( p_m \) is the price of money in terms of the consumption good.

They hold the two assets in proportions that depend on their relative yield. The yield on capital is the profit rate, that on money
is the negative of the expected rate of change of prices. The higher the profit rate the greater the demand for capital and, therefore, the lower the demand for money; similarly with the rate of inflation. Equilibrium in the assets markets may, therefore, be compactly represented thus:

$$\beta(\pi^e, r) = \frac{M_s p_m}{pK}; \quad \beta_1 < 0; \quad \beta_2 < 0;$$  \hspace{1cm} (9)

where $\pi^e$ is the expected rate of inflation, i.e., the negative of the expected rate of change of the price of money: $\pi^e = -E(\frac{\dot{p}_m}{p_m})$. $\beta$ is the ratio of the demand for real money balances to the demand for capital; and the right-hand side of equation (9) is the ratio of the supply of money to that of capital.

Equation (9) captures the stock equilibrium in the assets markets. At each moment in time there is also a flow accumulation of assets by capitalists. Capitalists net disposable income, $Y_c$, is comprised of a) total payments to capital and b) the real value of Central Bank payments less the real value of its depreciation. Since the analysis focuses on comparative statics - studying different equilibria each with a constant $r$ and hence a constant $p$ - capital gains arising from changes in the relative price of capital goods are excluded.

$$Y_c = rpK + \theta M_s p_m - \pi^e M_s p_m$$  \hspace{1cm} (10)

Capitalists save a constant fraction $s_c$, $0 < s_c < 1$, of their net disposable income, $S = \dot{W}$, and savings plans are always realized; this implies, in particular, that investment is passive.

At any moment in time the flow accumulation of wealth by capitalists can be described as follows:

$$pK + (\dot{M}_{p_m}) = s_c rpK + s_c(\theta - \pi^e)M_{p_m}.\hspace{1cm} (11)$$
where $\dot{K}$ is the flow increase in the stock of capital. Collecting terms and dividing both sides by $pK$ yields the following expression for the rate of growth of the economy:

$$g = scr - (1-sc)(\theta - \pi^e)Mp_m/pK$$

(12)

II.C. Long Period Position (Steady State Equilibrium)

The underlying concept of Sraffa’s approach to value and distribution is that of ‘long period positions’: equilibria around which the economy is assumed to operate. The economy studied here is said to be in a long period position iff:

1) $\dot{K} = 0$: the capital to employed labor ratio is constant;
2) $\dot{\beta} = 0$: the ratio of real money balances to capital is constant;
3) $\pi_e = \pi$: the expected rate of inflation equals the actual rate;
4) $\dot{r} = 0$: the profit rate is constant.

In a long period position, the ratio of real money balances to capital is constant and the rate of inflation is simply the monetary growth rate less the growth rate of the economy:

$$\pi^e = \theta - g.$$  (13)

From equations (12)-(13) the behavior of the rate of accumulation in the economy in a long period position can be compactly summarized as follows:

$$g = scr - (1-sc)\beta(\theta - g, r)$$  (14)

The determination of prices and profits is separated from the determination of output. Once the profit rate (or the real wage rate) is given, this immediately determines the real wage (or profit rate) and relative prices. However, the flow supplies of goods are not determined by the profit rate; in particular, note that there is no ‘neoclassical’ inverse relation between the capital intensity and the
rate of profit. The capital intensity and the composition of output in the economy are determined by the rate of accumulation - the growth rate - of the economy.

However, the model developed thus far is underdetermined. I have not discussed labor market equilibrium and, as pointed out by Marglin (1984B), one of the key differences between the various approaches to economic growth concerns the treatment of the labor market: different labor market closures yield strikingly different models and results.

III. A Monetary Theory of Profits

I begin by assuming that the economy grows at the same rate as the labor force, $g = n$, and that $n$ is exogenously given. As noted above, this assumption means that the capital intensity and the composition of output are automatically fixed.

The comparative statics of this model are easily analyzed. The growth rate is assumed fixed at $n$, and equation (14) can, therefore, be written as follows:

$$n = s_c r - (1-s_c) n \beta (\theta - n, r) \quad (14')$$

Totally differentiating and collecting terms:

$$dr/d\theta = [n(1-s_c)\beta_1]/[s_c - n(1-s_c)\beta_2] < 0 \quad (15)$$

It follows that the profit rate is determined by the monetary growth rate: $r(\theta)$.

Since, from (8), $dw/dr < 0$, it follows that $dw/d\theta > 0$; and from (7), $dp/d\theta < 0$. Capital intensity is a function of the growth rate (equation (4)), and is, therefore, unaffected by changes in the profit rate*.
The intuition behind these results is straightforward. An increase in $\theta$ raises the inflation rate, thus increasing the opportunity cost of holding cash. This lowers the demand for real balances, hence lowers real disposable income (equation (10)) and real consumption falls. The growth rate of the economy remains unchanged (by assumption) which means that the profit rate must fall to reduce real savings and to induce capitalists to hold the supply of money (i.e. for portfolio balance to hold).

Figure 1 captures this result. The AA curve mirrors the system described by equation (14): the locus of $\langle r, g \rangle$ equating real savings to real wealth accumulation for any given value of $\theta$. It is upward sloping because an increase in the profit rate means that $g$ must rise to clear the saving-asset accumulation market and to maintain portfolio balance (from (14): $dr/dg > 0$). $g$ is assumed fixed at $n$. An increase in $\theta$ lowers the demand for real cash balances; it follows from equations (9) and (14) that for the savings-wealth accumulation market to clear, it must be that, for any given level of $g$, the profit rate is lower; the AA curve shifts down and to the right and since, $g$ is assumed fixed at $n$, the new equilibrium profit rate, $r^{**}$, must be lower.

The principal conclusion, then, is that the Central Bank controls the long run distribution of income and relative prices, and in this model, an increase in the monetary growth rate benefits workers by reducing the profit rate and thereby forcing up real wages. The consumption price of capital goods falls. The higher rate of monetary growth is, of course, inflationary; inflation rises by the same amount as the rate of monetary growth: $d\pi/d\theta = 1$. However, Central Bank policy does not affect accumulation or output in this regime with $g=n$.

IV. A Monetary Theory of Growth

The Classics assumed that an unlimited supply of labor was available at a conventionally and historically determined subsistence wage. The Classical model, then, assumes that the supply of labor is
a horizontal line at a given real wage: \( w = w^* \). This assumption implies that the profit rate too is an exogenously given parameter, as is the relative price of capital goods.

An increase in \( \theta \) lowers the demand for real cash balances, which lowers capitalists’ income and, therefore, their real consumption. So net real savings rises, which means that investment goes up. Thus, the growth rate of the economy must rise.

Algebraically, equation (14) can be written as follows:

\[
g = s_c \bar{r} - (1-sc) g\bar{\beta}(\theta - g, \bar{r}) \tag{14’}
\]

Totally differentiating and collecting terms:

\[
dg/d\theta = -[(1-s_c)g\beta_1] / [1 + (1-s_c)\beta - (1-s_c)g\beta_1] > 0. \tag{16}
\]

Figure 2 captures this result. The upward sloping AA curve represents the system described by equation (14) for a given level of \( \theta \). The Classical assumption of a fixed real wage rate means that the profit rate is fixed at \( r^* \). An increase in \( \theta \) shifts the curve down and to the right: an increase in \( \theta \) decreases the demand for real cash balances and increases savings; in order to maintain portfolio balance and to ensure that the savings-asset accumulation market clears, it must be that for any given profit rate the growth rate is higher. Thus, since \( r \) is assumed fixed, the new equilibrium growth rate, \( g^{**} \), must be higher.

It is easy to see (using equation (4)) that the long run capital intensity of the economy rises with an increase in \( g^{10} \). Furthermore, the composition of output changes: the flow supply of investment goods rises. Inflation increases by less than the increase in \( \theta \): \( 0 < d\pi/d\theta < 1 \).

The principal conclusion of this comparative statics experiment is that in the Classical model, the Central Bank can control the long run rate of accumulation of the economy through its control of the money supply. It, therefore, controls the composition of output as well as employment growth. Thus, even though the bank does not
influence the distribution of income, class struggle over the control of Central Bank policy remains important.

IV. Conclusion

Foley (1989) challenges advocates of the monetary theory of profits to produce a coherent account of this view. This paper is a response to this challenge.

Several striking conclusions emerge from this theory. In a regime, first, with \( g = n \), Central Bank policy has no effect on output and growth, but the Central Bank does control the distribution of income and relative prices. The political conflict over the control of Central Bank policy is thus a central pillar of the theory. This differs sharply from neoclassical orthodoxy, Keynesian orthodoxy and Marxian orthodoxy.\(^1\)

Were the political struggle over the long run distribution of income to be shifted to the labor market, i.e., to the determination of the real wage, then the Central Bank would, of course, be unable to affect either the distribution of income or relative prices. However, the struggle over the control of Central Bank policy remains important: in a Classical regime with exogenously given real wages, Central Bank policy determines the growth rate of the economy and hence the growth of employment.
FOOTNOTES


3. See Foley (1989) for an extensive discussion of these issues.

4. Sraffa’s approach allows a key role for banks in the creation and distribution of money: money supply is defined as the sum of state, central bank and bank money. He emphasizes the importance of institutional constraints on central bank money creation: ‘new money is created to serve the financial requirements of the state’ rather than to meet the ‘real needs of trade’. (Sraffa (1920, p.12), also quoted in Panic0 (1988)). He also points out the role played by the conflicts within the capitalist class in affecting monetary policy. See Panic0 (1988) for a discussion.

5. Rao (1991) demonstrates that this result holds in a one sector economy.

6. This interpretation follows Kurz (1985), who provides a detailed argument.

7. Marglin (1984B) convincingly argues that the other difference between the major schools of thought lie in their different visions of the savings process.

8. If investment goods are more labor intensive than consumption goods, $D < 0$, then it is easy to see that the principal conclusion-$r (0)$; $r'(\theta) < 0$- remains unchanged. However, it is clear, from equation (7), that $p'(r) < 0$. It follows then that $dp/d\theta < 0$.

9. Note the paradoxical nature of this result: workers benefit from increasing transfers to capitalists.
10. Alternatively, if $D < 0$, it follows from equation (4) that the capital-employed labor ratio of the economy is a decreasing function of the rate of growth of the economy: $k'(g) < 0$. It is then easy to see that $\frac{dk}{d\theta} < 0$: the long run capital intensity of the economy falls. The rest of the results remain unchanged.

FIGURE 1: A monetary theory of the profit rate. An increase in $\theta$ shifts the AA curve out and to the right, lowering the equilibrium profit rate.
FIGURE 2: A monetary theory of growth. In a Classical regime, an increase in $\theta$ shifts the AA curve out and to the right, raising the equilibrium growth rate.
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