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A Simplified "Benchmark" Stock-flow Consistent (SFC) Post-Keynesian Growth Model

by

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ABSTRACT

Despite being arguably one of the most active areas of research in heterodox macroeconomics, the study of the dynamic properties of stock-flow consistent (SFC) growth models of financially sophisticated economies is still in its early stages. This paper attempts to offer a contribution to this line of research by presenting a simplified Post-Keynesian SFC growth model with well-defined dynamic properties, and using it to shed light on the merits and limitations of the current heterodox SFC literature.

Keywords: Post-Keynesian Growth, Stock-flow Consistency, Real-financial Interactions

JEL Classifications: E12, E17, E44, E60

1. INTRODUCTION

In recent years, a significant number of "stock-flow consistent" (SFC) Post-Keynesian growth models and articles have appeared in the literature,[1] making it one of the most active areas of research in Post- Keynesian macroeconomics. Yet, it is fair to say that most of the discussion so far has been phrased in terms of relatively complex, and often exploratory, (computer-simulated) models and that this has prevented the dissemination of the main insights of this literature to broader audiences.[2] This paper attempts to ease this problem by presenting a simplified (and, we hope, representative) Post-Keynesian SFC growth model which, in our view, sheds considerable light on the merits and limitations of existing (and usually more complex) heterodox SFC models, and could conceivably be used as a "benchmark" to facilitate discussion among authors of these models and authors in various other Post-Keynesian and related traditions.

Most of the appeal of Post-Keynesian SFC models, as well as the difficulties associated with them, stem from two basic features of these constructs, i.e., the facts that: (i) they are, in a sense to be explained below, "intrinsically dynamic" (Turnovsky 1977); and (ii) they model financial markets and real-financial interactions explicitly. Therefore, the relative merits of the SFC literature are more easily appreciated in the context of the discussion of how Post-Keynesians have conceptualized dynamic trajectories of real economies in historical time and how these are affected by financial markets' behavior.

Beginning with the latter issue, we have noted elsewhere[3] that there is a widespread consensus among prominent Keynesians of all persuasions[4] about the role played by financial markets, notably stock and credit markets, in the determination of the demand price for capital goods (and, hence, of investment demand, via some version of "Tobin's q") and in the financing of investment decisions. The role played by banks in the financing of investment is acknowledged by Keynes, for example, in the famous passage in which he notes that "the investment market can become congested through a shortage of cash" (Keynes 1937). More emphatically, Minsky argues

[1] See Taylor (2004); Lavoie and Godley (2001–2002); Zezza and Dos Santos (2004); Foley and Taylor (2004); Dos Santos (2005) and (2006), among many others. The current literature builds on the seminal work of, among others, Tobin (1980) and (1982) and Godley and Cripps (1983). See Dos Santos (2006) for a detailed discussion of these authors' contributions and Dos Santos (2005) for a discussion of the related "Minskyan" literature of the 1980s–1990s. The seminal work of Moudud (1998) with SFC models in the tradition of classical economists is also worth mentioning. A recent major contribution has been provided by Godley and Lavoie (2007).

[2] A notable exception being the theoretical models in Taylor (2004).

[3] Dos Santos (2006).

[4] Such as Davidson (1972); Godley (1999); Minsky (1975); and Tobin (1982).

that investment theories which neglect the financing needs of investing firms amount to "palpable nonsense" (Minsky 1986).

This consensus is extensible to the idea that asset prices are determined by the portfolio decisions of the various economic agents, being only marginally affected—if at all—by current saving flows. In the words of Davidson, "in the real world, new issues and household savings are trifling elements in the securities markets (…). Any discrepancy between (…) [new issues] and (…) [the 'flow' demand for new securities] is likely to be swamped by the eddies of speculative movements by the whole body of wealth-holders who are constantly sifting and shifting their portfolio composition" (Davidson 1972).

In other words, most Post-Keynesians would agree that the size and the desired composition of the balance sheets of the various institutional sectors (i.e., households, firms, banks, and the government, in a closed economy) determine (short period) "equilibrium" asset prices which, in turn, crucially affect "real [macroeconomic] variables."

Few Post-Keynesians would also disagree that "Keynes's formal analysis dealt only with a period of time sufficiently brief (Marshall's short period of a few months to a year) for the changes taking place in productive capacity over that interval, as a result of net investment, to be negligible relative to the total inherited productive capacity" (Asimakopulos 1991). Accordingly, many Post-Keynesians have argued that extending Keynes's analysis to "the long period" involves "linking adjacent short periods, which have different productive capacities, and allowing for the interdependence of changes in the factors that determine the values of output and employment in the short period[s]" (Asimakopulos 1991).

Essentially the same view was espoused by Joan Robinson (1956) and by Michael Kalecki, in an often quoted passage in which he notes that "the long run trend is but a slowly changing component of a chain of short-period situations. It has no independent entity" (Kalecki 1971). Not all Post-Keynesians agree with it, though. Skott (1989), for example, criticizes this Asimakopulos-Kalecki-Robinson view on the grounds that, when coupled with the usual Keynesian assumption[5] that firms' short-period expectations are roughly correct, it implies—given constant animal spirits—that the economy is always in long-period equilibrium, as defined by Keynes in Chapter 5 of the *General Theory*. While this last point is certainly correct, we do not see it as a bad thing. In fact, we argue in Section 3 that a careful analysis of Keynes's long-period equilibrium is much more useful than conventional wisdom would make us believe.

[5] Keynes (1937).

It so happens that the careful modeling of stock-flow relations provides a natural and rigorous link between "adjacent short periods." In particular, it makes sure that the balance sheet implications of saving and investment flows and capital gains and losses in any given short period are fully taken into consideration by economic agents in the beginning of the next short period. This, in turn, is crucial in Post-Keynesian models, for if one assumes that asset prices are determined by the portfolio choices of the various economic agents, one must also acknowledge that dynamically miscalculated balance sheets would imply increasingly wrong conclusions about financial markets' behavior.

In sum, and despite its somewhat discouraging algebraic form, the broader goal of current SFC literature is very similar to the one stated by Davidson in the passage above.[6] In fact, most of the (simple, though admittedly tedious) algebra below is meant only to make sure we are getting the dynamics of the balance sheets right and, therefore, approaching Davidson's problem from a more explicitly dynamic standpoint.

The structural and behavioral hypotheses of our model are presented in Section 2 below, while Section 3 discusses (the meaning of) its short and long period equilibria. Section 4 briefly discusses how the model presented here relates to the broader heterodox SFC literature.

2. THE MODEL IN THE SHORT RUN

2.1 Structural Hypotheses and their Systemwide and Dynamic Implications

The economy assumed here has households, firms (which produce a single good, with price p), banks, and a government sector. The aggregated assets and liabilities of these institutional sectors are presented in Table 1 below.

[6] Though, in most cases, SFC models simplify Davidson's analysis by working with one sector models and merging commercial and investment banks in one large banking sector. See Davidson (1972).

Table 1. Aggregate Balance Sheets of the Institutional Sectors.					
	Households	**Firms**	**Banks**	**Gov't**	**Totals**
1 - Bank deposits	$+D$		$-D$		0
2 - Bank loans		$-L$	$+L$		0
3 – Gov't bills			$+B$	$-B$	0
4 - Capital goods		$+p \cdot K$			$+p \cdot K$
5 - Equities	$+pe \cdot E$	$-pe \cdot E$			0
Net worth	$+Vh$	$+Vf$	0	$-B$	$+p \cdot K$
Note: *pe* stands for the price of one equity					

Table 1 summarizes several theoretical assumptions. First, and for simplification purposes only, we assume a "pure credit economy," i.e., that all transactions are paid with bank checks. This hypothesis is used only to simplify the algebra and can easily be relaxed without changing the essence of the argument. It is important to notice, however, that the financial structure assumed above rules out financial disintermediation (and, therefore, systemic bank crises) by hypothesis. Therefore, allowing for cash holdings will be necessary in more realistic settings.

Households are assumed not to get bank loans and to keep their wealth only in the form of bank deposits and equities. The reason why households do not care to buy government bills is that banks are assumed to remunerate deposits at the same rate the government remunerates its bills.[7] Banks are also assumed to: (i) always accept government bills as means of payment for government deficits; (ii) not pay taxes; and (iii) to distribute all its profits, so its net worth is equal to zero.

We will thus be working with the conventional case in which the government is in debt ($B > 0$), noting that not too long ago—in the Clinton years, to be precise—analysts were discussing the consequences of the United States paying all its debt. A negative B, i.e., a positive government net worth, can be interpreted in this model as "net government advances" to banks. We are also simplifying away banks' and government's investment in fixed capital, as well as their intermediary consumption (wages, etc.). These assumptions are made only to allow for simpler mathematical expressions for household income and aggregate investment.

[7] So that lending to firms is banks' only source of profits. According to Stiglitz and Greenwald (2003), a banking sector with these characteristics "is not too different from what may emerge in the fairly near future in the USA." In any case, this hypothesis allows us to simplify the portfolio choice of households considerably. More detailed treatments, such as the ones in Tobin (1980) or Lavoie and Godley (2001–2002), can easily be introduced, though only at the cost of making the algebra considerably heavier.

Firms are assumed to finance their investment using loans, equity emission, and retained profits.[8] Finally, the Modigliani-Miller (1958) theorem does not hold in this economy, so the specific way firms choose (or find) to finance themselves matters. As it has been pointed out that, "the greater the ratio of equity to debt financing the greater the chance that the firm will be a hedge financing unit"' (Delli Gatti, Gallegati, and Minsky 1994). This "Minskyan" point is, of course, lost in a Modigliani-Miller world, as in models in which firms issue only one form of debt.

Table 2. "Current" Transactions in our Artificial Economy
A (+) sign before a variable denotes a receipt, while a (-) sign denotes a payment

	Households	Firms		Gov't	Banks	Totals
		Current	***Capital***			
1 - Consumption	$-C$	$+C$				0
2 – Gov't expenditure		$+G$		$-G$		0
3 - Investment in fixed K		$+p\cdot\Delta K$	$-p\cdot\Delta K$			0
4 - Accounting memo: "Final" sales at market prices = $p\cdot X \equiv C + G + p\cdot\Delta K \equiv W + FT \equiv Y$						
5 - Wages	$+W$	$-W$				0
6 - Taxes	$-Tw$	$-Tf$		$+T$		0
7 - Interest on loans		$-il_{t-1}\cdot L_{t-1}$			$+il_{t-1}\cdot L_{t-1}$	0
8 - Interest on bills				$-ib_{t-1}\cdot B_{t-1}$	$+ib_{t-1}\cdot B_{t-1}$	0
9 - Interest on deposits	$+ib_{t-1}\cdot D_{t-1}$				$-ib_{t-1}\cdot D_{t-1}$	0
10 - Dividends	$+Fd +Fb$	$-Fd$			$-Fb$	0
11 - Column totals	$SAVh$	Fu	$-p\cdot\Delta K$	$SAVg$	0	0

The "current flows" associated with the stocks above are described in Table 2. As such, it represents very intuitive phenomena. Households in virtually all capitalist economies receive income in form of wages, interest on deposits, and distributed profits of banks and firms and use it to buy consumption goods, pay taxes, and save, as depicted in the households' column of Table 2. We simplify away household debt and housing investment. The government, in turn, receives money from taxes and uses it to buy goods from firms and pay interest on its lagged stock of debt, while firms use sales receipts to pay wages, taxes, interest on their lagged stock of loans, and dividends, retaining the rest to help finance investment. Finally, banks receive money from their loans to firms and holdings of government bills and use it to pay interest on households' deposits and dividends.

[8] As in Godley and Lavoie (2007) and Skott (1989), for example.

In a “closed system” like ours, every money flow has to “come from somewhere and go somewhere” (Godley 1999), and this shows up in the fact that all row totals of Table 2 are zero. Note also that firms’ investment expenditures in physical capital imply a change in their financial or capital assets and, therefore, is a “capital” transaction. As such it (re)appears in Table 3 below. The reason it is included in Table 2 is to stress the idea that firms buy their capital goods from themselves—an obvious feature of the real world, though a slightly odd assumption in our “one good economy.”

Table 3. Flows of Funds in our Artificial Economy
Positive figures denote sources of funds, while negative ones denote uses of funds

	Households	**Firms**	**Gov’t**	**Banks**	**Totals**
1 - Current saving	$+SAVh$	$+Fu$	$+SAVg$	0	$+SAV$
2 - Δ Bank deposits	$-\Delta D$			$+\Delta D$	0
3 - Δ Loans		$-\Delta L$		$-\Delta L$	0
4 - Δ Gov’t bills			$-\Delta B$	$+\Delta B$	0
5 - Δ Capital		$-p\cdot\Delta K$			$-p\cdot\Delta K$
6 - Δ Equities	$-pe\cdot\Delta E$	$+pe\cdot\Delta E$			0
Totals	0	0	0	0	0
Net worth *(accounting memo)*	$Vh = SAVh + \Delta pe\cdot E_{t-1}$	$Vf = Fu + +\Delta p\cdot K_{t-1} -\Delta pe\cdot E_{t-1}$	$+SAVg$	0	$SAV +\Delta p\cdot K_{t-1} = +p\cdot\Delta K +\Delta p\cdot K_{t-1}$

While it is true that beginning of period stocks necessarily affect income flows, as depicted in Table 2, it is also true that saving flows and capital gains necessarily affect end of period stocks, which, in turn, will affect next period’s income flows. This “intrinsic SFC dynamics” is shown in Table 3. Note that fluctuations in the price of the single good produced in the economy (for firms) and in the market value of equities (for firms and households) are the only sources of nominal capital gains and losses in this economy.

Given the hypotheses above, households’ saving necessarily implies changes in their holdings of bank deposits and/or stocks, while government deficits are necessarily financed with the emission of government bills, and investment is necessarily financed by a combination of retained earnings, equity emissions, and bank loans. As emphasized by Godley (1999), banks play a crucial role in making sure these interrelated balance sheet changes are mutually consistent.[9]

We finish this section reminding the reader that all accounts presented so far were phrased in nominal terms. All stocks and flows in Tables 1 and 2 above have straightforward "real" counterparts, given by their nominal value divided by p (the price of the single good produced in the economy), while the "real" capital gains in equities are given by

$$(\Delta pe_t \cdot E_{t-1} - \Delta p_t \cdot pe_{t-1} \cdot E_{t-1} / p_{t-1}) / p_t \qquad (1)$$

and the "real" capital gains in any other financial asset Z are given by[10]

$$-\Delta p_t \cdot (Z_{t-1} / p_{t-1}) / p_t \qquad (2)$$

We believe that the artificial economy described above—though not necessarily its accounting details—is quite familiar to most macroeconomists in the broad Post-Keynesian tradition. In order to keep things simple, we will try as much as possible to "close" it with (dynamic versions of) equally familiar Keynes/Kalecki hypotheses. Of course, given that modeling "economies as a whole" from a financially sophisticated Post-Keynesian standpoint implies making a relatively large number of simplifying assumptions about both the behavior and the composition of the various relevant sectors of the economy, very few people will agree with everything in our model. We do hope, however, that a sufficient number of Post-Keynesians will deem it representative enough of their own views to deserve attention or, at least, will find it illuminating to phrase their dissenting views as alternative structural or behavioral hypotheses about the obviously simplified artifical economy discussed above. If this turns out to be the case, we will consider ourselves successful in our main goal of providing a "benchmark" model in order to facilitate discussion among economists of the various Post-Keynesian and related traditions.

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[9] As is well known, most macroeconomic models assume that some sort of Walrasian auctioneer takes care of financial intermediation. This simplification is not faithful to the views of financially sophisticated Post-Keynesians, such as Davidson (1972); Godley and Cripps (1983); Minsky (1986); or Godley and Lavoie (2007), though.

[10] Given that ours is a "one good" economy, the real value of physical capital is not affected by inflation.

2.2 A Horizontal Aggregate Supply Curve

Following Taylor (1991), we assume that

$$p_t = w_t \cdot \lambda_t \cdot (1+\tau) \tag{3}$$

where p = price level, w = money wage per unit of labor, λ = labor-output ratio, and τ = mark-up rate.[11] From (1) it is easy to prove that the (gross, before tax) profit share on total income (π) is given by:

$$\pi = \frac{p_t \cdot X_t - W_t}{p_t \cdot X_t} = \frac{\tau}{1+\tau} \tag{4}$$

so that the (before tax) wage share on total income is

$$1-\pi = \frac{W_t}{p_t \cdot X_t} = \frac{1}{1+\tau} \tag{5}$$

and

$$W_t = (1-\pi) \cdot p_t \cdot X_t \tag{6}$$

We assume here also that the nominal wage rate, the technology, and the income distribution of the economy are exogenous, so all lower case variables above are constant, and therefore the aggregate supply of the model is horizontal. In other words, we work here with a fix-price model in the sense of Hicks (1965). All these assumptions can be relaxed, of course, provided one is willing to pay the price of increased analytical complexity. In particular, they allow us to avoid unnecessary complications related to inflation accounting.

2.3 Aggregate Demand

2.3.1 A "Kaleckian SFC" Consumption Function

The simplifying hypothesis here is that wages after taxes are entirely spent, while "capitalist households"—receiving distributed profits from firms and banks—spend a fraction of their lagged wealth—as opposed to their current income, as in Kalecki.[12] The presence of household's

[11] A more complex model may incorporate the effects of the interest rate on prices, if financial markets are able to affect production decisions.

[12] We have analyzed elsewhere (Zezza and Dos Santos, 2006) the relationship between income distribution and growth in this class of models, and we chose to adopt a simple specification in the present version.

wealth in the consumption function is, of course, compatible with Modigliani's (1954) seminal work. Formally,

$$C_t = W_t - Tw_t + a \cdot Vh_{t-1} = W_t \cdot (1-\theta) + a \cdot Vh_{t-1} \tag{7}$$

where θ is the income tax rate and a is a fixed parameter. Following Taylor, we normalize the expression above by the (lagged) value of the stock of capital[13] to get

$$C_t / (p_t \cdot K_{t-1}) = (1-\pi) \cdot (1-\theta) \cdot u_t + a \cdot vh_{t-1} \tag{8}$$

where $u_t = X_t/K_{t-1}$, and $vh_t = Vh_t/(p_t \cdot K_t)$.[14] Needless to say, equation (8) is compatible with the conventional, simplified Keynesian short-period specification ($C_t = C_0 + c \cdot Yd_t$), provided one makes

$C_0 = a \cdot Vh_{t-1}$ and $c = 1 - \pi$.

2.3.2 A "Neo-Kaleckian" Investment Function

The simplest version of the model presented here uses Taylor's (1991) "structuralist" investment function which, in turn, is an extension of the one used by Marglin and Bhaduri (1990) and a special case of the one used in Lavoie and Godley (2001–2002). Given that investment functions are a topic of intense controversy in heterodox macroeconomics—see, for example Lavoie, Rodriguez, and Seccareccia (2004)—it would be interesting to study the implications of "Harrodian" (or "Classical") specifications in which investment demand gradually adjusts to stabilize capacity utilization—as proposed, among others, by Shaikh (1989) and Skott (1989). Section 4 discusses this issue in greater detail, though space considerations have forced us to

[13] Taylor (1991) uses the current stock of capital because he works in continuous time. As both the formalization and the checking—through computer simulations—of stock-flow consistency requirements are reasonably complex in continuous time, and no proportional insight appears to be added, we work here in discrete time and assume—as Keynes—that the stock of capital available in any given "short period" is predetermined, i.e., that investment does not translate into capital instantaneously. We thus normalize all flows by the opening stock of capital and stocks by the current stock of capital.

[14] Note that getting from (7) to (8) implies an inflation correction on *vh,* which is simplified away in the current model where prices are fixed.

postpone a complete treatment to another occasion. Our current specification follows the broad structuralist literature in assuming, for simplification purposes, that the output-capital ratio is a good measure of capacity utilization. In symbols, we have

$$g_t = g_0 + (\alpha \cdot \pi + \beta) \cdot u_t - \theta_1 \cdot il_t \tag{9}$$

where $g_t = \Delta K_t / K_{t-1}$, il is the (real) interest rate on loans, and g_0, α, β, and θ_1 are exogenous parameters measuring the state of long term expectations (g_0), the strength of the "accelerator" effect (α and β), and the sensibility of aggregate investment to increases in the interest rate on bank loans (θ_1). In Section 4.2 we discuss what happens when one modifies this investment function along the lines suggested by Lavoie and Godley (2001–2002).

2.3.3 The "u" Curve

Assuming that both $\gamma_t = G_t/(p_t \cdot K_{t-1})$ and il are given by policy, the "short period" goods' market equilibrium condition is given by

$$p_t \cdot X_t = W_t \cdot (1-\theta) + a \cdot Vh_{t-1} + [g_0 + (\alpha \cdot \pi + \beta) \cdot u_t - \theta_1 \cdot il_t + \gamma_t] \cdot p_t \cdot K_{t-1} \tag{10}$$

or, after trivial algebraic manipulations,

$$u_t = \psi_1 \cdot A(il)_t + \psi_1 \cdot a \cdot vh_{t-1} \tag{11}$$

where

$$\psi_1 = 1/[1-(1-\pi)\cdot(1-\theta)-\alpha_1] \tag{12}$$

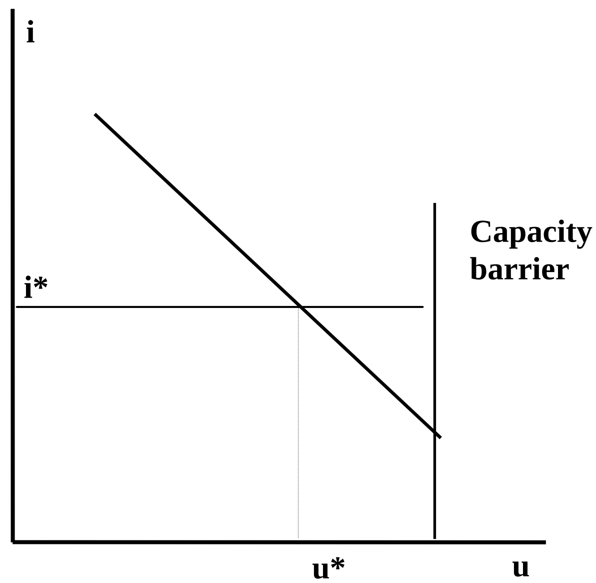
$$\alpha_1 = \alpha \cdot \pi + \beta \tag{13}$$

$$A(il)_t = g_0 - \theta_1 \cdot il_t + \gamma_t \tag{14}$$

Equation (11) is essentially the normalized "IS" curve of the model. Notice that, as for a textbook IS curve, the level of economic activity is determined by a multiplier ψ_1, times autonomous demand A, which here is given by the normalized government expenditure γ and autonomous growth in investment g_0, plus additional effects from the interest rate and the opening stock of wealth.

In fact, the "short period" equilibrium of the model, represented in Figure 1, has a straightforward "IS-LM" (of sorts) representation, which implies that "short period" comparative static exercises can be done quite simply. Note finally that, the (temporary, goods' market) equilibrium above only makes economic sense if the sum of the propensity to consume out of current income [i.e., $(1 - \pi)\cdot(1 - \theta)$] and the "accelerator" effect [i.e., $\alpha_1 = \alpha\cdot\pi + \beta$] is smaller than one. The (short period) consequences of having the income distribution impacting both the multiplier and the accelerator of the economy were examined in the classic paper by Marglin and Bhaduri (1990).

Figure 1. Short-Run Flow Equilibrium



2.4 Financial Markets

2.4.1 Financial Behavior of Households

The two crucial hypotheses here are that: (i) households make no expectation mistakes concerning the value of Vh, and (ii) the share δ of equity (and, of course, the share $1 - \delta$ of deposits) on total household wealth depends negatively (positively) on ib and positively (negatively) on the expectational parameter ρ.[15] Formally:

$$pe_t \cdot E_t^d = \delta \cdot Vh_t \qquad (15)$$

[15] Note that, as discussed in more detail in Section 3, the inclusion of expectation errors—say, along the lines of Godley (1999)—would imply the inclusion of hypotheses about how agents react to them, making the model "heavier."

$$D_t^d = (1-\delta) \cdot Vh_t \quad (16)$$

$$\delta = -ib + \rho \quad (17)$$

where ρ is assumed to be constant in this simplified "closure."[16] The simplified specification above follows Keynes in assuming that the demand for equities "… is established as the outcome of the mass psychology of a large number of ignorant individuals (..)" and, therefore, is "liable to change violently as the result of a sudden fluctuation in opinion due to factors that do not really much make difference to the prospective yield (…)" (Keynes 1936). The value of *Vh*, on the other hand, is given by the households' budget constraint (see Table 3 above):

$$Vh_t \equiv Vh_{t-1} + SAVh_t + \Delta pe_t \cdot E_{t-1} \quad (18)$$

while from Table 2 and equation (7) it is easy to see that

$$SAVh_t = +ib_{t-1} \cdot D_{t-1} + Fd_t + Fb_t - a \cdot Vh_{t-1} \quad (19)$$

so that

$$Vh_t = (1-a) \cdot Vh_{t-1} + ib_{t-1} \cdot D_{t-1} + Fd_t + Fb_t + \Delta pe_t \cdot E_{t-1} \quad (20)$$

2.4.2 Financial Behavior of Firms

For simplicity, we assume that firms keep a fixed *E/K* rate χ and distribute a fixed share μ of its (after-tax, net of interest payments) profits.[17] We thus have

$$E_t^s = \chi \cdot K_t = \chi \cdot K_{t-1} \cdot (1+g_t) \quad (21)$$

$$Fd_t = \mu \cdot [(1-\theta) \cdot \pi \cdot u_t \cdot p_t \cdot K_{t-1} - il_{t-1} \cdot L_{t-1}] \quad (22)$$

$$Fu_t = (1-\mu) \cdot [(1-\theta) \cdot \pi \cdot u_t \cdot p_t \cdot K_{t-1} - il_{t-1} \cdot L_{t-1}] \quad (23)$$

And, as the price of equity *pe* is supposed to clear the market, we have also that

$$E_t^d = E_t^s \quad (24)$$

[16] Though it plays a crucial role in Taylor and O'Connell's (1985) seminal "Minskyan" model.

[17] Varying χ and μ can be easily introduced, though only at the cost of making the algebra heavier. Note, however, that the hypothesis of a relatively constant χ is roughly in line with the influential New-Keynesian literature on "equity rationing." See Stiglitz and Greenwald (2003) for a quick survey.

so that from (15) and (21):

$$pe_t = \frac{\delta \cdot Vh_t}{\chi \cdot K_t} \tag{25}$$

Firms' demand for bank loans, in turn, can be obtained from their budget constraint (see Table 3). Indeed, from

$$\Delta L_t \equiv p_t \cdot \Delta K_t - pe_t \cdot \Delta E_t - Fu_t \tag{26}$$

it is easy to see that, by replacing equations (21) and (23) in (26):

$$L_t^d = [1+(1-\mu)\cdot il_{t-1}]\cdot L_{t-1} + [g_t \cdot p_t - pe_t \cdot g_t \cdot \chi - (1-\mu)\cdot(1-\theta)\cdot \pi \cdot u_t \cdot p_t]\cdot K_{t-1} \tag{27}$$

2.4.3 Financial Behavior of Banks and the Government

For simplicity, banks are assumed here—*a la* Lavoie-Godley (2001–2002) and Godley-Lavoie (2007)—to provide loans as demanded by firms. In fact, banks' behavior is essentially passive in the simplified model discussed here, for we also assume that: (i) banks always accept deposits from households and bills from the government; (ii) banks distribute whatever profits they make;[18] and (iii) the interest rate on loans is a fixed mark up on the interest rate on government bills. Formally:

$$L_t^s = L_t^d = L_t \tag{28}$$

$$D_t^s = D_t^d = D_t \tag{29}$$

$$B_t^s = B_t^d = B_t \tag{30}$$

$$il_t = (1+\tau_b)\cdot ib_t \tag{31}$$

$$Fb_t = il_{t-1}\cdot L_{t-1} + ib_{t-1}\cdot B_{t-1} - ib_{t-1}\cdot D_{t-1} \tag{32}$$

The government, in turn, is assumed to choose: (i) the interest rate on its bills (*ib*); (ii) its taxes (as a proportion θ of wages and gross profits); and (iii) its purchases of goods (as a

[18] Under this assumption, allowing banks to hold a fraction δ of its deposits in equities is one and the same thing of adding δ* to δ (hence our hypothesis that only households buy equities). Assuming that the banks' net worth can differ from zero would only make the algebra considerably more complex, however.

proportion γ of the opening stock of capital), while the supply of government bills is determined (as a residual) by its budget constraint:

$$G_t = \gamma_t \cdot p_t \cdot K_{t-1} \tag{33}$$

$$T_t = Tw_t + Tf_t = \theta \cdot W_t + \theta \cdot (p_t \cdot X_t - W_t) = \theta \cdot p_t \cdot X_t \tag{34}$$

$$B_t^s = (1 + ib_{t-1}) \cdot B_{t-1} + \gamma \cdot p_t \cdot K_{t-1} - \theta \cdot p_t \cdot X_t \tag{35}$$

3. COMPLETE "TEMPORARY" AND "STEADY STATE" SOLUTIONS

As noted above, the SFC approach allows for a natural integration of "short" and "long" periods. In particular, both Keynesian notions of "long-period equilibrium" and "long run" acquire a precise sense in a SFC context, the former being the steady-state equilibrium of the stock-flow system (assuming that all parameters remain constant through the adjustment process), and the latter being the more realist notion of a path-dependent sequence of "short periods," in which the parameters are subject to sudden and unpredictable changes. These concepts are discussed in more detail in Section 3.3 below. Before we do that, however, we need discuss the characteristics of the "short-period" (or "temporary") equilibrium of the model.

3.1 The "Short Period" Equilibrium

In any given (beginning of) period, the stocks of the economy are given, inherited from history. Under these hypotheses, and given distribution and policy parameters, we saw in Section 2.3.3 that the (normalized) level of economic activity—assuming that the economy is below full capacity utilization— is given by[19]

$$u_t = \psi_1 \cdot A(ib)_t + \psi_1 \cdot a \cdot vh_{t-1} \tag{11'}$$

But (demand-driven) economic activity is hardly the only variable determined in any given "short period." As noted above, the balance sheet implications of each period's sectoral income and expenditure flows and portfolio decisions are also (dynamically) crucial. Here the hypothesis that banks have zero net worth proves to be convenient, for it implies that the stock of bank loans (L) is determined by the stock of government debt (B) and to the stock of household wealth (Vh).

[19] We now use (31) into (14) to get autonomous demand relative to the interest rate on bills: $A(ib)_t = g_0 - \theta_1 \cdot (1 + \tau_b) \cdot ib_t + \gamma_t$.

Specifically, given that $L + B \equiv D$ (from Table 1) and $D = (1 - \delta)\cdot Vh$ from equations (16) and (29), we have that

$$L_t = (1-\delta)\cdot Vh_t - B_t \qquad (36)$$

It so happens that all other endogenous stocks and flows of the model are easily determined from u and (the normalized values of) B and Vh. Accordingly, the remains of this section will be spent computing the latter variables. The case of B is the simplest one. From equations (30) and (35) we have that

$$B_t = B_{t-1}\cdot(1+ib_{t-1}) + \gamma_t \cdot p_t \cdot K_{t-1} - \theta \cdot p_t \cdot X_t \qquad (37)$$

i.e., that the end-of-period government debt is given by beginning-of-period government debt plus the government's interest payments [$ib_{t-1}\cdot B_{t-1}$] and purchases of public goods [$\gamma_t \cdot p_t \cdot K_{t-1}$] minus its tax revenues [$\theta \cdot p_t \cdot X_t$]. Now, dividing the equation above by $\cdot p_t \cdot K_{t-1}$ and rearranging, we get

$$b_t = [b_{t-1}\cdot(1+ib_{t-1}) + \gamma_t - \theta\cdot u_t]/(1+g_t) \qquad (38)$$

where, $b_t = B_t/(p_t \cdot K_t)$.[20] In words, the *normalized* value of the government debt will increase (decrease) when the level of government debt increases faster (slower) than the value of the stock of capital.

To calculate the normalized value of the stock of households' wealth is a little trickier. We begin by noting that, from (18) above:

$$Vh_t \equiv Vh_{t-1} + SAVh_t + \Delta pe_t \cdot E_{t-1}$$

i.e., the nominal stock of household wealth in the end of the period is given by the sum of its value in the beginning of period, household saving in the period, and households' period capital gains in the stock market.

Now, note that equations (15), (21), (24), and (25) allow us to write

$$\Delta pe_t \cdot E_{t-1} = \delta \cdot Vh_t/(1+g_t) - \delta \cdot Vh_{t-1} \qquad (39)$$

and, replacing the expression above in (18), dividing everything by $p_t \cdot K_{t-1}$, and rearranging, we have that

[20] Equation (38) should include the real interest rate on bills. Since we assume inflation away in this version of the model, we keep the nominal interest rate *ib*.

$$vh_t = \frac{(1-\delta)\cdot vh_{t-1} + savh}{1+g_t-\delta} \tag{40}$$

where, $vh_t = Vh_t/(p_t \cdot K_t)$, and $savh_t = SAVh_t/(p_t \cdot K_{t-1})$.[21] Of course, vh increases whenever wealth grows faster than the value of the stock of capital and, as it turns out, this happens whenever the increase in nonequity household wealth [$(1-\delta)\cdot Vh_{t-1}$], represented by household saving [$SAVh$], is faster (slower) than the increase in the share of nonequity wealth in total household wealth [$1-\delta$] represented by the rate of growth of the capital stock g. This result has to do with the fact that increases in the rate of investment (g) also reduce the price of equities (for it increases their supply), creating relatively more capital losses the higher the proportion of total household wealth kept in equities(δ).

But we want an expression of vh in terms of b and u, not in terms of $savh$. In order to get one, recall equation (19):

$$SAVh_t = +ib_{t-1}\cdot D_{t-1} + Fd_t + Fb_t - a\cdot Vh_{t-1}$$

Now, replacing equations (16), (22), (29), and (32) in equation (19) and rearranging, we have that

$$SAVh_t = +il_{t-1}\cdot(1-\mu)\cdot L_{t-1} + ib_{t-1}\cdot B_{t-1} + \mu\cdot(1-\theta)\cdot\pi\cdot p_t\cdot X_t - a\cdot Vh_{t-1} \tag{41}$$

This result is intuitive. It says that—given our hypothesis that all after-tax wage income is spent—households' saving in a given period is given by banks' distributed profits and interest payments [$il_{t-1}\cdot L_{t-1} + ib_{t-1}\cdot B_{t-1}$], plus firms' distributed profits [$\mu\cdot[(1-\theta)\cdot\pi pt\cdot X_t - il_{t-1}\cdot L_{t-1}]$], minus the part of households' beginning of period wealth that is spent in consumption [$a\cdot Vh_{t-1}$].

Now, replacing (36) in the expression above one gets

$$\begin{aligned} SAVh_t = &+il_{t-1}\cdot(1-\mu)\cdot[(1-\delta)\cdot Vh_{t-1} - B_{t-1}] + \\ &+ ib_{t-1}\cdot B_{t-1} + \mu\cdot(1-\theta)\cdot\pi\cdot p_t\cdot X_t - a\cdot Vh_{t-1} \end{aligned} \tag{42}$$

or, equivalently, using (31):

$$\begin{aligned} SAVh_t = &Vh_{t-1}\cdot[ib_{t-1}\cdot(1+\tau_b)\cdot(1-\mu)\cdot(1-\delta) - a] + \\ &+[1-(1+\tau_b)\cdot(1-\mu)]\cdot ib_{t-1}\cdot B_{t-1} + \mu\cdot(1-\theta)\cdot\pi\cdot p_t\cdot X_t \end{aligned} \tag{43}$$

[21] The term vh_{t-1} should also be divided by 1 plus the rate of change in prices. Again we simplify this away.

i.e., households' saving in any given period can also be written as the sum of the dividends they receive from firms $\mu\cdot(1\text{-}\theta)\cdot p_t\cdot X_t\text{-}\mu\cdot il_{t\text{-}1}\cdot[(1\text{-}\delta)\cdot Vh_{t\text{-}1}\text{-}B_{t\text{-}1}]$, the interest income they receive (via banks) from the government $ib_{t\text{-}1}\cdot B_{t\text{-}1}$, and the interest income they would receive from banks if all their deposits were used to finance loans to firms $il_{t\text{-}1}\cdot(1\text{-}\delta)\cdot Vh_{t\text{-}1}$ minus the amount that is "lost'" due to the fact that part of this nonequity wealth is used to finance the government $ib_{t\text{-}1}\cdot B_{t\text{-}1}$, minus the part of households' beginning of period wealth which is spent in consumption $a\cdot Vh_{t\text{-}1}$. Our final *vh* equation (44) is then obtained dividing the expression above by $p_t\cdot K_{t\text{-}1}$ and replacing it in (40):

$$\begin{aligned} vh_t = \{[(1-\delta)\cdot(1+ib_{t-1}\cdot(1+\tau_b)\cdot(1-\mu)-a]\cdot vh_{t-1} + \\ +[1-(1+\tau_b)\cdot(1-\mu)]\cdot ib_{t-1}\cdot b_{t-1} + \mu\cdot(1-\theta)\cdot\pi\cdot u_t\}/(1+g_t-\delta) \end{aligned} \tag{44}$$

In sum, the intensive dynamics of the model above can be described, conditional on *ib*, by the following system of four equations determining the rate of growth in the stock of capital (9), government debt (38), households' wealth (44), and capacity utilization (11):

$$g_t = g_0 + (\alpha\cdot\pi+\beta)\cdot u_t - \theta_1\cdot il_t$$

$$b_t = [b_{t-1}\cdot(1+ib_{t-1}) + \gamma_t - \theta\cdot u_t]/(1+g_t)$$

$$\begin{aligned} vh_t = \{[(1-\delta)\cdot(1+ib_{t-1}\cdot(1+\tau_b)\cdot(1-\mu)-a]\cdot vh_{t-1} + \\ +[1-(1+\tau_b)\cdot(1-\mu)]\cdot ib_{t-1}\cdot b_{t-1} + \mu\cdot(1-\theta)\cdot\pi\cdot u_t\}/(1+g_t-\delta) \end{aligned}$$

$$u_t = \psi_1\cdot A(il)_t + \psi_1\cdot a\cdot vh_{t-1}$$

which determine other financial stocks in the economy, namely from (16):

$$d_t = (1-\delta)\cdot vh_t \tag{45}$$

where *d* is the stock of bank deposits normalized by the stock of capital; from (36):

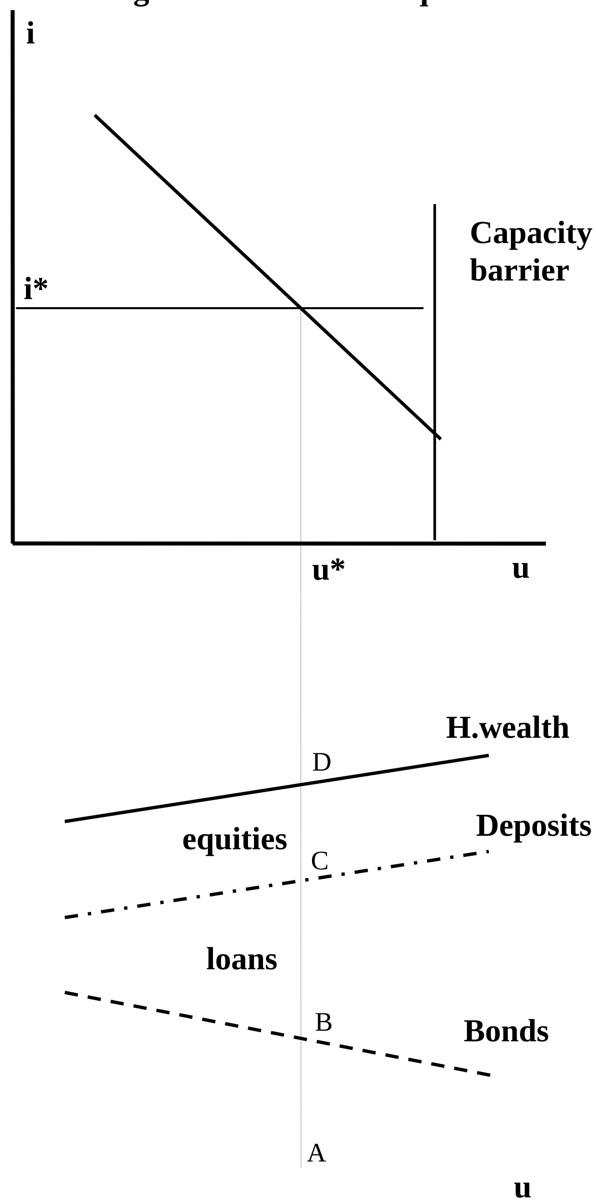
$$l_t = (1-\delta)\cdot vh_t - b_t \tag{46}$$

where *l* is the normalized stock of loans, and finally from (15):

$$e_t = \delta\cdot vh_t \tag{47}$$

where *e* is the normalized value of the stock of equities, eg $e = pe\cdot E/(p\cdot K)$.

Figure 2. Short-Run Equilibrium



The temporary equilibria of the system therefore has the clear-cut graphic representation in Figure 2, and, again, the (period) comparative statics exercises are straightforward. Note that the distance between the stock of wealth and the stock of deposits in the bottom part of Figure 2 (the CD segment) measures the (normalized) stock of equities which household holds for a given interest rate, while the distance between the stocks of deposits and the stock of bills (the BC segment) measures the stock of loans.

3.2 Model Properties

The positions of the curves in Figure 2 are determined by history and, therefore, change every period. We note, however, that the *vh* curve will be higher than the *b* curve in all relevant cases. Indeed, consolidating the balance sheets in Table 1 tells us that $b + 1 \equiv vh + vf$. Since the maximum (relevant) value of *vf* is 1 (assuming that both loans and the price of equity go to zero, and firms do not accumulate financial assets), it is easy to see that *vh* has to be bigger than *b*. The *d* curve will always be below the *vh* curve and above the *b* curve, in order to imply a nonnegative stock of loans, and its position will depend on *ib*.

The model admits a convenient recursive solution. Given *u* (which can be calculated directly from the initial stocks, monetary and fiscal policies, and distribution and other parameters), one can easily get *g*, *b*, and *vh* and, given these last two variables, one can then calculate *l*, *d*, and *vf*, and, therefore, Tobin's *q*, for $q \equiv 1 - vf$.

Since the links among variables are often formed by combinations of parameters, it is useful to summarize the sign of links, as in the rows of Table 4.

Table 4. Short-Run Links among Model Variables

	u_t	g_t	vh_t	vh_{t-1}	b_{t-1}
u_t		-	-	> 0	-
g_t	> 0		-	-	-
vh_t	> 0	< 0		> 0	?
b_t	< 0	< 0	-	-	> 0

Most results are straightforward while some require a few additional comments.

Since wealth *vh* depends positively on *u* but negatively on *g*, the overall impact on wealth of an increase in the utilization rate which increases the growth rate is ambiguous. The first derivative shows that *vh* will increase with *u* if

$$vh > \mu \cdot (1 - \theta) \cdot \pi / (\alpha \cdot \theta + \beta)$$

Therefore "small" values of wealth imply a positive link between the utilization rate and wealth itself, which is convenient for model stability: starting from small values for *vh*, a shock to the utilization rate cumulates through higher wealth, which feeds back into higher utilization rates and faster growth, but eventually, as wealth gets large enough, further increases in *u* will reduce *vh* stabilizing the model.

The graph in Figure 2 is drawn under the assumption that vh (and therefore d) depend positively on u.

The direct link between vh_t and vh_{t-1} is positive, unless households spend in each period a fraction of their wealth $a \cdot vh_{t-1}$ which exceeds their stock of deposits given by $(1-\delta) \cdot vh_{t-1}$, which is implausible. The overall impact of a change in vh_{t-1} on vh_t is ambiguous, as shown in Table 4, since it depends on the relative size of the effects operating through u (see above); vh_{t-1} (positive) and g (negative). From the discussion above, the size of the overall impact of the opening stock of wealth on the closing stock of wealth will decrease with the wealth level.

An increase in the opening stock of bills b_{t-1} will decrease the stock of wealth when the banks' mark-up τ_b is "high," e.g., when

$$\tau_b > \mu/(1-\mu)$$

Model stability requires that $\delta b_t/\delta b_{t-1} < 1$, which will be true when the growth rate g is smaller than the interest rate ib. Again, stability requires that $\delta vh_t/\delta vh_{t-1} < 1$, which cannot be ensured, but is likely to hold when the share of wealth spent on consumption is not too small relative to the size of household deposits.

More formally, model stability can be analyzed with the help of phase diagrams, reducing our system of four equations to a system in (the changes of) vh and b. Using (11) and (9) in (44) and (38) we get

$$\Delta vh_t = f_1(vh_{t-1}; b_{t-1}) \tag{48}$$

$$\Delta b_t = f_2(vh_{t-1}; b_{t-1}) \tag{49}$$

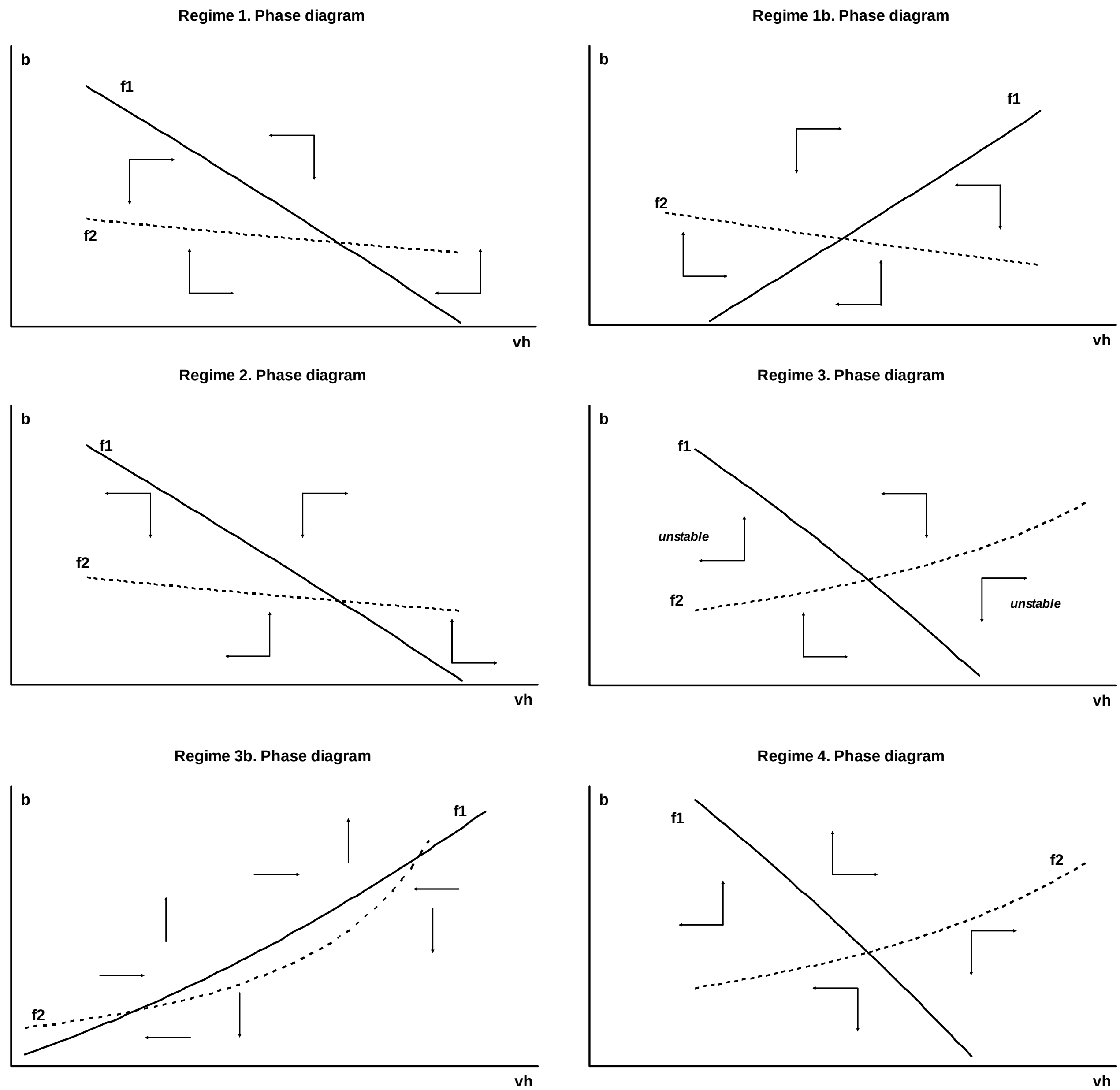
Inspection of the explicit functions in (48) and (49) reveals different regimes, which may be either stable or unstable. Noting that $\delta b_t/\delta vh_{t-1} < 0$ always holds, we have six different regimes summarized in Table 5.

Table 5. Possible Model Regimes

		$\delta b_t/\delta b_{t-1} < 0$ ib < g	$\delta b_t/\delta b_{t-1} > 0$ ib > g
"Small" reaction of wealth to its opening value	$\delta vh_t/\delta vh_{t-1} < 0$ $\delta vh_t/\delta b_{t-1} < 0$	**Regime 1** (stable)	**Regime 3** (multiple equilibria, potentially unstable)
	$\delta vh_t/\delta vh_{t-1} < 0$ $\delta vh_t/\delta b_{t-1} > 0$	**Regime 1b** (stable)	**Regime 3b** (multiple equilibria, potentially unstable)
"Large" reaction of wealth to its opening value	$\delta vh_t/\delta vh_{t-1} > 0$ $\delta vh_t/\delta b_{t-1} < 0$	**Regime 2** (unstable)	**Regime 4** (unstable)

The phase diagrams for all regimes are shown in Figure 3. We note that, although Regimes 2 and 4 are theoretically possible, we have not been able to generate them with any combination of parameters, since—as discussed above—the relationship between vh_t and vh_{t-1} is likely to be small and decreasing in *vh*. When the interest rate is small enough relative to the growth rate, Regimes 1 or 1b will apply, and we can obtain Regimes 3 or 3b for shocks which increase the interest rate, or decrease the growth rate in the economy.

Figure 3. Model Phase Diagrams under Alternative Regimes



Summing up, the model admits at least one solution for economically sensible values of parameters, and can produce multiple equilibria under Regime 3. In most cases the model will thus converge to a long-run equilibrium, and in the next section we will investigate the properties of such equilibrium.

3.3 The Long-Period Equilibrium of the Model and its Interpretation

3.3.1 The Long-Period Equilibrium

Define long-period equilibrium as the situation where our stock ratios *b*, *l*, *u*, and *vh* are constant for given interest rates. By virtue of (9), this will imply steady growth. Applying these conditions to equations (9), (11), (38), and (44) above, let us derive a system of two equations, either in *vh* and *u*, or in *vh* and *b*. Both solutions are, of course, equivalent, but their graphical representation, and their formal derivation, provide different insights which are worth exploring.[22]

Deriving equations in *vh* and *u* The first long-run equilibrium condition can be obtained directly from (11), solving for *vh*. In the long-run, there is a strictly positive relation between wealth and the utilization rate, which does not depend on the composition of wealth, but only on the effect of the interest rate on investment:

$$vh = [u / \psi_1 - A(ib)] / a \tag{50}$$

The second equilibrium condition is obtained by substituting *u* into *b*, and both into *vh:*

$$vh = \left[\frac{\psi_4 \cdot ib \cdot (\gamma - \theta \cdot u)}{\psi_2 + \alpha_1 \cdot u} + \alpha_2 \cdot u \right] \Big/ \left(\psi_3 + \alpha_1 \cdot u \right) \tag{51}$$

where α_1 is the accelerator from equation (13) and

$$\alpha_2 = \mu \cdot (1 - \theta) \cdot \pi \tag{52}$$

$$\psi_2 = g_0 - [1 + (1 + \tau_b) \cdot \theta_1] \cdot ib \tag{53}$$

$$\psi_3 = g_0 - [\theta_1 + (1 - \delta) \cdot (1 - \mu)] \cdot (1 + \tau_b) \cdot ib + a \tag{54}$$

$$\psi_4 = 1 - (1 + \tau_b) \cdot (1 - \mu) \tag{55}$$

The system of equations (50) and (51) yields a cubic expression in *vh*, which confirms the possibility of multiple equilibria discussed above. Under Regime 1, our numerical analysis under a wide choice of parameters has shown that only one solution implies economically meaningful values for all variables, while under Regime 3, more than one (economically meaningful) solutions are possible.

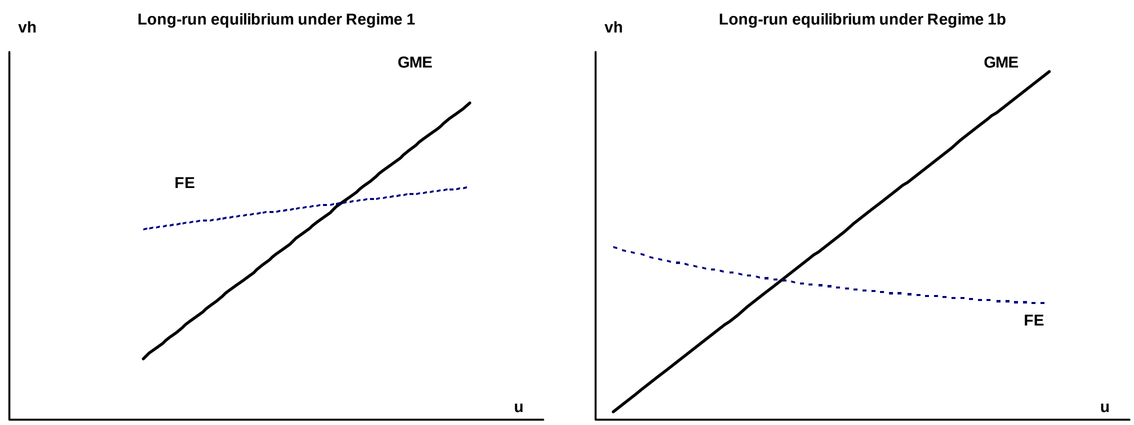
Our equation (50) has been derived directly from our equations for growth equilibrium in the goods market, which is influenced by total wealth *vh*, but does not depend on the composition

[22] See the Appendix for details on deriving the two sets of equilibrium conditions.

of wealth or by the levels of government debt or by the stock of loans. We therefore label this curve GME for Goods Markets Equilibrium, since it gives the combination of utilization rates *u* and wealth to capital ratio *vh* which imply steady growth for any distribution of wealth.

Since our second equation for long-run equilibrium has been derived, for given (equilibrium) values of *u* and *g*, through the equilibrium values for *b* and *vh*, we will label this curve FE for Financial Equilibrium. Equation (51) has a negative slope under Regime 1b, and a positive slope under Regime 1. Under Regime 3, the slope of the FE curve varies with *u*, yielding multiple equilibria.

Figure 4. Long-Run Equilibrium under Different (Stable) Regimes



Deriving equations in *vh* and *b* An alternative derivation of the long-run solution in the *b*, *vh* space is also of interest. Applying the steady-state conditions to equations (38) and (44) above gives us the following two new long-period equilibrium conditions:

$$(ib \cdot b + \gamma - \theta \cdot u) / b = g \tag{56}$$

$$[ib \cdot (1+\tau_b) \cdot (1-\delta) \cdot (1-\mu) - a] \cdot vh + [1-(1+\tau_b) \cdot (1-\mu)] \cdot ib \cdot b + \mu \cdot (1-\theta) \cdot \pi \cdot u = g \tag{57}$$

The equations above simply state that steady-growth is only possible when the rate of growth of both government debt (implied by the government deficit) and household wealth (implied by household saving) are equal to the rate of growth of the capital stock *g*.[23]

That is to say, the long period equilibrium is such that

$$-SAVg_t / B_{t-1} = SAVh_t / Vh_{t-1} = \Delta K_t / K_{t-1}$$

Now note that replacing (9) in (56) and (57) gives us the following conditions, which are an explicit form of the equations analyzed earlier for phase diagrams:

$$b = (\zeta_1 - \zeta_2 \cdot vh)/(\zeta_3 + \zeta_4 \cdot vh) \tag{58}$$

$$b = (\zeta_4 / \zeta_7) \cdot vh^2 + (\zeta_5 / \zeta_7) \cdot vh + (\zeta_6 / \zeta_7) \tag{59}$$

where

$$\zeta_1 = \gamma - \theta \cdot A \cdot \psi_1 \tag{60}$$

$$\zeta_2 = \theta \cdot a \cdot \psi_1 \tag{61}$$

$$\zeta_3 = A \cdot (1 + \alpha_1 \cdot \psi_1) - \gamma - ib \tag{62}$$

$$\zeta_4 = \alpha_1 \cdot a \cdot \psi_1 \tag{63}$$

$$\zeta_5 = \psi_0 \cdot \psi_1 \cdot A - ib \cdot (1 + \tau_b) \cdot (1 - \delta) \cdot (1 - \mu) + a \cdot [1 - \psi_1 \cdot (1 - \theta) \cdot \mu \cdot \pi] - \gamma \tag{64}$$

$$\zeta_6 = (1 - \theta) \cdot \mu \cdot \pi \cdot \psi_1 \cdot A \tag{65}$$

$$\zeta_7 = ib \cdot [1 - (1 - \mu) \cdot (1 + \tau_b)] \tag{66}$$

where *A* is the normalized autonomous expenditure defined in (14).

As one would expect, the long-period steady-state of the model above is the point in which both equilibrium conditions are simultaneously valid, i.e., in which all stocks and flows are growing at the rate of growth of the capital stock, *g*. Note that increases in *vh* lead to increases in *u* and, therefore, on *g*, that is to say, higher values of *vh* imply higher equilibrium rates of growth. Note also that multiplying *vh* and *b* by *(1+g)/u* gives us the stocks of household wealth and government debt relative to total product, which are often used in policy discussions.

[23] Note that the fact that $pe = \delta \cdot Vh/(\chi \cdot K)$, from equation (25) above, implies that the price of equity will be constant in steady growth equilibrium.

A great part of the complexities associated with the system above stems from its first equation (58). To see why this is the case, note first that a positive (long-period equilibrium) b is only possible when both the equation's numerator ζ_1-$\zeta_2 \cdot vh$, which is a proxy of the government's normalized "primary deficit," and its denominator $\zeta_3+\zeta_4 \cdot vh$, which is the difference between the rate of growth of the stock of capital and the interest rate on government debt, have equal signs. Indeed, if, say, the numerator is negative (i.e., if there is a primary surplus) while the denominator is positive (i.e., the growth rate of the stock of capital is larger than the interest rate on government debt), then it is intuitively clear that the government debt/capital value ratio must be continuously declining (Regime 1). If, on the other hand, the numerator is positive (i.e., if there is a primary deficit) and the denominator is negative (i.e., the growth rate of the stock of capital is smaller than the interest rate on government debt), then the contrary is true, i.e., the government debt/capital value ratio is continuously increasing (Regime 3).

It is, then, intuitively clear that a constant government debt/capital value ratio requires either the combination of primary deficits and an interest rate on government debt smaller than the rate of growth of the capital stock (a situation similar to that of, say, the United States in 2004) or a combination of primary surpluses and an interest rate on government debt larger than the rate of growth of the stock of capital (a situation similar to that of, say, Brazil in 2004). These two regimes imply radically different slopes for the equation above, as discussed earlier. An increase in vh in the "American" regime, for instance, decreases the primary deficit and increases the rate of growth in the stock of capital, therefore implying a smaller equilibrium b, so that the positive difference between $b \cdot g$ (the deficit required to keep b constant) and $b \cdot ib$ (the interest payments on public debt) is reduced to compensate for the smaller primary deficit. The contrary is true in the "Brazilian" regime, because an increase in vh implies an increase in the primary surplus, therefore implying a larger b so that the negative difference $b \cdot g$ and $b \cdot i$ is increased to compensate for the higher primary surplus.

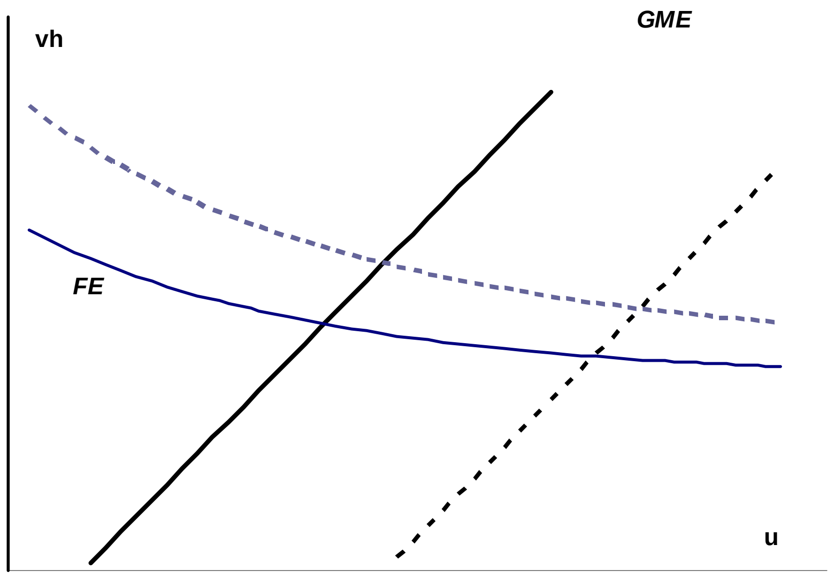
Note also that the denominator of equation (58) cannot be zero, i.e., the interest rate on the government debt cannot be equal to the rate of growth in the stock of capital (or, algebraically, $vh \neq -\zeta_3/\zeta_4$). Indeed, in such a situation, a constant government debt/value of capital ratio is incompatible with a nonzero primary deficit (which forces b to infinite or minus infinite, depending on the case, whenever vh approaches the critical value). We can, therefore, get a better understanding of the different regimes analyzed earlier: there is one in which $\zeta_3 > 0$ (so that the "critical" value of vh is negative, for ζ_4 is always positive) and one in which $\zeta_3 < 0$ (so that the "critical" value of vh is positive).

The second equation (59) in the system is slightly easier to understand. The key point here is that, as emphasized by Minsky (1982), government deficits help corporations to reduce their debt. This is easy to see in the balance sheet of banks: given Vh (and, therefore, D), higher values of B imply smaller values of L. This, in turn, has two effects. First, banks' profits—and therefore, (capitalist) households' disposable income and saving—are reduced. Second, firms' debt service burden is reduced, leading to higher dividend payments to households (given firms' higher profits net of interest payments) and, therefore, higher (capitalist) households' disposable income and saving. The overall result of these opposing effects is summarized in ζ_7. Given that ζ_4 is unambiguously positive, a positive ζ_7 implies that the equation will have a positive slope, i.e., that a higher value of b will lead to higher values of vh. The contrary is true, of course, when ζ_7 is negative.

3.3.2 Model Long Run Properties

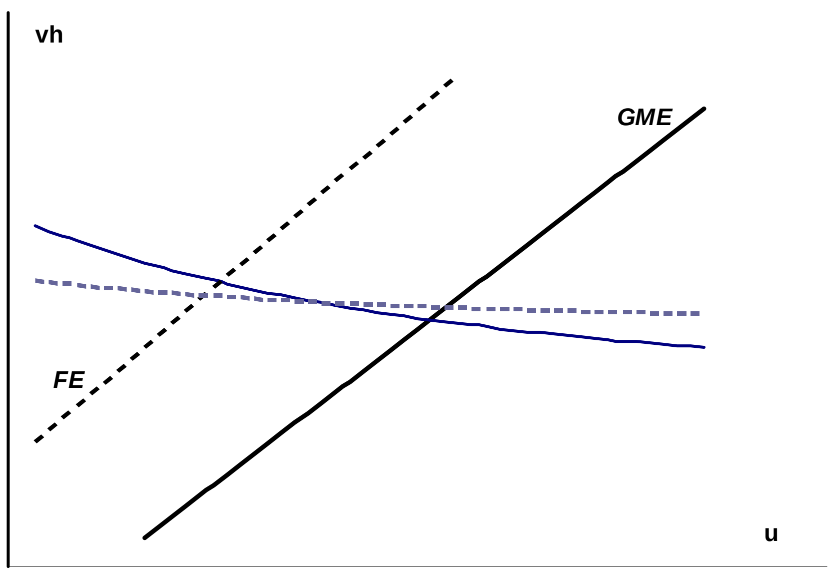
The equilibrium conditions derived above can be used to assess the response of the model to shocks. For instance, a standard Keynesian shock to government expenditure γ, and hence to government deficit, has an impact on growth even in the long run, as shown in Figure 5, where the dashed curves represent the equilibrium conditions after the shock. The major effects operate through the multiplier-accelerator effect, i.e., through a shift to the right of the ***GME*** curve, with a final increase in growth, the utilization rate, and the stock of households' wealth. A second-order effect in the same direction is obtained through a change in the composition of wealth, i.e., an upward movement of the ***FE*** curve, originated by the now higher level of government debt b.

Figure 5. A shock to govt. expenditure (Regime 1b)



Models that neglect the impact of a shift in the composition of wealth, and thus calculate the results of the shock for a given *vh*, will thus underestimate the impact on growth of an expansionary fiscal policy.

Figure 6. A shock to wages (Regime 1b)



A second exercise illustrates the effects of a change in the distribution of income towards wages, obtained by increasing the share of profits on income π. Results are reported in Figure 6. It is interesting to note in this case that, although the increase in consumption generates an increase in the utilization rate, which stimulates growth through the accelerator effect, the drop in the value of the accelerator outbalances the former effect, so that the model turns out to be profit driven for our choice of parameters.

Figure 7. A shock to the interest rate (Regime 1b)

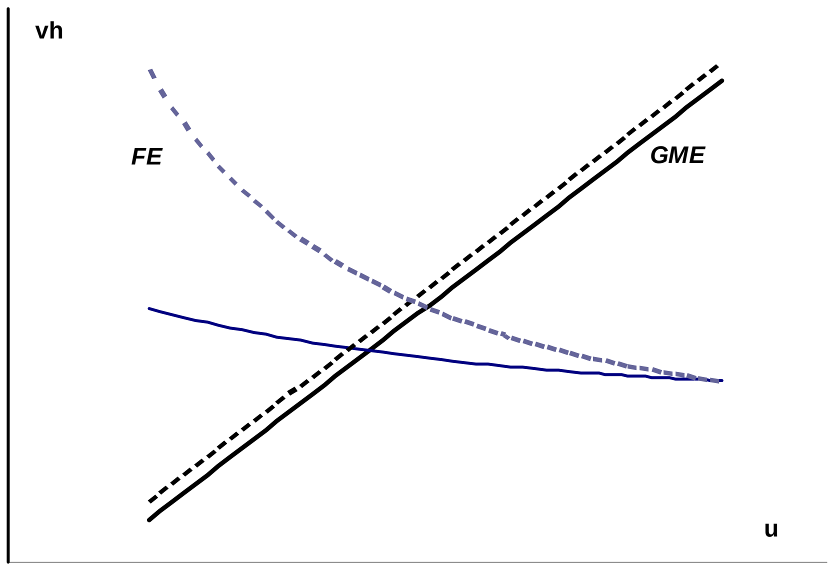
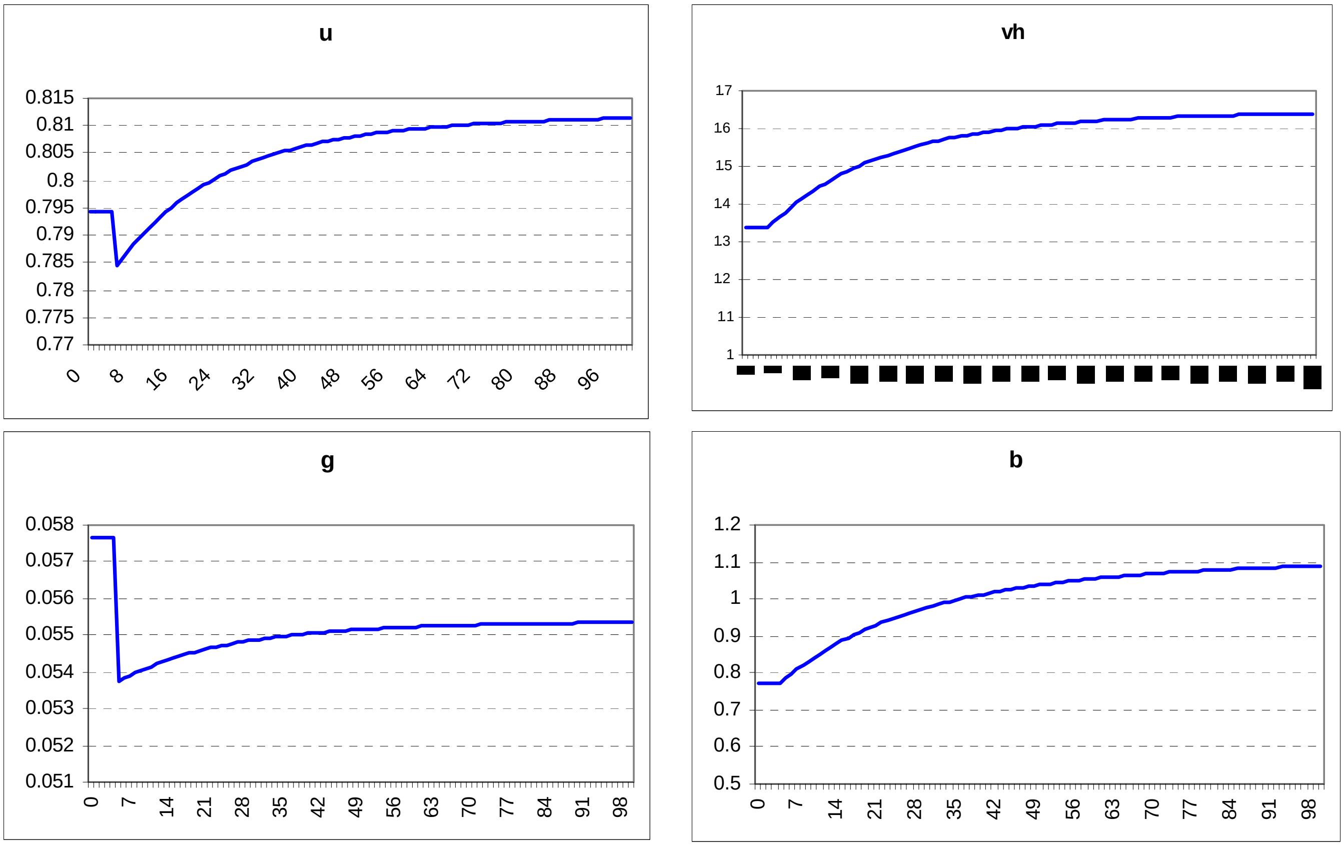


Figure 8. Dynamic Response of Model Variables to a Shock to the Rate of Interest (Regime 1b)



We finally analyze the effects of an increase in the interest rate on government bills *ib*, which implies a higher interest rate on loans. In this experiment, the shocked solution is such that Regime 1 will still hold, e.g., *ib* < *g*. It is interesting to note that the negative impact on growth coming from the increase in the cost of borrowing is partially balanced by the increase in household income and wealth arising from interest payments, so that the utilization rate turns out to be higher in the new long-run growth path, although the increase in *u* is insufficient to balance the impact of the interest rate on growth, which is lower than in the previous growth path. In Figure 8 we show the dynamic response of model variables to the shock.

Figure 9. A shock to the interest rate (switch to Regime 3)

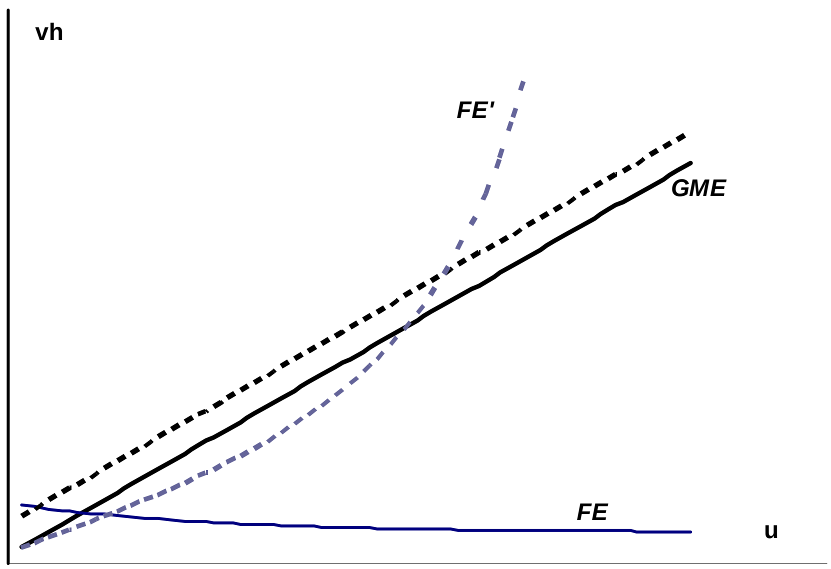
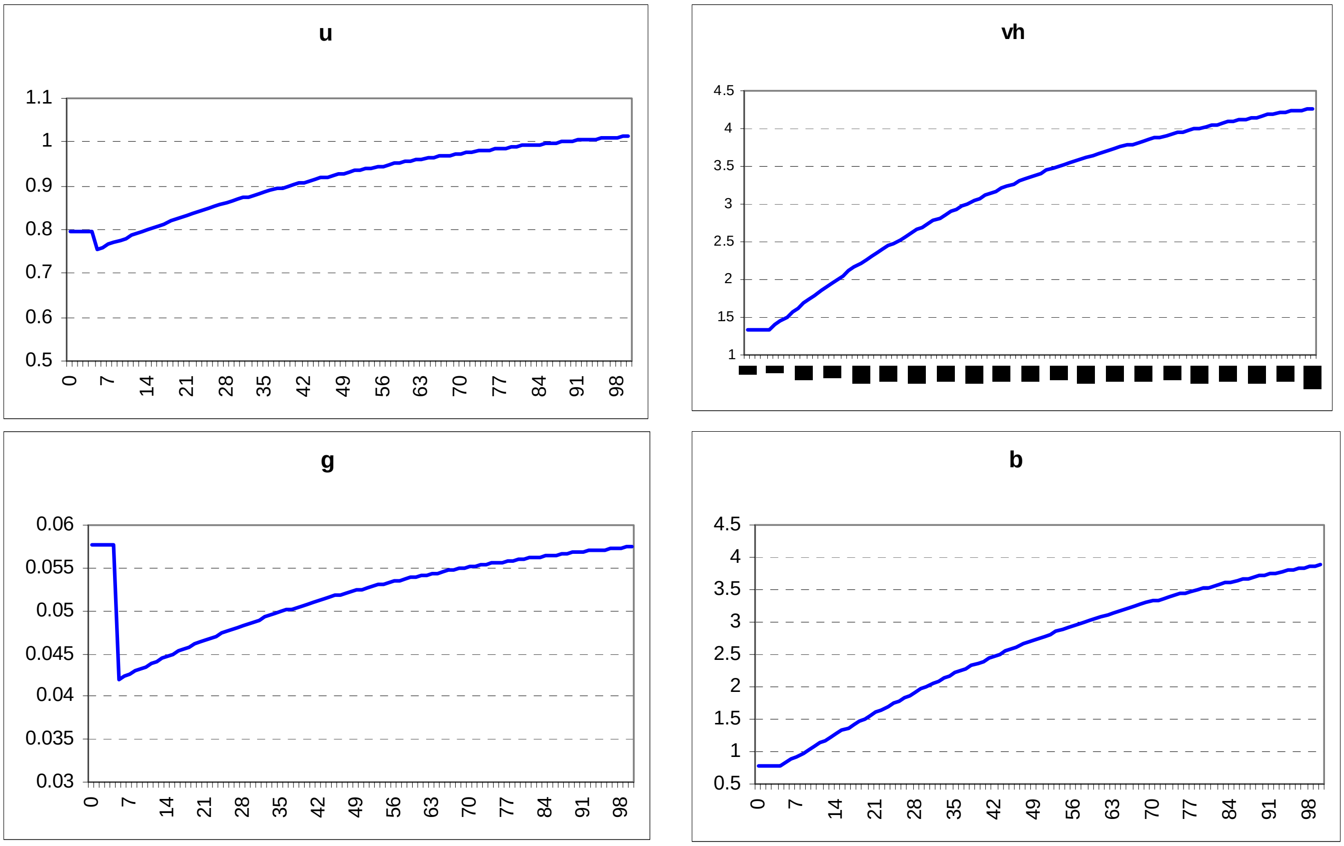


Figure 10. Dynamic Response of Model Variables to a Shock to the Rate of Interest (Regime 3)



If the size of the shock to the interest rate is such to generate a switch from the stable Regime 1 to the (potentially unstable) Regime 3, the result will be as in Figure 9. Note that the slope of the FE curve is now positive, as in Regime 3. It is interesting to note, from Figure 10, that the dynamic response of model variables to the shock is not explosive, but the rate of convergence is very slow. This implies that an unstable situation where government debt is constantly increasing relative to output may last for a very long period, and will eventually generate a financial crisis, e.g., a change in some other parameters in the economy. In our numerical simulations of the shock in Figures 9 and 10, at one point the stock of loans becomes negative, so that the final steady-state equilibrium, although feasible algebrically, would never be reached by a real economy.

3.4 Why Bother with the Long Period Equilibrium? Some Methodological Considerations

The model presented above uses "a Keynes/Marshallian short period with satisfied short-period expectations as the basic unit of analysis" (Skott 1989). Skott (1989) objects to this procedure, correctly noting that "if animal spirits stay constant [as they do in the derivations above, for we assume that both g_0 and δ are constant] and short term expectations are always fulfilled [as it is also the case in the model above], then the economy must evolve a time path which is consistent with the initial long-period expectations; effectively, the economy must be in long-run equilibrium." This is seen as a problem because Skott, like many Post-Keynesians, does not believe that Keynes's long period equilibrium is a particularly useful concept. For example, Asimakopulos argues that "[the concept of long-period equilibrium] cannot have a significant role on a theory that embodies Keynes's vision of the volatile nature of capitalist economies" (Asimakopulos 1991). In fact, he notes that even the "…notion of short-period equilibrium may be of limited use (…) [for] disappointed short-term expectations—with direct effects on production and employment decisions and cumulative effects on long term expectations and investment—would appear to be among the most important factors driving the economy" (Asimakopulos 1991). In light of these arguments, Skott concludes that "the possibility of disappointed expectations must be allowed for in an extension of Keynesian theory to cover long run developments" (Skott 1989).

We do not disagree with this general point. We note, however, that it implies neither that the long-period equilibrium is useless concept nor that it is necessarily useful to try to do justice to "the possibility of disappointed expectations" in formal models such as the one above.

Beginning with this latter point, we note that it is awkward to assume that some agents' expectations can be disappointed while other agents' expectations are always "right." In the context of the (simple) model above, this implies assuming that firms' demand expectations could be wrong, so that inventories would need to be explicitly taken into consideration—unless one is willing to assume that prices adjust to guarantee that markets always clear (Skott 1989). The same could also happen to firms' (and households') expectations about stock market outcomes (and, therefore, firms' total debt burden and households' total wealth), firms' (and households' and banks') expectations concerning government behavior (and, therefore, interest and tax rates), households' expectations concerning labor demand (and, therefore, wage income), banks' expectations of total deposits supply and loans demand, and so on. Moreover, assuming disappointed expectations in the context of a formal model implies having to say something also about how agents (form expectations and) react to these disappointments, and we know no developed Post-Keynesian theory of agents' "reaction functions."[24] In sum, trying to do justice to disappointed expectations in the context of formal models of "complete" monetary economies implies working with very complex constructs with a large number of variables (and reaction functions), many of which are without clear empirical counterparts (including previous periods' expectation errors, and the parameters of the expectation formation and reaction functions assumed).[25] As a consequence, implementing Skott's (and, for that matter, Godley's) "ideal" approach is extremely difficult at best, and ultimately unfeasible at worst. Such a pessimistic view was articulated by Asimakopulos—in a slightly different context—as follows: "[Keynes] recognizes that allowances must be made for the interactions among the independent variables [in the sense of Chapter 18 of the *General Theory*] of his analysis. Changes in one variable can lead to changes in other variables, and the full effects of any initial change depend on these interactions. The complexity of these interrelations means that the analysis of changes over time cannot be adequately handled by mathematical equations" (Asimakopulos 1991).

Turning now our attention to the former point, we note that Keynes's long-period equilibrium as interpreted above is nothing more than a useful ceteris paribus view of where the economy is (or, at least, could be) heading at any given point in time. To be sure, parameters are bound to change continuously and there is no reason to believe the economy will, in fact, remain in any long-period equilibrium trajectory. Still, we believe that the analysis of the properties of

[24] Though the efforts of Backus et al. (1980); Skott (1989); Moudud (1998); and Godley (1996) and (1999), among others, are worth mentioning.

[25] See, for example, the seminal efforts by Godley (1996) and (1999).

the "long-period equilibrium" so defined has important normative implications, for it allows one to study the characteristics of internally consistent dynamical trajectories—pioneered, as far as we know, by Mrs. Robinson and her various ages—and sheds considerable light on what will have to happen in true historical time. For example, if "long-period analysis" shows that the economy is heading to a situation in which debt-income ratios will be very high—or even explode, if the model turns out to be unstable in this way—one knows for sure that sooner or later the parameters of the system will have to change so as to prevent this outcome. If, on the other hand, the economy is close to a virtuous long-period path, one might suspect that abrupt parametric changes might have disruptive dynamical implications—so that policy makers, and the society as a whole, can debate whether or not to try to counterbalance them. In other words, rather than being ahistorical, long-period equilibrium analysis (and, in particular, the study of a sequence of ever changing long-period equilibria and their stability properties) as described above should help the construction of convincing historical narratives about sequences of short periods with continuous (but not directly modeled) parametric changes. While this may strike some macroeconomists as too modest a goal, it surely has the advantage of being a feasible one. In the same spirit, Godley and Cripps wrote:

> "we do not ask the reader to believe that the way economies work can be discovered by deductive reasoning. We take the contrary view. The evolution of whole economies is a highly contingent historical process. We do *not* believe it is possible to establish precise behavioural relationships (…) by techniques of statistical inference. Few laws of economics will hold good across decades and or between countries. On the other hand, we must exploit logic so far as we possibly can. Every purchase implies a sale: every money flow comes from somewhere and goes somewhere: only certain configurations of transactions are mutually compatible [or sustainable]. The aim here is to show how logic can help to organize information in a way that enables us to learn as much from it as possible. That is what we mean by macroeconomic theory (…)"
>
> Godley and Cripps 1983, emphasis in the original

4. HOW DOES THE MODEL ABOVE RELATE TO THE HETERODOX SFC LITERATURE?

As noted above, many heterodox SFC papers have appeared in the last years, and a major contribution by Godley and Lavoie (2007) has just been published. This increasingly large and diverse literature has tried to do several things, including: (i) checking the logical consistency of "incomplete" models;[26] (ii) extending the approach to deal with open economy issues;[27] (iii) discussing the theoretical compatibility of SFC models with the views of important authors who phrased their views in literary form;[28] (iv) producing applied models which can be used to study actual economies;[29] and (v) exploring the properties of models with different financial architectures and supply specifications.[30] Space considerations force us to focus here only on how the model above compares to the one proposed by Lavoie and Godley (2001–2002) and Godley and Lavoie (2007), which are particularly close to ours in spirit. Before we do that, however, we must say a few words on what seems to be the most controversial issue in the current heterodox debate on macrodynamics,[31] i.e., whether or not one should assume that the economy tends to some sort of "normal capacity utilization" in the long run.

4.1 Harrod versus Kalecki

Much has been written for and against the specific investment function used in the model above.[32] Those who criticize it (mostly economists working in the classical tradition of Ricardo and Marx) point out that the long-period equilibrium is a position in which firms' capacity utilization is consistent with firms' expected profitability and there is nothing in equation (9) that ensures that this is the case. Alternatively, they prefer to assume, *a la* Harrod, a given (in the sense of being static or determined by fixed parameters) optimum capacity utilization level u and to impose as a necessary condition for the long-period equilibrium either that $u = u^*$ or that u fluctuates around u^*. Those who support the specification we used above, in turn, argue, *a la* Kalecki, that firms are comfortable with a relatively wide range of capacity utilization levels (as depicted above).

[26] E.g., Taylor (2004); Dos Santos (2005).
[27] E.g., Godley and Lavoie (2003); Lequain (2003); Taylor (2004).
[28] E.g., Dos Santos (2006); Moudud (1998).
[29] E.g., Foley and Taylor (2004); Godley (1999).
[30] E.g., Godley (1996) and (1999); Kim (2005); Lavoie and Godley (2001–2002); Godley and Lavoie (2007); Moudud (1998).
[31] E.g., Lavoie et al. (2004); Moudud (1998).
[32] See Lavoie et al. (2004) for a nice survey of the arguments.

We believe it is illuminating to see this debate—as pointed out to one of us by Robert Blecker in an informal conversation—as a controversy about the “size of the comfortable range.” On one hand, classical economists do understand that issues such as firm heterogeneity, aggregation problems, and barriers to entry competition cast considerable doubt on the existence of one single and magical optimum aggregate capacity utilization figure. On the other hand, Post-Keynesians do understand that capacity utilization cannot be anything in the long period. The point, then, is whether or not this comfortable range is better described as a point (as it would be the case if it is really narrow) or as a relatively wide range (as depicted above). We have no a priori reason to believe either one is the case.

We do believe that the heat of the debate will decrease in time, after more empirical evidence becomes available and the broad messages of each type of model become clearer. In fact, we see this paper as an attempt at clarification of the Post-Keynesian/Kaleckian model. On this respect, we point out that, if we were to assume a fixed utilization rate $ u* $ in the long run, then growth in the stock of capital would be uniquely given by (9), implying a unique equilibrium value for *vh* and all other variables in the model. The model we have deployed would, in this case only, show the trajectory of the economy towards its long-run, unique equilibrium, and any shock other than to parameters in equation (9) would have only temporary effects.

4.2 Lavoie and Godley’s Model (2001–2002)

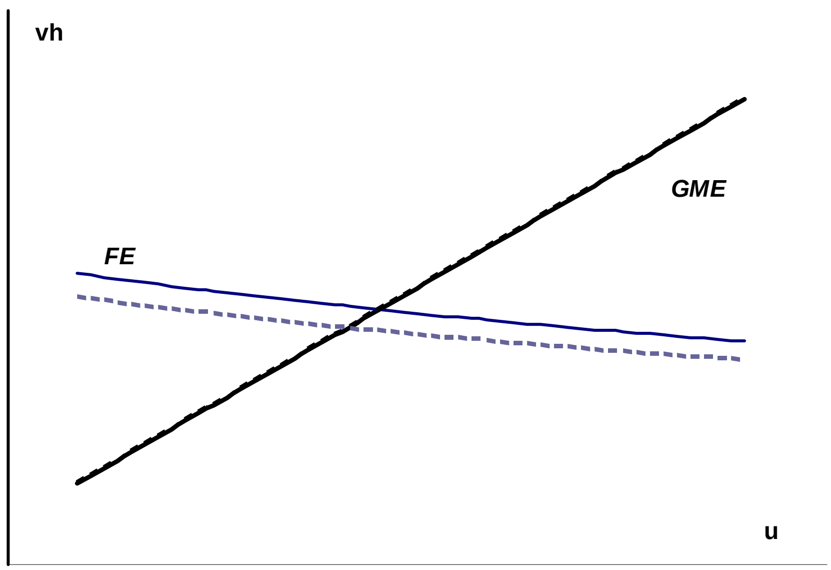
The model presented here has many things in common with Lavoie and Godley (2001–2002) [LG from now on] for a very good reason. We were, in fact, inspired by LG, and tried here both to simplify it (in order to get well defined long-period results) and extend it (so as to allow the discussion of fiscal and monetary policies).

Since we have no significant methodological differences with LG—with the possible exception of our lack of inclination to tackle disequilibrium dynamics directly, at least in simplified theoretical models—we will limit ourselves here to discuss why it is so difficult to understand the nature of LG’s long-period equilibria, let alone its dynamics.

There are a few important differences between the model presented here and LG’s. The most important are related to the feedbacks from financial markets to growth: to begin with, their investment function is affected also by Tobin’s *q* (positively) and firms’ loan to capital ratio (negatively).[33] Moreover, households' portfolio decisions are assumed to depend linearly on expected real rates of return of deposits and equities as, for example, in Tobin (1982). In the current version of our model, we had to simplify on both sides to achieve analytically tractable long-run solutions, but we believe that future extensions of our model that incorporate more complex interactions between the financial and the real sector in the spirit of LG will prove very interesting. To spell this out, notice that the only variables in our model which affect the financial equilibrium ***FE*** curve without affecting the ***GME*** curve directly are the dividends to profits ratio μ and, more importantly, the link ρ between the interest rate and the share of equities in households porfolio δ, which is our simplification for LG Tobinesque set of asset demand equations.

An increase in the desired share of equities in households portfolio δ will reduce the value of wealth in the steady growth path, since it increases the δ_3 parameter—see equations (51) and (54), shifting the ***FE*** curve downwards, as in Figure 11.

Figure 11. A shock to financial markets (Regime 1b)



The possible feedbacks from financial decisions to growth are therefore limited in this version of the model with respect to LG: a price we had to pay to keep the model tractable without resorting to numerical simulations. This is even more true for the most recent growth

[33] However, in Godley and Lavoie (2007) they adopt our own simplification of the investment function.

model presented in Godley and Lavoie (2007), which is far more sophisticated than ours, and yet shares some features, such as the permanent effects of fiscal policy on output.

5. FINAL REMARKS

In the sections above, we presented a very simplified SFC Post-Keynesian growth model and related it to the existing Structuralist/Post-Keynesian literature(s).[34]

The long-run properties of the model have been derived from sequences of short-period equilibria, and we have discussed the conditions under which the model will converge to a stable growth path. Moreover, stable long-run growth paths have been discussed using two complementary approaches: the first focusing on equilibrium conditions in the goods and the financial markets, and the second stemming from the requirements of stable expansion of both government debt and household's wealth.

We have no doubt that the model discussed above can be developed in several ways, many of which have already been suggested in the literature discussed above. We hope, however, to have clarified the structure and the limitations of this family of models. Though our specific derivations depend on the simplifying assumptions we made, we hope also to have convinced the reader that understanding the dynamics of (normalized) balance sheets and, therefore, the nature of long-period equilibria, such as the one discussed above, is generally more useful than the conventional wisdom leads us to believe.

[34] The relation of this kind of modeling with mainstream macroeconomics was discussed in Dos Santos (2006) and, more generally, in Taylor (2004).

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APPENDIX 1. DERIVATION OF THE LONG RUN SOLUTIONS

Let's start from the u equation in (11), setting $vh_t = vh_{t-1}$.

$$u = \frac{a \cdot vh + g_0 - \theta_1 \cdot il + \gamma}{1 - (1-\pi)\cdot(1-\theta) - (\alpha \cdot \beta + \pi)}$$

Define ψ_1 as in (12):

$$u = \psi_1 \cdot [a \cdot vh + g_0 - \theta_1 \cdot il + \gamma]$$

Solving for vh yields the first equilibrium condition (50) in the text:

$$vh = [u/\psi_1 - A(ib)]/a$$

with

$$\delta vh/\delta u = 1/(\psi_1 \cdot a)$$

Now move to the equation for b in (38), where we assume again long-run equilibrium, i.e., $b_t = b_{t-1}$.

$$b = [b \cdot (1+ib) + \gamma - \theta \cdot u]/(1+g)$$

$$b = (\gamma - \theta \cdot u)/(g - ib)$$

Using (9) we get

$$b = (\gamma - \theta \cdot u)/[g_0 + (\alpha \cdot \pi + \beta)\cdot u - \theta_1 \cdot il - ib]$$

which can be simplified using (31):

$$b = (\gamma - \theta \cdot u)/\{g_0 + (\alpha \cdot \pi + \beta)\cdot u - [1 + \theta_1 \cdot (1+\tau_b)]\cdot ib\}$$

$$b = (\gamma - \theta \cdot u)/(\psi_2 + \alpha_1 \cdot u)$$

where ψ_2, α_1 are defined as in (53), (13) respectively.

Note that b will be increasing in u as long as the growth rate g is greater than the interest rate on bills ib. More precisely,

$$\delta b/\delta u = -(\theta + b \cdot \alpha_1)/(g - ib)$$

which is negative when $g > ib$ and

$$b > -\theta / \alpha_1$$

Turning to vh in (44) and setting $vh_t = vh_{t-1}$

$$vh \cdot (1 + g - \delta) = [(1-\delta) \cdot (1 + ib \cdot (1+\tau_b) \cdot (1-\mu) - a] \cdot vh +$$
$$+ [1 - (1+\tau_b) \cdot (1-\mu)] \cdot ib \cdot b + \mu \cdot (1-\theta) \cdot \pi \cdot u$$

$$vh \cdot \{g - [(1-\delta) \cdot (ib \cdot (1+\tau_b) \cdot (1-\mu) - a]\} = \psi_4 \cdot ib \cdot b + \mu \cdot (1-\theta) \cdot \pi \cdot u$$

using the definition of ψ_4 in (55). Substitute for g using (9):

$$vh \cdot \{g_0 + \alpha_1 \cdot u - \theta_1 \cdot (1+\tau_b) \cdot ib - (1-\delta) \cdot ib \cdot (1+\tau_b) \cdot (1-\mu) + a\} =$$
$$\psi_4 \cdot ib \cdot b + \mu \cdot (1-\theta) \cdot \pi \cdot u$$

Using the definitions of ψ_3 from (54):

$$vh \cdot (\psi_3 + \alpha_1 \cdot u) = \psi_4 \cdot ib \cdot b + \mu \cdot (1-\theta) \cdot \pi \cdot u$$

and finally using the result above for b:

$$vh \cdot (\psi_3 + \alpha_1 \cdot u) = \psi_4 \cdot ib \cdot \frac{\gamma - \theta \cdot u}{\psi_2 + \alpha_1 \cdot u} + \mu \cdot (1-\theta) \cdot \pi \cdot u$$

we get the second equilibrium condition (51) in the text.

APPENDIX 2. LIST OF SYMBOLS

Symbol	Description
A	Normalized autonomous expenditure
B	Stock of government bills
C	Households' consumption
D	Stock of bank deposits
E	Stock of equities
Fb	Banks' profits
Fd	Firms' distributed profits
FT	Total firms' profits
Fu	Firms' undistributed profits
G	Government expenditure
K	Stock of capital goods
L	Stock of bank loans to firms
SAV	Total savings
$SAVg$	Government savings
$SAVh$	Households' savings
T	Total tax receipts
Tf	Taxes on profits
Tw	Taxes on wages
Vf	Firms' net worth
Vh	Households' net worth
W	Wages
X	Output
Y	Total private sector income
A	Propensity to consume out of wealth
b	Government bills to capital ratio
g	Growth in the stock of capital
g_0	Autonomous growth in the stock of capital
ib	Interest rate on government bills
il	Interest rate on bank loans
l	Bank loans to capital ratio
p	Price level
pe	Market price of equities
$savh$	Households' savings to capital ratio
u	Output capital ratio
vh	Households' wealth to capital ratio
w	Money wage per unit of labor
α	Accelerator effect through profits
α_1	Overall accelerator
α_2	Parameter - see (52)
β	Exogenous accelerator effect
γ	Government expenditure to capital ratio
δ	Ratio of equities in households' wealth
ζ_1	Parameter–see (60)
ζ_1	Parameter–see (61)
ζ_1	Parameter–see (62)
ζ_1	Parameter–see (63)
ζ_1	Parameter–see (64)
ζ_1	Parameter–see (65)
ζ_1	Parameter–see (66)
θ	Tax rate

θ_1	Effect of the interest rate on loans on investment
λ	Labor-output ratio
μ	Dividends to profits ratio
π	Profits share on income, before tax
ρ	Link between the interest rate and the share of equities in h. wealth
τ	Mark-up rate
τ_b	Banks' mark-up rate
χ	Ratio of equities to capital
ψ_1	Multiplier
ψ_2	Parameter–see (53)
ψ_3	Parameter–see (54)
ψ_4	Parameter–see (55)