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Do the Innovations in a Monetary VAR Have Finite Variances?

by

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ABSTRACT
Since Christopher Sims's "Macroeconomics and Reality" (1980), macroeconomists have used structural VARs, or vector autoregressions, for policy analysis. Constructing the impulse-response functions and variance decompositions that are central to this literature requires factoring the variance-covariance matrix of innovations from the VAR. This paper presents evidence consistent with the hypothesis that at least some elements of this matrix are infinite for one monetary VAR, as the innovations have stable, non-Gaussian distributions, with characteristic exponents ranging from 1.5504 to 1.7734 according to ML estimates. Hence, Cholesky and other factorizations that would normally be used to identify structural residuals from the VAR are impossible.

Keywords: Vector Autoregressions; Stable Distributions; Stable-Paretian Distributions; Infinite Variance; Monetary Policy

JEL Classifications: C32, E52

I. INTRODUCTION

Since Sims (1980), economists have been using monetary structural vector autoregressions (VARs) to measure the effects of policy changes and test models.[1] This paper provides estimates of the characteristic exponents of the distributions of the innovations in the reduced-form of one such VAR.

This introduction describes the properties of stable distributions and briefly describes the mathematics of VARs. It then shows that when at least one error term in a VAR has a stable, non-Gaussian distribution, it is impossible to construct meaningful impulse response functions and variance decompositions, the key tools of structural VAR analysis. The reason is simple: stable, non-Gaussian distributions do not have finite variances, making structural factorizations nonsensical. Finally, the introduction outlines the remainder of the paper.

A random variable X has a stable distribution if it has a domain of attraction, i.e., if there is a sequence of i.i.d. random variables Y_1, Y_2,…. and sequences of positive numbers $\{d_n\}$ and real numbers $\{a_n\}$, such that

$$\frac{Y_1 + Y_2 + \dots + Y_n}{d_n} + a_n \overset{d}{\Rightarrow} X$$

where the arrow symbol means "converges in distribution" (Samorodnitzky and Taqqu 1994: 5). If the Y's have a finite variance, X has a normal distribution, which is the most well-known stable distribution.

Furthermore, there is an equivalent definition: a random variable X has a stable distribution if for each n greater than or equal to 2, there is a positive number C_n and a real number D_n such that

$$X_1 + X_2 + \dots + X_n \overset{d}{=} C_n X + D_n$$

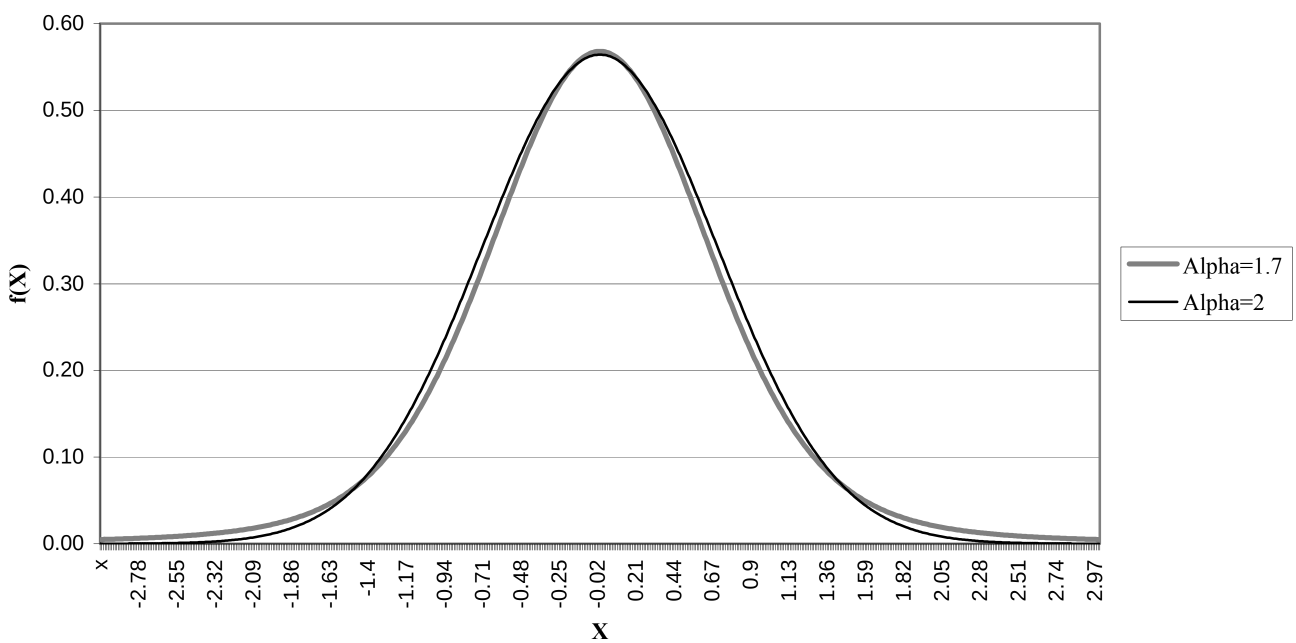
[1] See, for example, Galí (2008: 8–9) and the references therein.

where $X_1, X_2, \ldots, X_n$ are independent copies of X and the C_n and D_n are constants, and where the arrow symbol means "equals in distribution" (Samorodnitzky and Taqqu 1994: 3). It turns out that

$$C_n = n^{1/\alpha}$$

where α is known as the characteristic exponent of the distribution. Clearly then, when the X_i are normally distributed, $\alpha = 2$. Alpha takes on values in the interval (0, 2], with lower α's indicating more high-peaked and thick-tailed distributions. Only two stable distributions with $\alpha < 2$ have explicit density formulas: the Cauchy and Lèvy distributions. The stable distributions are a four-parameter family: α, β for skew, γ for scale, and δ for location. Figure 1 shows the standard normal distribution and a symmetric, stable distribution of the same scale with $\alpha = 1.7$.

Figure 1. Densities of Standard Normal Distribution and Symmetric Stable with Alpha = 1.7



The most important feature of these distributions from the point of view of this paper is that when $\alpha < 2$, the variance does not exist, and for α less than or equal to 1, neither the mean nor the variance exist. Equivalently, these moments do not converge, or are infinite.

This paper examines the implications of infinite variances of innovations for structural monetary VARs. To see these implications, recall that the structural form of a VAR of order p is

$$AY_t = B_1 Y_{t-1} + B_2 Y_{t-2} + ... + B_p Y_{t-p} + \eta_t$$

where A and the B_is are n-by-n matrices of parameters, with A nonsingular; the Y_ts are n-vectors of economic and monetary variables at time t; and η is a n-vector of disturbances. As we will see, the problem is to identify A, and that is not always possible, even with the usual identifying conditions. It is assumed that

$$\begin{aligned} &E(\eta_t) = 0 \\ &E(\eta_t | Y_{t-1}, Y_{t-2}, ..., Y_{t-p}) = 0\,. \\ &E(\eta\eta') = I \end{aligned}$$

where I is the n-by-n identity matrix. The reduced form of the VAR can be written

$$Y_t = C_1 Y_{t-1} + C_2 Y_{t-2} + ... + C_p Y_{t-p} + \varepsilon_t \tag{1}$$

where

$$\begin{aligned} &\forall\ j\ and\ t, \\ &C_j = A^{-1} B_j \\ &\varepsilon_t = A^{-1} \eta_t \end{aligned} \tag{2}$$

The variance-covariance matrix of ε_t is

$$V = E(\varepsilon_t \varepsilon_t') = E(A^{-1} \eta\eta' A^{-1}{}') = A^{-1} A^{-1}{}' \tag{3}$$

To find the needed parameters, one first estimates each equation in (1) using least squares.[2] The residuals from the regressions are consistent estimates of the ε_t, but most important uses of structural VARs require that we recover the η_t. To find η_t, one first obtains V*, the sample variance-covariance matrix of the ε_t. Then, assuming A is lower triangular, we can get an estimate of it by decomposing V* into the product of a lower triangular matrix A^{-1} and its transpose $A^{-1\prime}$ (the Cholesky factorization). Once the factorization has been accomplished, the η_t (the structural disturbances) can be identified from

$$\eta_t = A\varepsilon_t$$

Subsequent to Sims's (1980) article, other forms of identification for the structural innovations have been developed. These use different restrictions on the VAR, but also usually involve a factorization of V (e.g., Blanchard and Quah 1989).

The key uses of structural VAR are impulse response functions (i.e., moving average representations), which measure the effects over time of a given one-time shock to one element in η_t, and forecast error variance decompositions, which show the proportion of the variation of each variable in Y_t that is due to random shocks in each element of η_t. The moving average representation is an equation such as

$$Y_t = D_1\eta_t + D_2\eta_{t-1} + D_3\eta_{t-1} + \ldots..$$

which is obtained by inverting the VAR (equation 1) and transforming the ε_t to η_t.

Through the use of appropriate restrictions on A, the structural shocks η_t can be interpreted as monetary policy shocks, money demand shocks, and so on. These exercises cannot be done with the ε_t, because these reduced-form innovations are correlated.

One implication of infinite diagonal elements in the variance-covariance matrix V for structural VAR is that the decomposition $V = A^{-1}A^{-1\prime}$ is nonsensical, so there are no structural innovations defined by $\eta_t = A\varepsilon_t$. All elements of the sample variance-covariance matrix V* will of course be finite, but V* will be an "estimate" of a matrix V with some infinite diagonal

[2] Given the assumptions above, equation-by-equation estimation yields a consistent estimate of the regression parameters. Equation-by-equation least squares is identical to the seemingly unrelated regressions estimator in this case, so it is also the efficient generalized least squares (GLS) estimator. See Davidson and MacKinnon (2004: especially 595–597) or Hamilton (1994: 291–350), and the references therein for more details.

entries.[3] The econometrician invokes the method of moments by setting V = V*, and in doing so, she is setting some finite sample moments equal to infinite population moments. Hence, if the characteristic exponent α of the distribution of any element of ε_t is less than 2, we cannot find a meaningful A and proceed with impulse response functions and forecast error variance decompositions.[4]

The rest of the paper is organized as follows. Section II discusses the existing economic literature on stable distributions and monetary VARs. Section III is a discussion of this paper's monetary VAR, including the data, the specification, and the results. It includes findings on heteroskedasticity, which is an alternative explanation of thick-tailed distributions. Section IV provides estimates of the characteristic exponents of all innovations for both the full sample and two subsamples and reports diagnostics to assess the fit of the estimated stable distributions. Section V draws together the key conclusions of the paper.

II. REVIEW OF THE LITERATURE

The use of stable distributions for economic variables began with Mandelbrot's (1963) analysis of securities price changes, where he had noticed thick-tailed distributions. Fama (1963, 1965a, 1965b) and Mandelbrot (1963, 1967) reported evidence that characteristic exponents of the distributions they studied were usually less than two. Blattberg and Gonedes (1974) and Clark (1973) countered that certain nonstable distributions better fit financial data. Blattberg and Sargent (1971) tested robust estimators of regression coefficients that were more efficient than least squares when the error terms were stable non-Gaussian. Granger and Orr (1972) analyzed the implications of stable distributions for time series analysis. Other papers, including Bhansali (1993), have studied the properties of estimates of impulse response functions for autoregressive processes with non-Gaussian stable distributions. These articles have not dealt with structural identification. More recently, Rachev, Kim, and Mittnik (1997) and DasGupta and Mishra (2004) reviewed findings on the econometrics of non-Gaussian stable distributions. Tsionas (1999) showed how Markov chain Monte Carlo methods could be used to estimate

[3] Also, if more than one innovation has infinite variance, some off-diagonal entries in the variance-covariance matrix will be infinite.

[4] Another issue is the efficient estimation of the parameters when some residual variances are infinite. This point seems moot in the structural VAR setting, for the reasons stated in this paragraph, but references to articles on robust estimation in the presence of infinite-variance errors are included in Section 2.

models with stable, non-Gaussian disturbances. Mirowski (1990) discussed the history of these distributions in economics.

Christiano, Eichenbaum, and Evans (1999) gives an account of what was learned about monetary VAR models. The VAR studied in this paper was chosen to be similar to many of those in the existing literature. Galí (2008: 8–9) is an example of a textbook that uses a VAR of this type as an empirical benchmark for a new Keynesian macro model. Some articles that present VARs similar to the one below are Bernanke and Mihov (1998b), Christiano, Eichenbaum, and Evans (1996), and Strongin (1995). These articles are discussed within a common framework by Leeper, Sims, and Zha (1996: 29–39).

This paper is not meant as an analysis or discussion of any of these articles in particular. Rather, the paper is meant to illustrate problems that can occur in a VAR that is representative of many of those in the literature. Rudebusch (1998) undertook a detailed analysis of the innovations in monetary VARs, though the issues he raised are unrelated to those studied in this paper.

Many recent articles have modeled thick-tailed behavior of monetary VAR residuals with various forms of heteroskedasticity (variances that change over time), including stochastic volatility, autoregressive conditional heteroskedasticity (ARCH), and Markov regime-switching models. Some of these approaches do not permit the researcher to compute the kinds of impulse response functions and variance decompositions that are central to the papers in the preceding paragraph. Also, many of these models result in a marked increase in the number of parameters, subperiods, and/or impulse response functions. The approach here is to describe the residuals parsimoniously with one four-parameter distribution. Heteroskedasticity is studied in more depth in the remaining sections of the paper.

Some references to the literature on methods of estimating α are provided at the beginning of section IV.

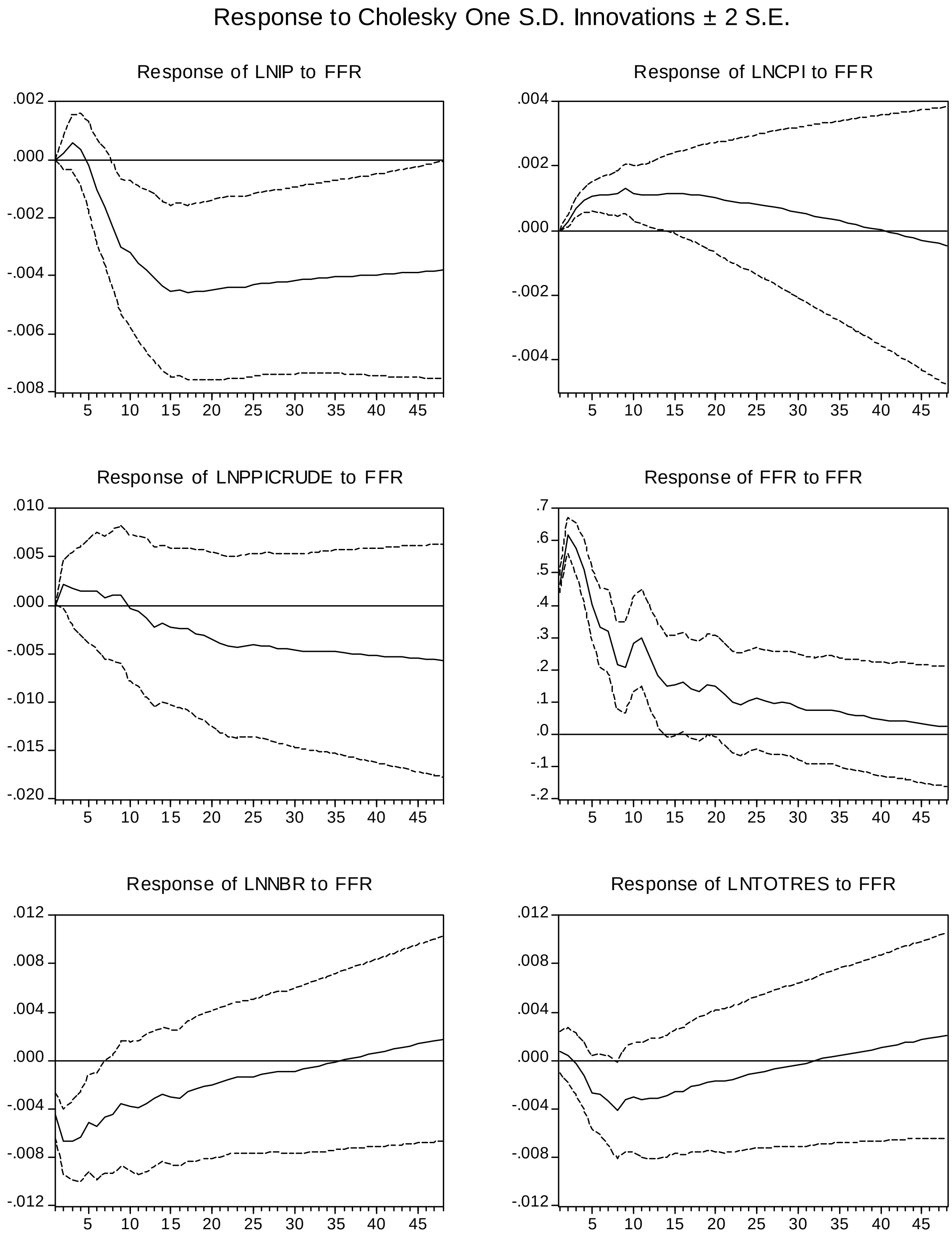
III. THE RESERVES VAR: DATA, MODEL, RESULTS, AND PROPERTIES OF THE RESIDUALS

The data are monthly and span the period January 1959–November 2007. The included variables are a constant, industrial production (IP), the consumer price index for all urban consumers (CPI), the crude materials producer price index (PPI), the federal funds rate (FFR), and the Federal Reserve's nonborrowed reserves (NBR) and adjusted total reserves (TR) series. All variables other than FFR were used in their seasonally adjusted forms and transformed into logs. Twelve lags of each variable were used in each equation.[5]

First, this section presents a few results from the model. Since the purpose of this paper is not to present qualitatively new impulse response functions or variance decompositions, one set of impulse response functions will be shown merely to demonstrate that the VAR is fairly typical. Figure 2 (see next page) shows impulse response functions over a 48-month horizon for a positive, one-standard-deviation shock to FFR,[6] which we will assume is the policy variable. The ordering of the variables in the Cholesky decomposition was IP, CPI, PPI, FFR, NBR, TR. Two-standard-deviation error Monte Carlo error bands are shown in the figure.[7]

[5] The NBR variable, described below, fell to negative levels after November 2007, making the log transformation impossible. Therefore, the sample was truncated at that date.
[6] The standard deviation was adjusted for degrees of freedom.
[7] The error bands were calculated based on 3,000 replications.

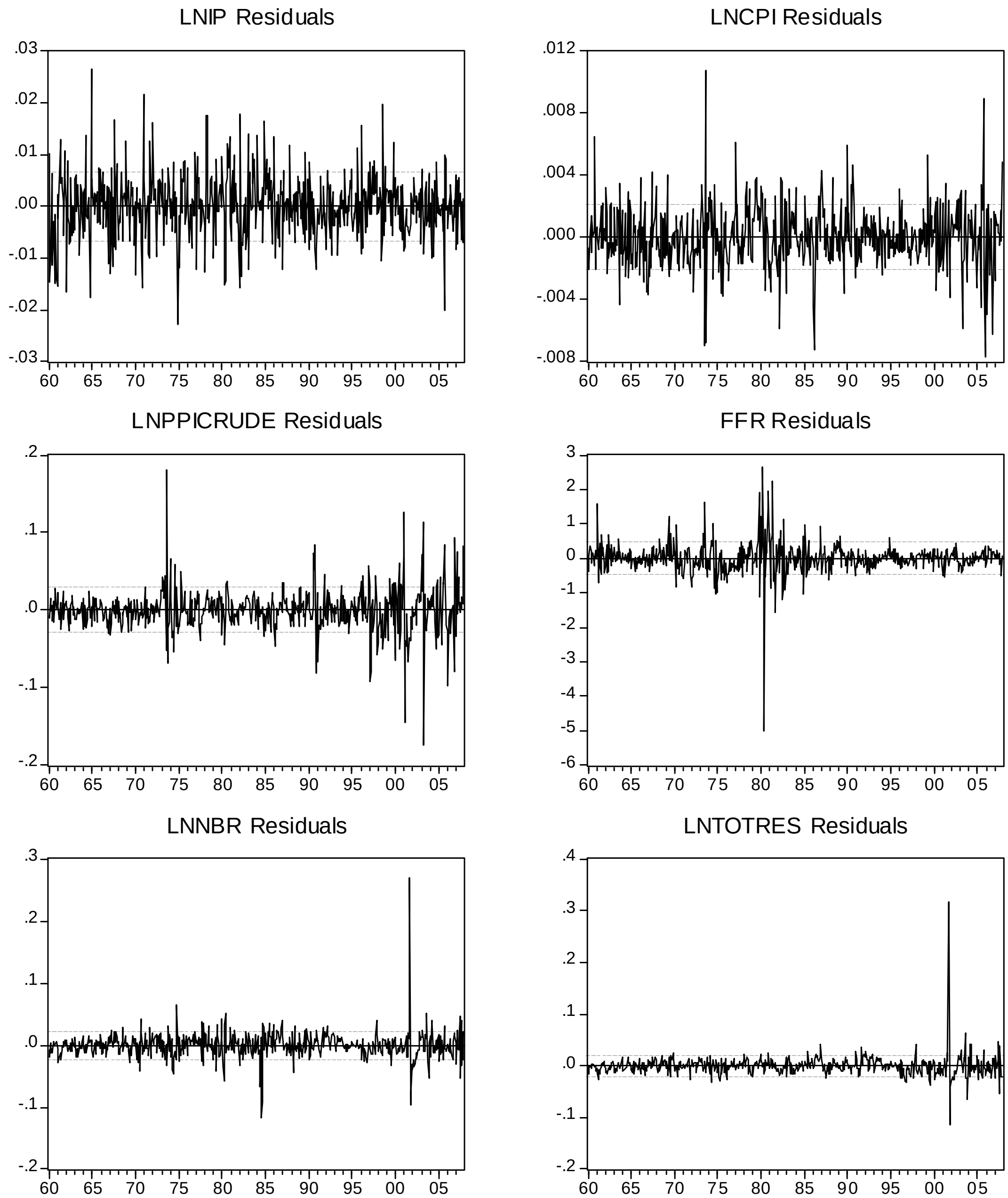
Figure 2. Response to Cholesky One S.D. Innovations 2± S.E.



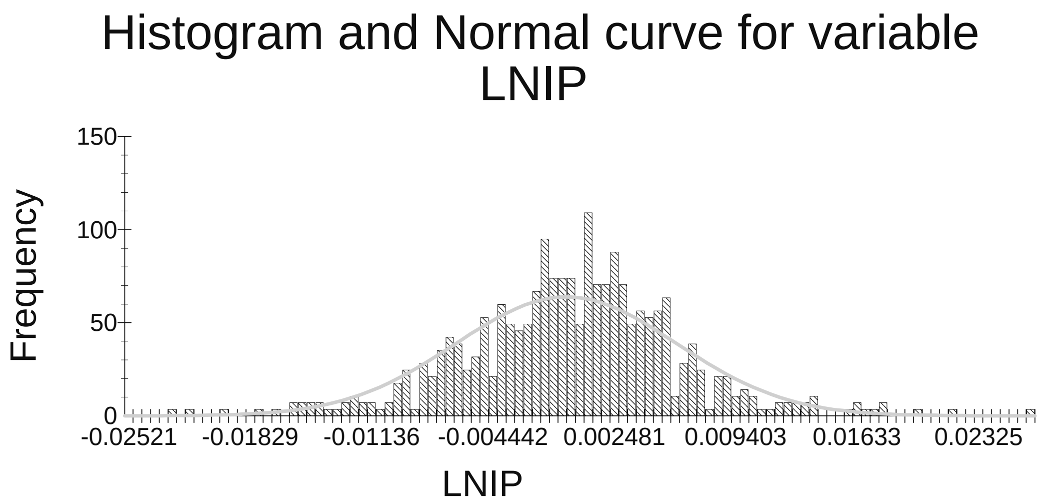
The responses are mostly typical for a monetary VAR. The response of IP (industrial production) to a contractionary FFR (federal funds rate) shock is long-lived, negative, and statistically significant. (Since the variables other than FFR were used in log form, the numbers on the ordinates can be interpreted as approximations of percentage differences.) There appears to be a "price puzzle," a phenomenon that appears in some VARs of this type (Sims 1992): CPI actually rises after a positive FFR shock, and this effect lasts for more than three years.

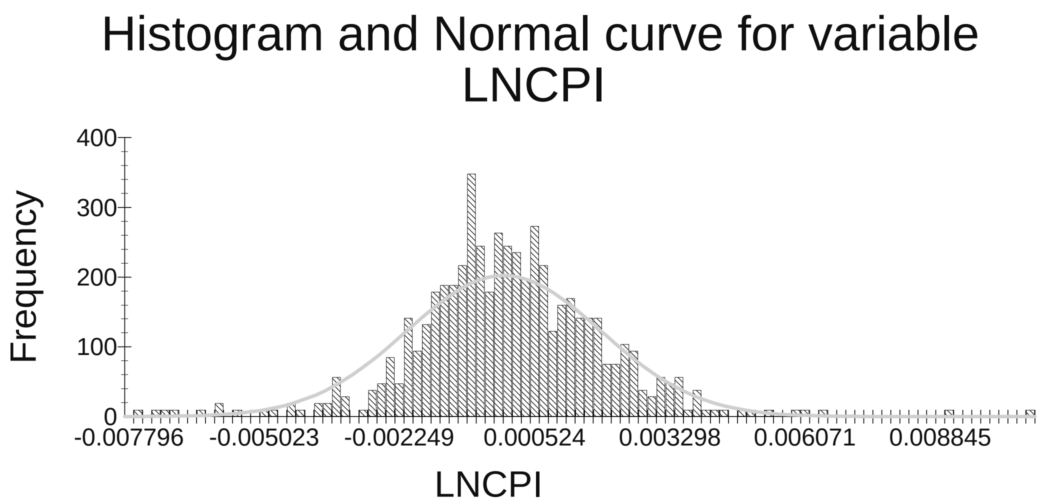
The primary concern of this paper is the distribution of the innovations of the reduced-form VAR. The innovations ε_t for each regression are charted in figure 3, a set of histograms follows in figure 4, and regression residual diagnostics appear in table 1 (below the figures). The dotted lines above and below the zero line in each innovation time series chart are one standard deviation from the mean. The histograms also show normal densities for comparison purposes.

Figure 3. Reduced-Form VAR Innovations ε_t

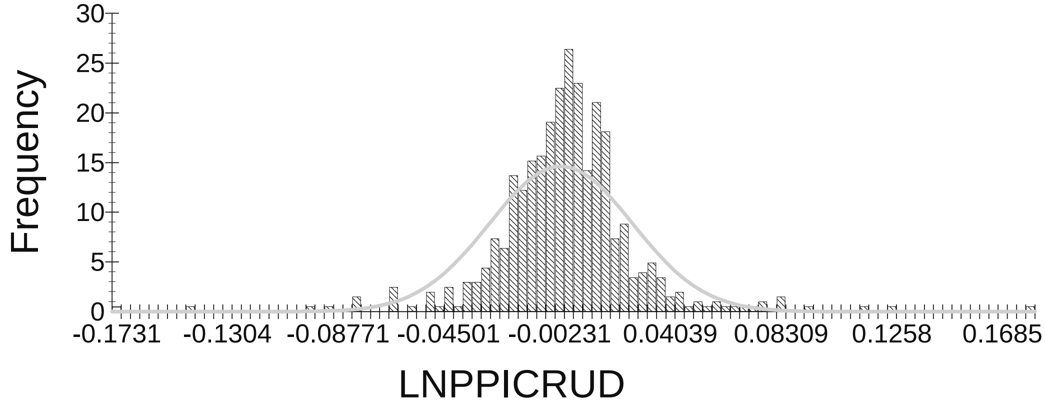


Figures 4–9. Histograms for ε_t

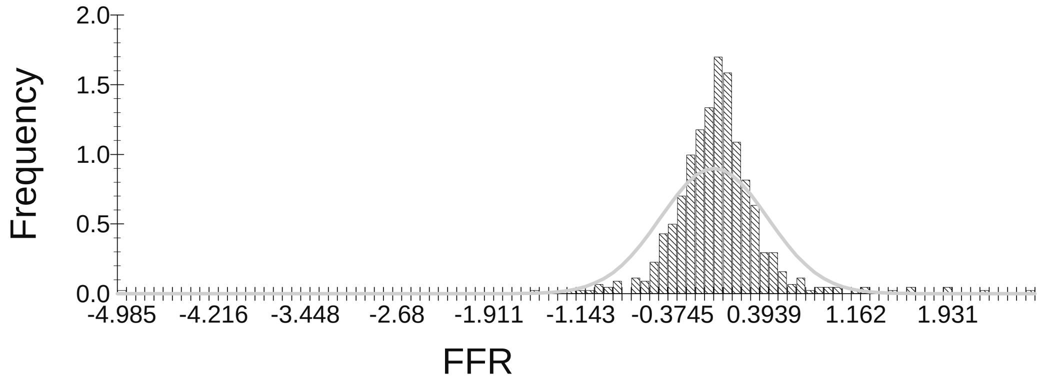




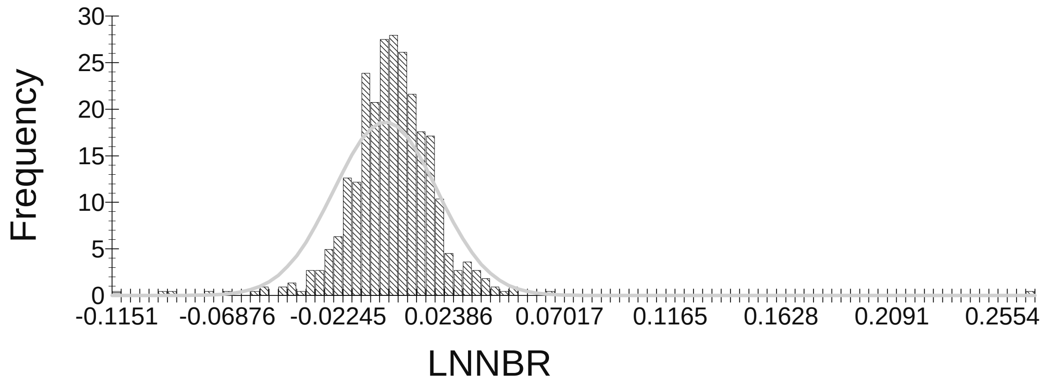
Histogram and Normal curve for variable
LNPPICRUD



Histogram and Normal curve for variable
FFR



Histogram and Normal curve for variable
LNNBR



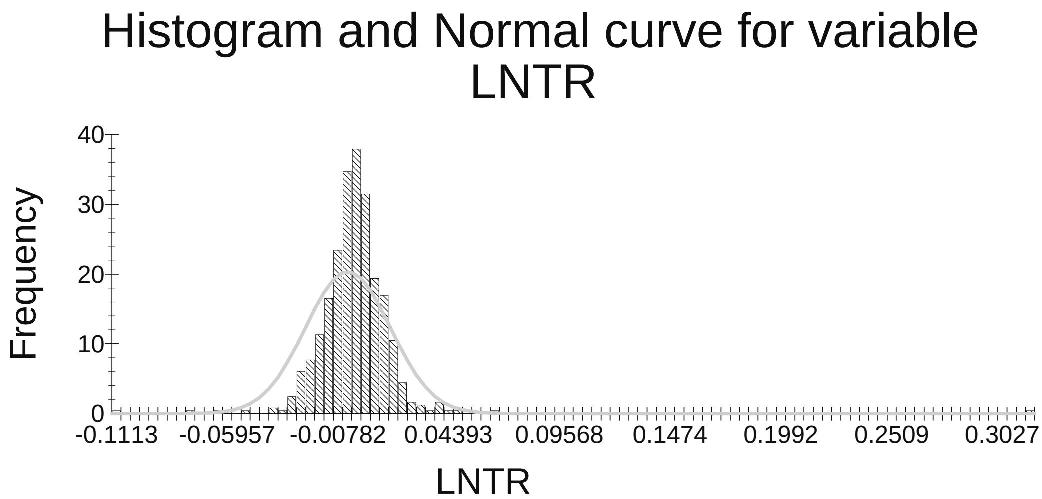


Table 1. Sample Statistics for Reduced-Form Innovations ε_t

	RESID01	RESID02	RESID03	RESID04	RESID05	RESID06
Mean	-2.77E-16	8.26E-15	-9.01E-15	3.02E-13	-7.75E-15	1.61E-14
Median	0.000183	-4.85E-05	0.000605	-6.00E-05	0.000203	-0.000347
Maximum	0.026462	0.010786	0.180953	2.660516	0.268910	0.317765
Minimum	-0.022984	-0.007704	-0.174898	-5.023082	-0.116996	-0.113474
Std. Dev.	0.006249	0.001972	0.027257	0.443849	0.021413	0.019618
Skewness	0.022306	0.175226	0.070495	-1.509032	2.794186	6.973937
Kurtosis	4.464086	6.285196	12.24979	35.74013	47.98869	123.6040
Jarque-Bera	51.40351	261.5130	2050.320	25899.56	49239.45	353142.5
Probability	0.000000	0.000000	0.000000	0.000000	0.000000	0.000000
Sum	-1.58E-13	4.75E-12	-5.18E-12	1.74E-10	-4.45E-12	9.27E-12
Sum Sq. Dev.	0.022413	0.002232	0.426443	113.0792	0.263201	0.220917
Observations	575	575	575	575	575	575

The histograms in figures 4–9 give the impression that a non-Gaussian distribution of some type is likely. Table 1 and the histograms indicate that each set of residuals has excess kurtosis, and some are very skewed. Each Jarque-Bera test rejects the null of normality. Excess kurtosis and skew are consistent with a stable, non-Gaussian distribution. Autocorrelations are not reported, but each disturbance term tends to be very weakly autocorrelated.

Figure 3 gives the impression of clusters of volatility. Mandelbrot (1963) observed such behavior in many financial time series, and it is certainly consistent with stable, non-Gaussian conditional and unconditional distributions (deVries 1991). On the other hand, the clusters of elevated or low volatility are also consistent with an ARCH (autoregressive conditional heteroskedasticity) or generalized ARCH (GARCH) process. Such processes have thick-tailed unconditional distributions, but not infinite variances (Engle 1982: 992). Therefore, an ARCH model is one possible alternative to a stable, non-Gaussian process. Tables 2 and 3 give the results of Engle (1982) tests for ARCH, first using 3 lags of the residuals, then 12 lags. Chi-squared test statistics (third column) above the .05 critical value, which are marked with asterisks, reject the null of no ARCH effects.

Table 2. Engle Test for ARCH, 3 Lags of Residuals, Sample Period 1959–2007

Equation	R^2	R^2 X T
IP	.051	29.172*
CPI	.091	52.052*
PPI	.123	70.356*
FFR	.061	34.892*
NBR	.025	14.300*
TR	.019	10.868*

Table 3. Engle Test for ARCH, 12 Lags of Residuals, Sample Period 1959–2007

Equation	R^2	R^2 X T
IP	.071	39.973*
CPI	.109	61.367*
PPI	.131	73.753*
FFR	.137	77.131*
NBR	.025	14.075
TR	.019	10.697

While the null is rejected in each case for the first specification, and for four of the residuals in the second, the very small R^2s indicate a fairly weak, though precisely estimated, effect. ARCH is clearly part of the story if the residuals have finite variances. However, ARCH does not exist under infinite variance, rendering the Engle test statistic meaningless in that case.

Several articles have investigated heteroskedasticity in VARs similar to the one reported here. Some of these have found certain subperiods of homoskedasticity. Bernanke and Mihov (1998a: 163) find no evidence of a structural break in the policy block of their structural disturbances variance-covariance matrix in the periods January 1966–September 1979 and April 1988–April 1996. Tables 4 and 5 show the results of Engle tests for the first of these subperiods and for April 1988 to the end of the sample, which were performed after re-estimating the model over these shorter periods.

Table 4. Engle Test for ARCH, 3 Lags of Residuals, Sample Period 1966–1979

Equation	R^2	R^2 X T
IP	.012	1.944
CPI	.144	23.328*
PPI	.103	16.686*
FFR	.072	11.664*
NBR	.001	.162
TR	.025	4.05

Table 5. Engle Test for ARCH, 12 Lags of Residuals, Sample Period 1966–79

Equation	R^2	R^2 X T
IP	.047	7.191
CPI	.169	25.857*
PPI	.135	20.655
FFR	.124	18.972
NBR	.037	5.661
TR	.094	14.382

The innovations in the equations for IP, NBR, and TR seem to be free of ARCH or GARCH effects in the period 1966–79. Tables 6 and 7 refer to the 1988–2007 subperiod.

Table 6. Engle Test for ARCH, 3 Lags of Residuals, Sample Period 1988–2007

Equation	R^2	R^2 X T
IP	.076	17.708*
CPI	.009	2.097
PPI (raw mat.)	.077	17.941*
FFR	.007	1.631
NBR	.001	.233
TR	.001	.233

Table 7. Engle Test for ARCH, 12 Lags of Residuals, Sample Period 1988–2007

Equation	R^2	R^2 X T
IP	.104	23.296*
CPI	.040	8.960
PPI	.085	19.040
FFR	.020	4.480
NBR	.001	.224
TR	.001	.224

For this subperiod, the CPI, FFR, NBR, and TR residuals appear to have no ARCH or GARCH effects for each of the two lag lengths tested. Among other questions, the next section investigates both subperiods for signs of non-Gaussian stable shocks.

IV. THE RESERVES VAR: ESTIMATES OF THE CHARACTERISTIC EXPONENTS

Akgiray and Lamoureux (1989), Garcia, Renault, and Veredas (2006), Kogon and Williams (1998), Lombardi and Calzolari (2008), and McCulloch (1997) discuss the relative merits of some methods for estimating stable parameters. DuMouchel (1973) shows that except for some "exceptional parameter values," the maximum likelihood (ML) estimates of α, β, γ, and δ are consistent and

$$n^{1/2}(\hat{\alpha}-\alpha,\hat{\beta}-\beta,\hat{\gamma}-\gamma,\hat{\delta}-\delta)$$

has a limiting normal distribution with mean (0, 0, 0, 0) and covariance matrix I^{-1}, where I is the information matrix and the parameters with circumflexes are the estimates.

Here, we begin with the estimation and diagnostics approach suggested by Nolan (1999, 2001). Three estimates are used here: the quantile method of McCulloch (1986), the characteristic function regression method of Koutrovelis (1980) and Kogon and Williams (1998), and the ML estimate (DuMouchel 1973; Nolan 2001).[8] Table 8 reports estimates of the characteristic exponents (α) for the innovations in each equation of the reduced-form VAR.

Table 8. Estimates of α for Innovations in Six-Variable VAR

Equation	Estimator	Characteristic Exponent Estimate (α) (2 times asymptotic standard deviation).
IP	Quantile	1.6875
	Char. function	1.8664
	ML	1.7734 (.1165)
CPI	Quantile	1.7280
	Char. Function	1.8189
	ML	1.7325 (.1208)
PPI	Quantile	1.5987
	Char. Function	1.6141
	ML	1.5504 (.1265)
FFR	Quantile	1.5668
	Char. Function	1.5884
	ML	1.5623 (.1295)
NBR	Quantile	1.7167
	Char. Function	1.7391
	ML	1.7201 (.1221)
TR	Quantile	1.6864
	Char. Function	1.7543
	ML	1.7606 (.1180)

[8] All three estimates were computed using the STABLE program, version 3.14.02, developed by John Nolan of American University and available online at academic2.american.edu/~jpnolan.

The last column of table 8 shows the estimates, and, in parentheses, two times the asymptotic standard deviations for the maximum likelihood estimates. The results are fairly consistent across estimators for each set of residuals. In each case, the normal distribution ($\alpha = 2$) is more than two standard deviations above the estimate. One note of caution is that for α close to the Gaussian value of two, the normal asymptotic distribution of the estimate of α is not a good approximation, with the likelihood function falling more steeply to the right of the estimate than to the left for relatively small samples (DuMouchel 1983: 1021). Also, asymptotic distribution theory simply does not apply when $\alpha = 2$ (DuMouchel 1983: 1021).

Having fitted stable distributions to each set of residuals, the next question is whether the distributions are stable at all. Nolan notes that "As with any other family of distributions, it is not possible to prove that a given data set is stable" (2001: 388). Nonetheless, some diagnostic tools can help determine if the data are consistent with a hypothesis of stability (Nolan 2001: 388). Figures 10–15 are modified P-P plots[9] (percent-percent plots) for the ML estimates above of the distributions of each innovation. The closer the thick, gray line is to the thin, straight line, the better the ML stable estimate fits the data.

[9] Modified P-P plots, introduced in Michael (1983), are also known as stabilized P-P plots, though the latter term is not connected to the stable family of distributions. Modified P-P plots apply an arcsin transformation to standard P-P plots in order to equalize the variance of all of the points on the plot. The resulting plot enables a better assessment of the fit at the extremes of the distribution (Nolan 2001: 388). Let F_0 be the ML estimate of the distribution of one of the disturbances, using the stable model. Also, let e_i , i = 1, 2,….., n-1, n be the order statistics of the residuals. Then, the i^{th} abscissa of the modified P-P plot is

$$r_i = (2/\pi)\arcsin\left[\left([i-1/2]/n\right)^{1/2}\right]$$

and the ith ordinate is

$$s_i = (2/\pi)\arcsin\left[F_0^{1/2}(e_i)\right]$$

The modified P-P plots in this paper are constructed from 200 points.

Figure 10. Modified PP Plot for IP Stable Fit (ML Estimate)

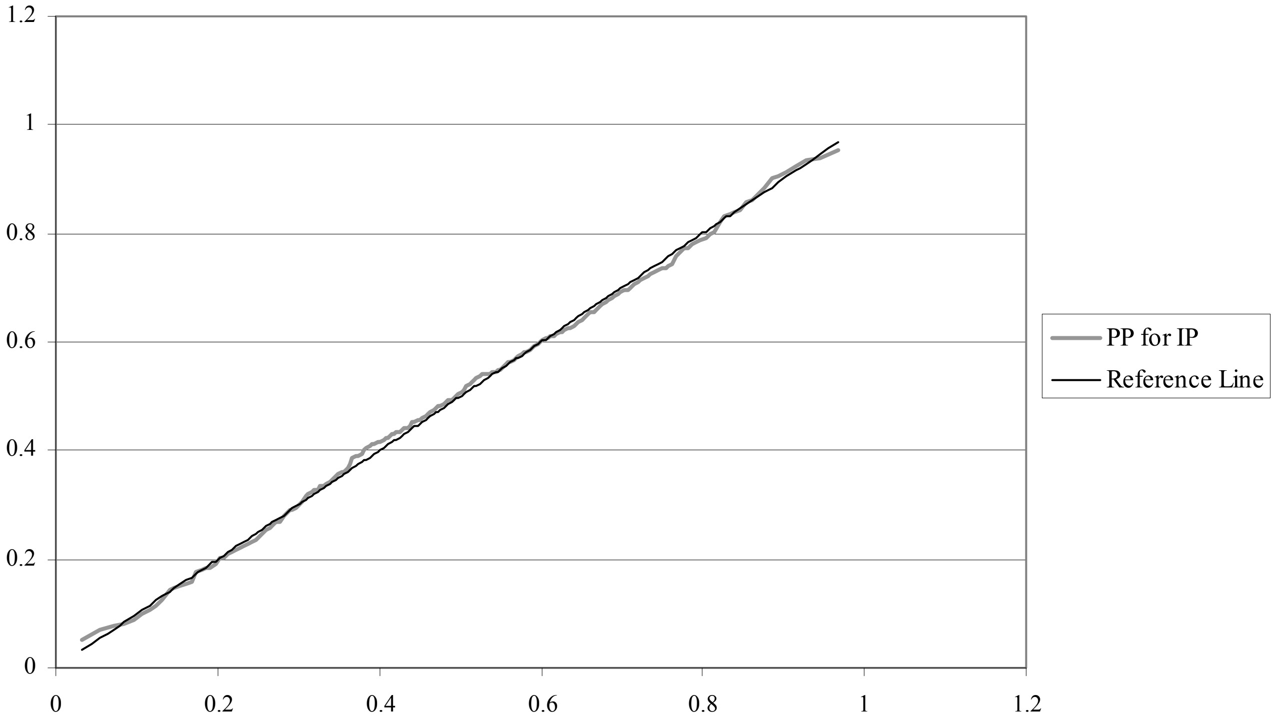


Figure 11. Modified PP Plot for CPI Stable Fit (ML Estimate)

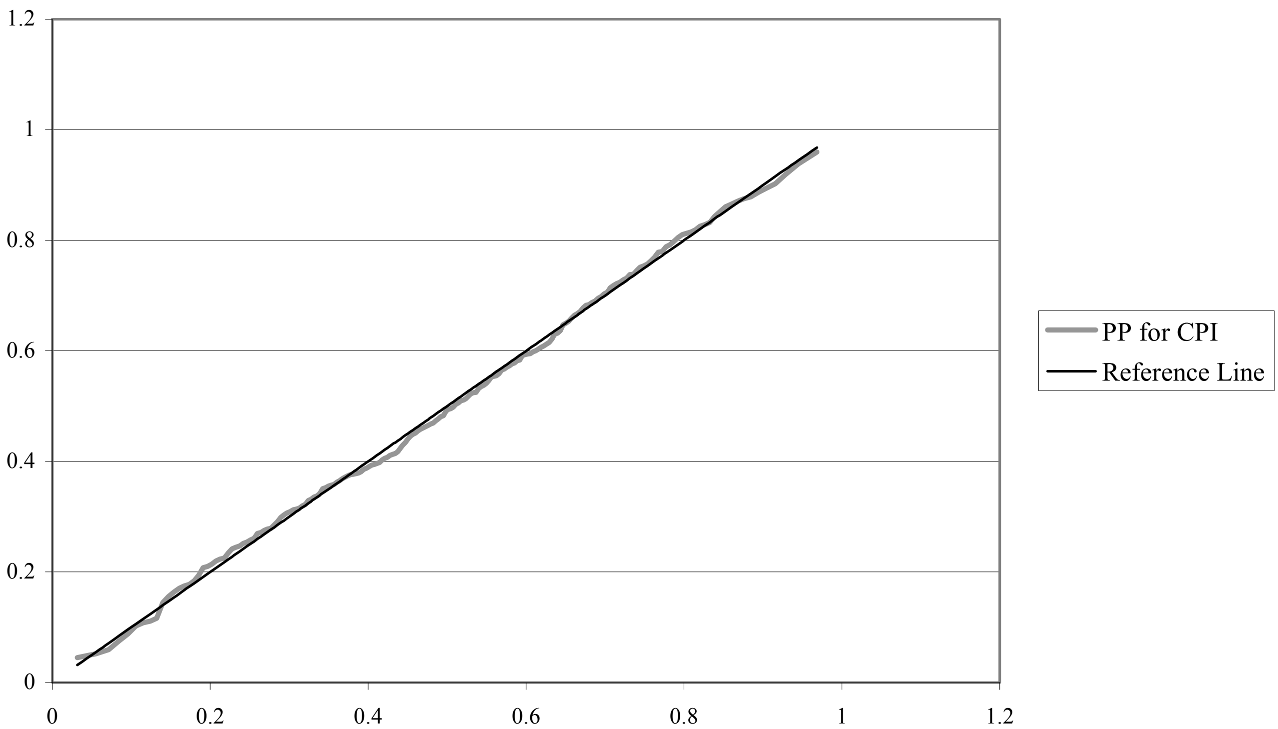


Figure 12. Modified PP Plot for PPI Stable Fit (ML Estimate)

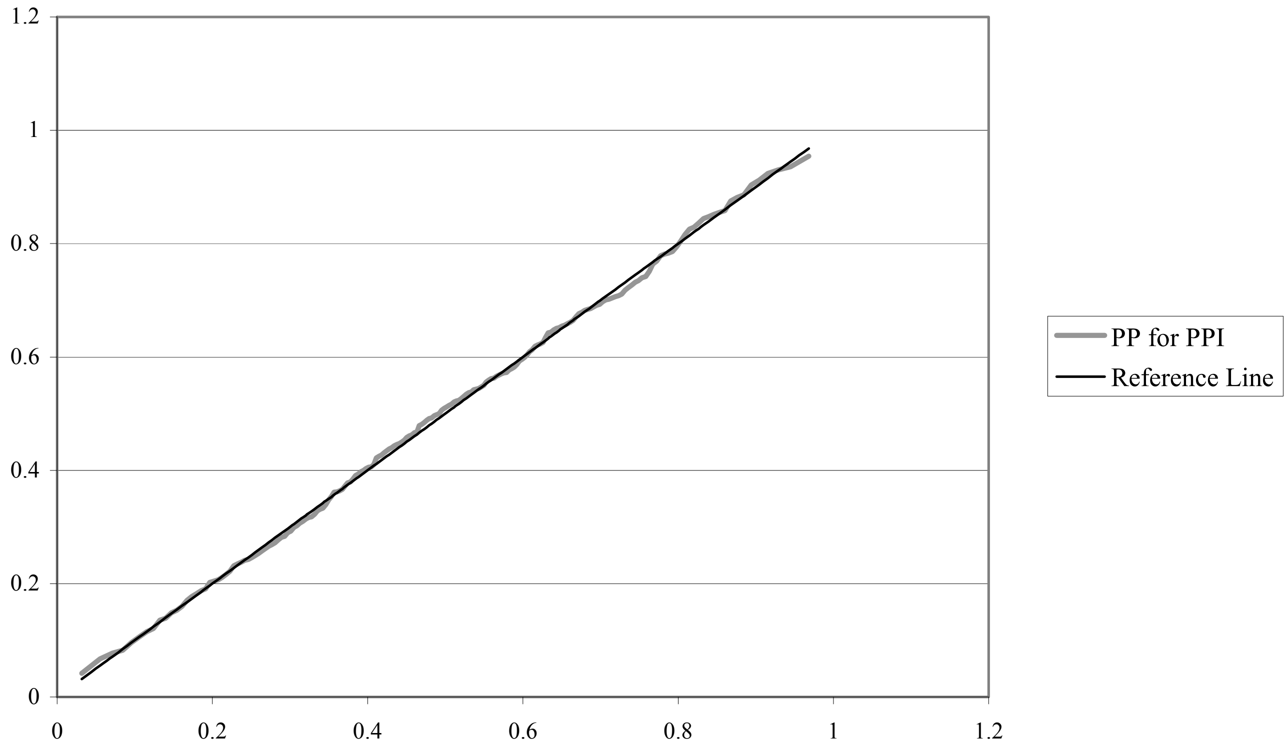


Figure 13. Modified PP Plot for FFR Stable Fit (ML Estimate)

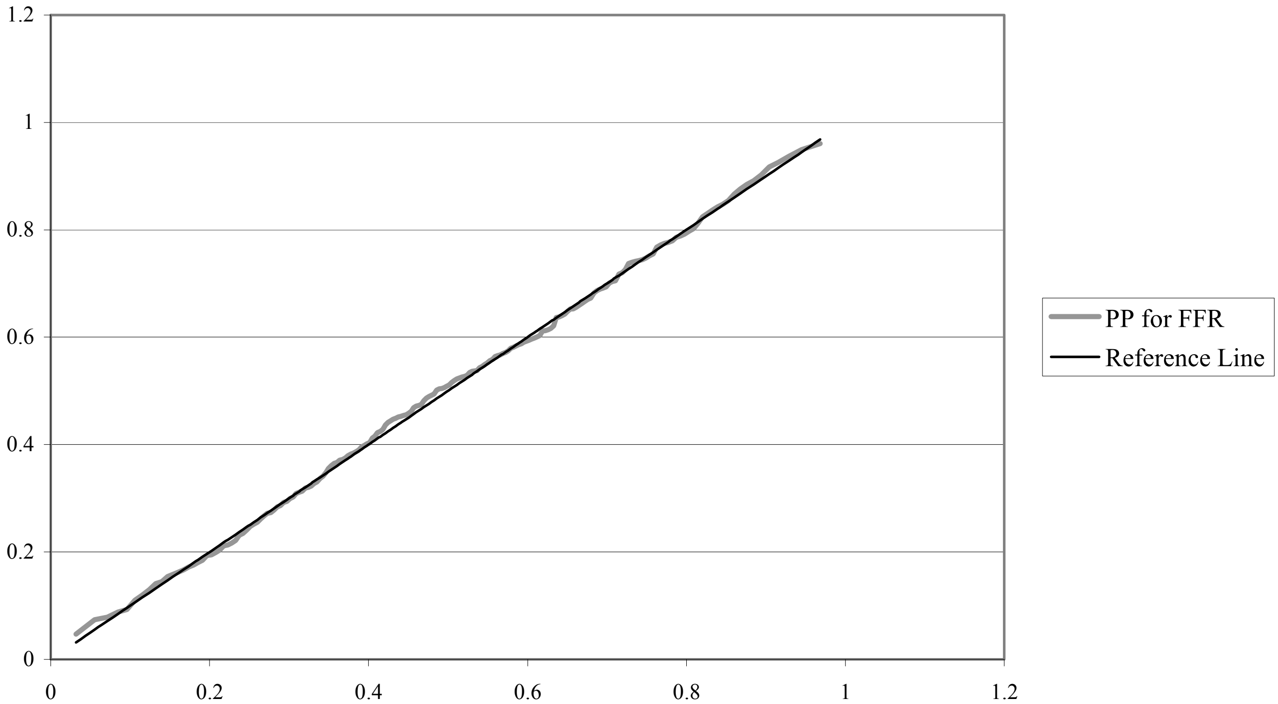


Figure 14: Modified PP Plot for NBR Stable Fit (ML Estimate)

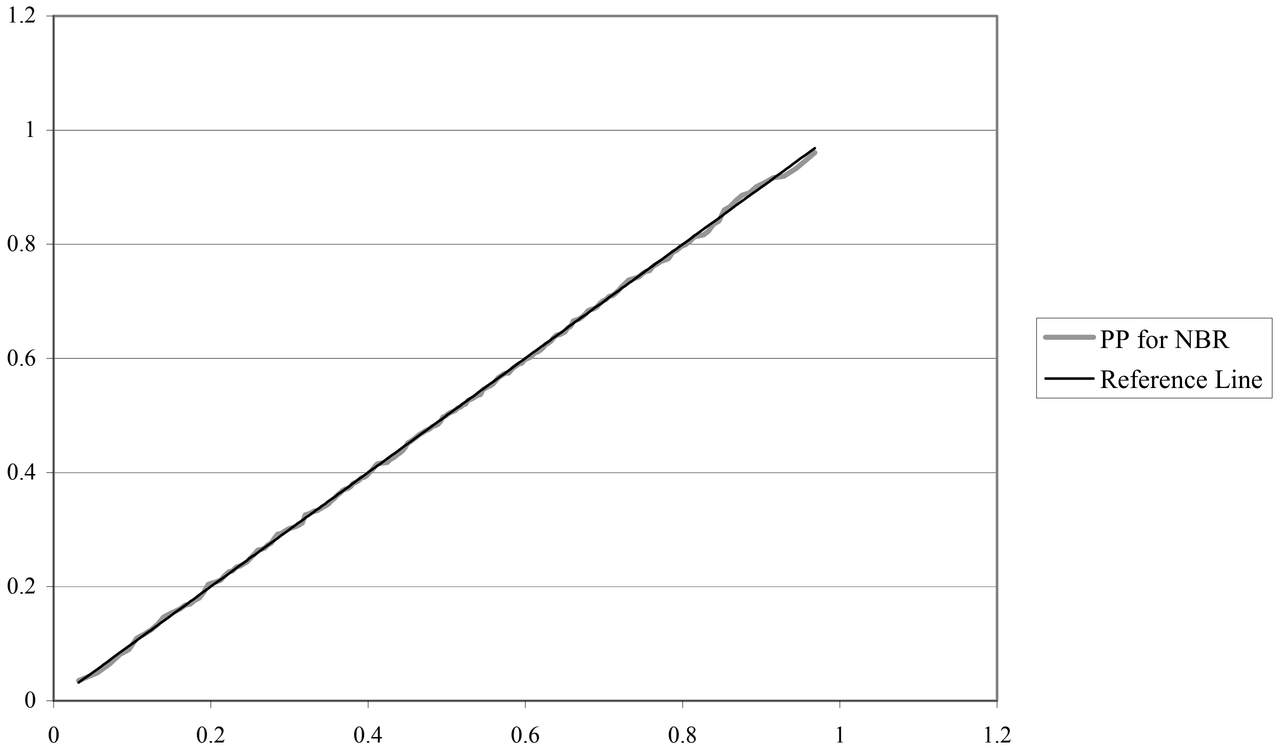
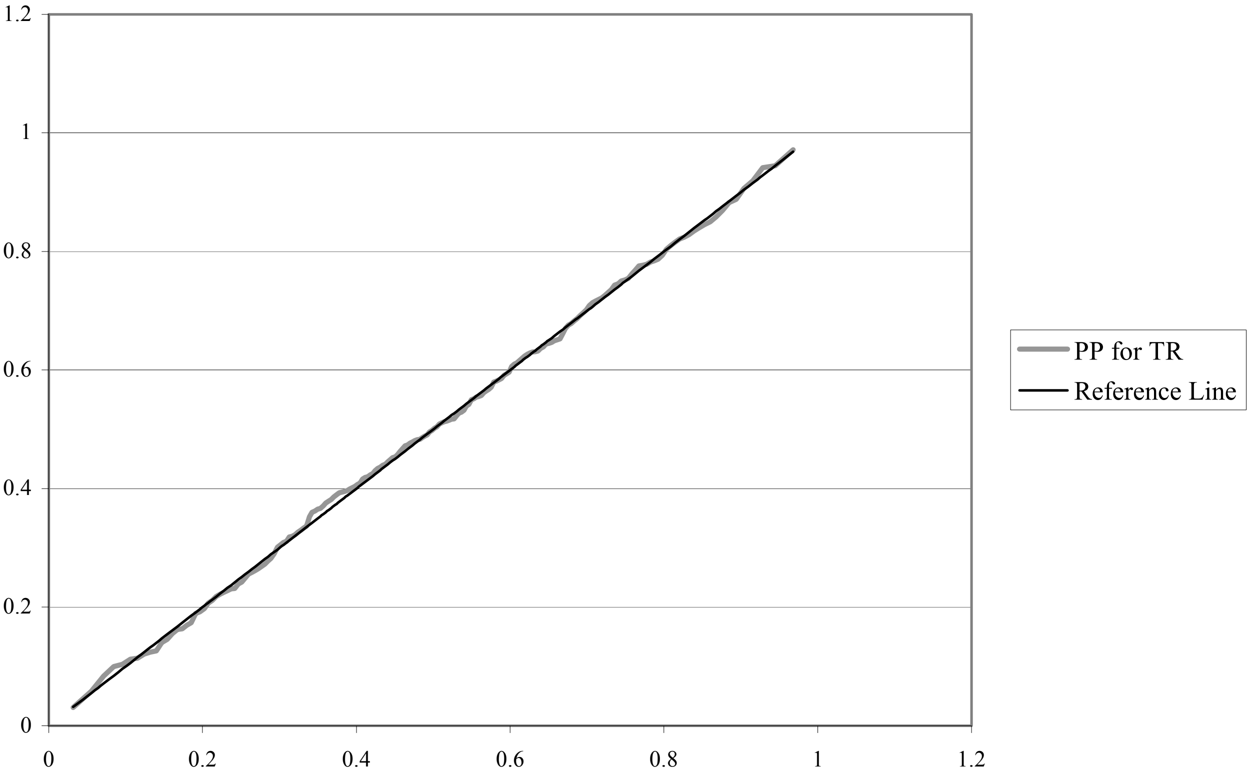


Figure 15. Modified PP Plot for TR Stable Fit (ML Estimate)



These figures show that the ML estimates appear to result in very good fits for all six series of innovations.[10]

One way of testing the hypothesis that heteroskedasticity is responsible for the appearance of non-normality is to focus on estimates over subsamples that appear relatively homoskedastic. For the 1966–79 subsample, three sets of residuals appeared to be free of ARCH effects in the tests of the previous section: IP, NBR, and TR. For the 1988–2007 subsample, CPI, FFR, NBR, and TR appeared homoskedastic according to the test. Tables 9 and 10 show estimates of α for these subperiods, with the homoskedastic innovations highlighted in gray.

Table 9. Estimated Characteristic Exponents (α) for 1966–79 Subsample

Equation	Estimator	Characteristic Exponent Estimate (α)
IP	Quantile	1.8892
	Char. function	1.9626
	ML	2.0000 (#)
CPI	Quantile	1.7983
	Char. Function	1.8648
	ML	1.8557 (.1869)
PPI	Quantile	1.5987
	Char. Function	1.8071
	ML	1.7351 (.2241)
FFR	Quantile	1.6319
	Char. Function	1.8449
	ML	1.7630 (.2166)
NBR	Quantile	1.8935
	Char. Function	1.9432
	ML	1.8673 (.1815)
TR	Quantile	1.9271
	Char. Function	1.9813
	ML	2.0000 (#)

Notes: # Confidence interval not shown for IP and TR because asymptotic theory does not apply at α = 2.
Residuals that were homoskedastic according to the tests of the previous section are highlighted with a gray background.

[10] Among 40 other tested distributions, the one that appeared to fit the residuals most closely and consistently was a log-logistic distribution.

Table 10. Estimated Characteristic Exponents (α) for 1988–2007 Subsample

Equation	Estimator	Characteristic Exponent Estimate (α)
IP	Quantile	2.0000
	Char. Function	1.9401
	ML	1.8807 (.1489)
CPI	Quantile	1.8258
	Char. Function	1.8773
	ML	1.8581 (.1559)
PPI	Quantile	1.7713
	Char. Function	1.8619
	ML	1.8520 (.1595)
FFR	Quantile	1.7428
	Char. Function	1.9431
	ML	1.9046 (.1401)
NBR	Quantile	1.8169
	Char. Function	1.7341
	ML	1.7588 (.1839)
TR	Quantile	1.7204
	Char. Function	1.7121
	ML	1.7252 (.1882)

Notes: Residuals that were homoskedastic according to the tests of the previous section are highlighted with a gray background.

The sample splits are unevenly effective in removing the non-normality of the data. Given the small sample sizes for the two subperiods, the results—including the asymptotic standard deviations—should be interpreted with great caution. For each variable in a given subperiod, the three estimators give more divergent results than in the full sample, which makes us less confident of the results. This uncertainty is also reflected in the larger two-standard-deviation asymptotic intervals reported in parentheses in the last column of each table than for the full sample. For the first subsample, the Gaussian case ($\alpha = 2$) lies outside the confidence interval for the maximum likelihood α's for FFR and PPI. For the second subsample, the estimates for NBR and TR are likewise significantly different from two. In that subsample, these last two variables both had R^2s of .001 in the Engle ARCH tests reported above. To sum up, a model that divided the sample into these two subperiods plus a third subperiod for the intervening years would succeed in removing non-normality in all subperiods for two variables at most—IP and CPI. The effort to explain away the excess kurtosis in the distributions with time-varying variances does not completely succeed, at least when heteroskedasticity is modeled with an ARCH or GARCH process.

V. SUMMARY AND CONCLUSION

This paper reports estimates of the characteristic exponents α of the innovations in a six-variable monetary VAR. The reason for finding these estimates is that for $\alpha < 2$, stable distributions have infinite variances, making structural factorizations of innovation variance-covariance matrices impossible.

This paper's VAR appears to lead to impulse response functions that are typical in the monetary VAR literature. However, diagnostics show that the innovations have thick-tailed distributions. Also, Engle (1982) tests indicate weak but statistically significant ARCH effects, which could potentially account for the thick-tailedness of the innovation distributions. Pursuing a hypothesis that the innovations have stable, non-Gaussian unconditional distributions, the paper finds ML estimates of the α's ranging from 1.5504 for the innovations in the equation for the crude materials producer price index (PPI) to 1.7734 for the industrial production (IP) equation. Using the asymptotic confidence intervals, all of these estimates of α are significantly different from two, the value for the Gaussian case. P-P plots give a visual impression that the estimated stable distributions fit the innovations well. Following through on the earlier observation of ARCH effects, the paper re-estimates the VAR for subperiods that appear free of heteroskedasticity based on Engle (1982) tests. For the 1988–2007 subsample, two variables without statistically significant ARCH effects—the innovations in the NBR and TR equations—had estimated α's that were very close to the estimates for the full sample. Moreover, for most of the key purposes of VARs such as the one in this paper, it is the *unconditional* distribution of the innovations that is relevant for the purpose of identifying the structural residuals.

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