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Infinite-variance, Alpha-stable Shocks in Monetary SVAR: Final Working Paper Version

by

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ABSTRACT

This paper adumbrates a theory of what might be going wrong in the monetary SVAR literature and provides supporting empirical evidence. The theory is that macroeconomists may be attempting to identify structural forms that do not exist, given the true distribution of the innovations in the reduced-form VAR. The paper shows that this problem occurs whenever (1) some innovation in the VAR has an infinite-variance distribution and (2) the matrix of coefficients on the contemporaneous terms in the VAR's structural form is nonsingular. Since (2) is almost always required for SVAR analysis, it is germane to test hypothesis (1). Hence, in this paper, we fit α-stable distributions to VAR residuals and, using a parametric-bootstrap method, test the hypotheses that each of the error terms has finite variance.

Keywords: Vector Autoregression; Lévy-stable Distribution; Infinite Variance; Monetary Policy Shocks; Heavy-tailed Error Terms; Factorization; Impulse-Response Function

JEL Classifications: C32, C46, C50, E30, E52

1. INFINITE-VARIANCE, ALPHA-STABLE SHOCKS IN MONETARY SVAR

Following a seminal work by Sims (1980), economists often estimate vector autoregression (VAR) of the following form

$$Y_t = C(L)Y_{t-1} + \varepsilon_t \qquad (1)$$

where Y_s is a vector of economic variables, C(L) is a matrix polynomial in the lag operator L, and ε_t is a vector of serially independent disturbances with covariance matrix Σ.

Frequently, one uses such a reduced-form VAR to identify a structural or semi-structural VAR (SVAR) such as

$$AY_t = B(L)Y_{t-1} + \eta_t \qquad (2)$$

where A is a square, nonsingular, positive definite matrix (Bernanke 1986; Blanchard and Quah 1989; Blanchard and Watson 1986; Sims 1986).[1]

SVARs of this form are used by macroeconomists to answer research questions such as: Do central banks cause recessions (Sims and Zha 2006a)? Could shocks to the supply of oil have something to do with these recessions (Bernanke, Gertler, and Watson 1997; Hamilton and Herrera 2004)? Could contractionary monetary policy shocks increase inflation (Barth and Ramey 2001)? Have the Fed's policymaking rules changed over time, and if so, has the economy performed better as a result of such changes (Sims and Zha 2006b; Benati and Surico 2009)? Are the properties of a particular dynamic stochastic general equilibrium model consistent with macro data (Smets and Wouters 2003)? Is the business cycle driven mostly by technology shocks, as opposed to monetary shocks or other "real" shocks (Galí 1999, Galí and Rabanal 2004; Francis and Ramey 2005)? What are the effects of fiscal-policy shocks (Romer and Romer 2010)? The use of SVAR techniques is almost ubiquitous in macroeconomics. Hannsgen (2008) argued that the disturbances in one or more of the equations in (1) might well have infinite unconditional variance when estimated using macro data and typical specifications. In fact, the paper reports results suggesting that infinite-variance stable distributions of the type discovered by Paul Lévy (1925) in the early 20th century fit the residuals from a standard monetary VAR model quite well. This empirical issue is crucial for

[1] Two key reference works that cover SVAR are Lütkepohl (2006, especially 357–386) and Hamilton (1993, especially 324–340). Breitung, Brüggemann, and Lütkepohl (2004) focus on SVAR. Watson (1994) is an early handbook article on VARs, and SVARs in particular, while Christiano, Eichenbaum, and Evans (1999) and Stock and Watson (2001) are surveys that emphasize applied SVAR work in macroeconomics. Qin (2010) surveys VAR research since the late 1970s, providing an historical account of the "rise of VAR modeling approach," and Sims (2010) provides a retrospective on the SVAR literature.

SVAR analysis using equations (2). Most crucially, perhaps, if the true VAR reduced-form shock vector ε_t does not have a finite covariance matrix Σ, no structural representation such as (2) exists for system (1), unless we allow A to be singular. In particular, the vector of orthogonal shocks $\eta_t = A\varepsilon_t$ cannot be constructed when one or more components of ε_t has infinite variance (See section 3 and appendix 1 for more on this issue.)

In a statistics journal, Hill (2006) has drawn attention to a broader array of problems with the use of VARs on data possessing fat-tailed distributions. Zarepour and Roknossadati (2008) have studied a VAR with infinite-variance non-Gaussian shocks. Long ago, Sims himself noted thick-tailed residual distributions in one of his first important articles on VARs (1980, p. 17). Nonetheless, very few articles have even pondered this issue, and it remains an important empirical question whether the shocks in VARs with typical specifications and variables in fact have finite second moments.

Any test of the hypothesis that a standard VAR model had infinite-variance innovations would presumably rest on the basis of existing tests for the normality of residuals and raw data. However, most normality tests were not designed to be implemented with alternative hypotheses involving infinite-variance distributions. Saniga and Miles (1979) were among the first to study the performance of standard normality tests when the alternative hypothesis was a stable, non-Gaussian distribution. Bera and McKenzie (1986) focused on the performance of the Jarque-Bera moment-based test against a stable non-Gaussian alternative. A more recent study by Frain (2007) considers simulation evidence on normality tests for stable variates. These and other articles have shown that some standard normality tests are fairly robust to problems that sometimes arise with heavy-tailed data. In an explicitly alpha-stable framework, DuMouchel (1983) and McCulloch (1997) explored the distributions of ML stable-distribution parameter estimators and related log-likelihood ratio test statistics for the null hypothesis $\alpha = 2$. In the context of multi-equation time series econometrics, though, little or no work has been done on normality tests with a non-Gaussian stable alternative hypothesis.

Lately, however, a great deal of thought has been given to non-Gaussian, but finite-variance, models in the SVAR context. Kilian (1998) found evidence of skew and excess kurtosis in the residuals of a monetary VAR. In an article that is highly relevant to this study, Kilian and Demiroglu (2000) showed that a parametric bootstrap could successfully correct severe size distortions in Jarque-Bera normality tests for VAR residuals and also has the advantage of reasonable power. Once the universe of alternative shock models for VARs is expanded to time-varying, and/or dependent processes, a wide array of possibilities has been discussed, though these models also have finite-variance shocks (for example, Cogley and Sargent (2005), Gambetti, Pappa, and Canova (2008), Primiceri (2005), Lanne and Lütkepohl (2008a,b; 2010), Sims, Waggoner, and Zha (2008), and Sims and Zha (2006b)).

Whether a given univariate distribution has infinite variance depends in practice only on the tails of the distribution, which determine if the expression for the population variance converges. However, numerous authors have shown in various ways that tail index estimators require very large sample sizes indeed—perhaps in excess of 10,000 observations with many common heavy-tailed distributions (Fofack and Nolan 1999; McCulloch 1997; Paolella 2001; Weron 2001).[2] Hence, a test of the composite hypothesis that a particular VAR's residuals have infinite variance could be very biased when the VAR was fitted to typical macroeconomic data sets.

An alternative approach is to condition our test on an assumption that the innovations have a stable distribution. Under this condition, the null hypothesis that a distribution has a finite variance is equivalent to the hypothesis that the stable-distribution parameter $\alpha = 2$. Arguing in favor of this approach, there are numerous *a priori* reasons why a stable distribution is likely to be at least a good approximation for many datasets. Notably, the generalized central limit theorem places stable distributions at the center of modern statistics. This theorem establishes that stable distributions are the only ones that arise as the limit of a normalized sum of independently and identically distributed variates (Embrechts, Klüppelberg, and Mikosch 1997, 79–80; Feller 1971). As Paolella points out, "although there are numerous other distributions which possess heavier tails than the normal, if one wishes to interpret the error term as a random variable representing the sum of many external effects which cannot be realistically captured by the model, the stable Paretian is the only valid candidate" (2001, 1095–1112). Moreover, many dependent processes have infinite-variance stable unconditional distributions (see, for example, Bartkiewicz, Jakubowski, Mikosch, and Wintenberger 2010). As a final example, Theorems 1 and 2 in Tucker (1968, p. 1386) show that when k random variates converge toward differing stable distributions with stable indexes α_i , $i = 1,2,3,\ldots k$, the convolution of all k variates converges to a stable distribution with $\alpha = \min(\alpha_1, \alpha_2, \alpha_3, \ldots, \alpha_k)$. Hence, a weighted sum of random variables can have an infinite-variance limiting distribution if even one of the summands has such a limit. All of the results in this paragraph

[2] The bias of some tail-index estimators is often very large for stable distributions with $\alpha > 1.5$.

demonstrate in various ways that one can in principle interpret a stably distributed error term as a sum of non-included variables with negligible effects, even without requiring that the non-included variables be independently and identically distributed.

In the main part of this study, we adopt a classical hypothesis-testing approach to infer whether α = 2 in the shocks in a standard monetary VAR model, using residuals from VARs with two alternative lag-length specifications and several different sample periods. Hence, one would wish for a test that could demonstrate the empirical plausibility of a hypothesis that one or more innovations in a given VAR had (possibly asymmetric) stable distributions.

Unfortunately, though, as Borak, Misiorek, and Weron report, "there are no standard, widely accepted tests for assessing stability" (2011, 10). In fact, tests based on increasing sums of observations such as the one used by Fama and Roll (1971) have proven somewhat fragile in simulation studies and may even be unreliable for sample sizes typical in macroeconomic research (Fielitz and Rozelle 1983; Lau and Lau 1993, Lau and Lau 1997; Paolella 2001). Hence, following some of the suggestions of Nolan (2001) and Weron (2001), we will rely partly on a "visual inspection" method to discern how well the estimated stable distributions fit the residuals. Additionally, in a separate section of this paper, we report the results of an informal comparative analysis in the spirit of Blattberg and Gonedes (1974), Rachev and Mittnik (2000, chapter 4), and Tucker (1992). More specifically, we compare log-likelihoods and goodness-of-fit measures for our estimated stable distributions with those of fitted Student's t distributions and generalized autoregressive heteroscedasticity (GARCH(1,1)) models. (We return to the GARCH(1,1) model in the last paragraph of this section.) This comparative exercise gives us some confidence that the stable model fits the error terms in our standard VAR model reasonably well.

Since this study is motivated largely by an infinite-variance critique of SVARs, statistical inference about the parameter α, often known as the characteristic exponent or stable index, is a key part of this paper. If the true stable parameter vector $(\alpha, \beta, \gamma, \delta)$ for a given VAR equation i lies in the interior of the parameter space, the distribution of the ML estimate $(\hat{\alpha}_{ML}, \hat{\beta}_{ML}, \hat{\gamma}_{ML}, \hat{\delta}_{ML})$ asymptotically approaches a multivariate normal law with a known covariance matrix equal to the inverse of the population Fisher information matrix (DuMouchel 1973). However, the use of large-sample confidence intervals to make inferences about the stable-distribution parameters in a VAR data-generating process is complicated by the fact that they are valid only conditionally on our estimates of the VAR's coefficients. These coefficients would have to be known to assure that the residuals were equal to the error terms, which are of course unobservable. Hence, DuMouchel's asymptotic standard errors cannot be used in a straightforward way to construct confidence intervals for the stable parameters in a VAR model with stable shocks.

In this paper, we conduct tests of the null hypothesis $\alpha = 2$. Unfortunately, the distribution of the ML estimator fails to meet a key regularity condition in the Gaussian region of the parameter space, preventing the use of standard asymptotic distribution theory as the basis for such a test (DuMouchel 1973; 1983, 1022–1023). Nonetheless, convergence of the ML estimator is actually faster when $\alpha = 2$. Michael Woodroofe showed that the ML estimator is superconsistent under normality, i.e., when $\alpha = 2$, $P(\hat{\alpha}_{ML} = 2) \to 1$ as the sample size $n \to \infty$ (DuMouchel 1983, 1022–1023 and appendix). Hence, "the asymptotic behavior of a test of $\alpha = 2$ is non-regular in a way that favors making a correct decision" under the maintained hypothesis of a general stable model (DuMouchel 1983, 1028).

To carry out our tests, we begin by estimating our VARs, making use of techniques and specifications that are somewhat standard in the macro SVAR literature. Then, for each VAR equation, we conduct parametric bootstrap tests of the null hypothesis that $\alpha = 2$ under the alternative hypothesis $\alpha \in (0, 2)$. (Our parametric bootstrap technique is similar to that of Kilian and Demiroglu (2000)[3]; see appendix 2 for details.) Our tests make use of (1) the ML estimator (Nolan 2001; DuMouchel 1973); (2) the empirical characteristic function estimator (Koutrouvelis 1980; Kogon and Williams 1998); and (3) the quantile estimator (McCulloch 1986). Each of these estimators also serves as a bootstrap test statistic in this study. The fourth test is a likelihood-ratio (LR) test that is conservative relative to the LR test one could hypothetically conduct if one possessed a full ML estimator for a VAR model with stable shocks. To wit, unrestricted estimates for LR tests are usually executed using a fully maximized likelihood function, which we lack for $\alpha < 2$, because of our use of the least-squares estimator. In this case, however, the asymptotic LR tests overcome our reliance on VAR coefficient estimates that may be suboptimal under the alternative hypothesis, by settling for a lower bound on the test statistic $-2LLR = -2(\ell(\hat{\theta}_{ML\ RESTRICTED}; Y) - \ell(\hat{\theta}_{MLE}; Y))$ that is usually needed for an LR test. (For more details, see also section 5.)

[3] The fact that the parameters lie on the boundary of the parameter space does not preclude a valid bootstrap, because we test only an equality restriction (Andrews 2000).

Given that we are using the correct specifications, numerous tests imply that $\alpha < 2$ in various VAR equations, especially those for nonborrowed reserves (NBR) and total reserves (TR), and for most of our tested sample periods. These findings uphold the notion that infinite-variance innovations might present serious problems for SVARs, as suggested above and further explained in section 3 of this paper. The thrust of our conclusions generally persists under both of our lag-length specifications.

Our tests are based on a VAR model with shocks that are i.i.d. The i.i.d. assumption is probably not accurate for most of the error terms, at least in our full-sample VARs. In particular, standard ARCH tests reveal at least mild heteroscedasticity in most of the residuals from our full-sample VARs. This departure can complicate inference in two ways. First, the unconditional distributions of heteroscedastic datasets or residuals can appear to be fat-tailed, sometimes even when they are not. In fact, heteroscedastic models with finite variance have been key rivals for i.i.d. non-Gaussian, stable models, particularly in the field of finance (Clark 1973; Ghose and Kroner 1995). Hence, it seems likely that heteroscedastic shocks would lead to downward bias in our estimates of α and probably to overrejection in our tests of normality. Second, serial dependence in the squares or absolute values of the shocks would reduce the efficiency of the coefficient estimates in VARs such as the ones in this paper.

Hence, in essence, it is useful to disentangle the effects of fat-tailed shocks from those of time-varying scale or variance. To some extent, this issue is resolved in this paper by examining residuals from VARs estimated on subsamples for which the null of homoscedasticity is not rejected. In addition, we re-estimate stable parameters for the full-sample VARs after filtering (standardizing) their residuals using a GARCH(1,1) model (Borak, Misiorek, and Weron 2011, 24–28). Tests on the σ_t-filtered residuals yield estimates of α that almost all fall well below 2, though filtering the residuals generally increases $\hat{\alpha}$. It thus appears that some of the VAR error terms in our models might combine standard stable, non-Gaussian shocks with time-varying scale, as in deVries (1991), Haas, Mittnik, Paolella, and Steude (2005), Liu and Brorsen (1995), and Mittnik, Paolella, and Rachev (2002). Nonetheless, we remain interested primarily in the unconditional distributions of the error terms in both full and partial samples. The reason for this focus is that most of the key results from the monetary VAR literature involve unconditional distributions. For example, more modern techniques such as the Markov-switching models introduced in Sims, Waggoner, and Zha (2008) do not allow one to obtain time-invariant impulse response functions, even for short sample periods.

This study analyzes these issues as follows: section 2 provides background on stable distribution theory; section 3 presents an infinite-variance critique of SVARs; section 4 discusses the data and estimation procedures used in the VARs from which we obtain our residuals; section 5 presents and discusses our estimates of stable parameters for the error terms in our full-sample VARs and our tests of the null hypothesis $\alpha = 2$; section 6 extends our case to the error terms in VARs estimated for subperiods of our sample; section 7 uses a GARCH filtering technique to obtain signals regarding the conditional distributions of our error terms; section 8 compares the fits of our estimated stable distributions with the fits of t distributions and those of our estimated GARCH shock models from the previous section; finally, section 9 further discusses the findings of this paper.

2. ALPHA-STABLE DISTRIBUTIONS

The many special statistical properties of alpha-stable random variables offer some theoretical reasons for the use of alpha-stable error terms in an econometric model (Bartels 1977) and suggest why alpha-stable distributions have been found in many kinds of scientific and financial data, starting in the early 1960s with the work of Mandelbrot and Fama (Mandelbrot 1963, 1967; Fama 1963, 1965a and b; Palágyi and Mantegna 1999).[4]

Stable distributions, sometimes referred to as stable-Paretian or Lévy-stable distributions, are the only possible limiting distributions for sums of i.i.d. shocks. That is, a random variable X has a stable distribution if it has a domain of attraction, i.e., if there is a sequence of i.i.d. random variables Y_1, Y_2,…. and sequences of positive numbers $\{d_n\}$ and real numbers $\{a_n\}$, such that

$$\frac{Y_1 + Y_2 + \ldots + Y_n}{d_n} + a_n \stackrel{d}{\Rightarrow} X$$

where the arrow symbol means "converges in distribution to" as the sample size $n \to \infty$ (Samorodnitsky and Taqqu 1994: 5). If the Y's have a finite variance, X is normally distributed.

[4] Stable distributions were largely discovered by Paul Lévy (1925). Two references on stable distributions and processes are Samorodnitsky and Taqqu (1994). More applied introductions can be found in Adler, Feldman, and Taqqu (1998), Borak, Misiorek, Weron (2011), Embrechts, Klüppelberg, and Mikosch (1997), Nolan (forthcoming), and Rachev and Mittnik (2000). Econometric results and issues involving stably distributed variables are discussed in Rachev, Kim, and Mittnik (1999a and b). Andrews, Calder, and Davis (2009) is a contribution in the area of autoregressive processes.

Furthermore, an alternative definition is available. Except for a few special cases, stable distributions have no closed-form CDFs or PDFs. But a random variable Z has a stable distribution iff it has the same distribution as aZ + b, where Z can be characterized by the characteristic function

$$\varphi(u) = E(\exp(iuZ)) = \exp\left(-|u|\left[1 - i\beta\tan\left(\frac{\pi\alpha}{2}\right)(sgn\, u)\right]\right)$$

with $\alpha \in \{(0,2]\backslash 1\}$ and $\beta \in [-1,1]$, or by

$$\varphi(u) = E(\exp(iuZ)) = \exp\left(-|u|[1 + i\beta(2/\pi)(sgn\, u))log|u|]\right)$$

with $\alpha = 1$ and $\beta \in \{[-1,1]\}$

The parameters in this characteristic function have the following interpretations:

α = characteristic exponent or stable index. This parameter affects the kurtosis of the distribution. Lower values of α are associated with higher peaks near the center of the distribution and with thicker tails. Figure 1 shows an example of how the value of α affects the shape of a stable distribution.

β = skew parameter. Negative values mean that the distribution is skewed to the left and positive values indicate skew to the right.

In addition, the following parameters can be used to make the distribution wider or narrower or to shift it horizontally along the real line:

γ = scale parameter (–∞ < γ < ∞)

δ = location parameter (–∞ < δ < ∞)

A normal distribution is a stable distribution with α = 2 and β = 0 and has no skew or excess kurtosis. Also, of course, normal distributions have finite variances. On the other hand, when α < 2, the variance is infinite and is sometimes said not to exist. As we see next, when one or more VAR error terms has a distribution with infinite variance, the consequences for SVAR analysis are serious indeed and go well beyond those caused by error terms with mildly thick-tailed distributions and finite variances.

3. VARs WITH ONE OR MORE INFINITE-VARIANCE ERROR TERMS DO NOT HAVE STRUCTURAL REPRESENTATIONS

This paper examines the implications for SVARs of infinite-variance innovations. To see these implications, recall that the structural form of a VAR model of order p is

$$AY_t = B_1Y_{t-1} + B_2Y_{t-2} + ... + B_pY_{t-p} + \eta_t \qquad (2')$$

where A and the B_is are k-by-k matrices of parameters, with A nonsingular; Y_t, t = 1, 2, 3….., T, are k-component vectors of economic variables at time t; and η_t is a k-component vector of structural shocks. Presample values $Y_{-(p-1)}$, $Y_{-(p-2)}$,…, Y_{-1} , Y_0 are taken as given. To keep the notation simple, we have not included a constant vector in this equation, though we use one in the specification described below.

In addition, SVAR uses a set of distributional assumptions about the structural shock vector like the following:

$$
\begin{aligned}
&E(\eta_t) = 0 \\
&E(\eta_t \mid Y_{t-1}, Y_{t-2}, \dots, Y_{t-p}) = 0 \\
&E(\eta_t \eta_s') = I \;\; for\ t = s \\
&\qquad\quad = 0 \;\; for\ t \neq s
\end{aligned}
.
$$

where I is the k-by-k identity matrix.[5] An estimate of the structural form (2) is indispensible for much of the work that is done with VARs. The parameter matrices B_j and the structural shock vectors η_t, t = 1, 2, 3,…., T–1, T, of (2) are usually identified using the reduced form VAR[6]

$$Y_t = C_1 Y_{t-1} + C_2 Y_{t-2} + \dots + C_p Y_{t-p} + \varepsilon_t \qquad (1')$$

where

$$
\begin{aligned}
&\forall\ j\ and\ \ for\ t = 1, 2, 3, \dots T \\
&C_j = A^{-1} B_j \qquad (3) \\
&\varepsilon_t = A^{-1} \eta_t
\end{aligned}
$$

The covariance matrix of ε_t is

$$\Sigma = E(\varepsilon_t \varepsilon_t') = E(A^{-1} \eta_t \eta_t' A^{-1}{}') = A^{-1} A^{-1}{}' \qquad (4)$$

To find the needed parameter and shock estimates, one first estimates the reduced form (1). The residuals $\hat{\varepsilon}_t$ from the estimated system are consistent estimates of the shocks ε_t, but the most important uses of SVARs require that we identify the η_t. To do this, one first obtains an estimate $\hat{\Sigma}$ of the error covariance matrix. One must then make use of identifying restrictions. For example, most early articles adopted the identifying condition that A is a lower triangular matrix. In this case, A can be identified by decomposing $\hat{\Sigma}$ into the product of a lower-

[5] Many studies make more specific distributional assumptions about the disturbance term η_t, especially for maximum likelihood estimation (Hamilton 1994, 291–302). Also, $E(\eta_t \eta_t')$ is sometimes assumed to be an arbitrary diagonal matrix D with strictly positive diagonal elements, rather than the identity matrix (Bernanke 1986; Sims 1986).

[6] The stability condition requires that the characteristic roots of the system (1) lie within the complex unit circle.

triangular matrix $\hat{A}^{-1}$ and its transpose $\hat{A}^{-1\prime}$ (the Cholesky factorization) and inverting the former to obtain $\hat{A}$. Estimates of η_t, t = 1, 2, 3…T–1, T, can then be obtained from the relationship

$\hat{\eta}_t = A\hat{\varepsilon}_t$

In the years since Sims's (1980) article, macroeconomists have developed various new ways of identifying SVARs, including long-run restrictions (Blanchard and Quah 1989), sign restrictions (Uhlig 2005), and nontriangular patterns of zero restrictions on the elements of A (Bernanke 1986; Blanchard and Watson 1986; and Sims 1986). Almost all of these identification schemes involve factorizations of Σ.[7]

The two main uses of the structural estimates are:

1. Impulse response functions (IRFs) based on the structural moving average representation

$$Y_t = D_0\eta_t + D_1\eta_{t-1} + D_2\eta_{t-2} + \cdots$$

which measure the effects over time of a one-unit or one-standard-deviation shock to a component of the structural shock vector η_t, and

2. Forecast error variance decompositions (FEVDs), which reveal the proportion of the random variation of each variable in Y_t that is due to variation in each component in the shock vector η_t.

With the use of various identifying restrictions, the structural shocks are interpreted as estimates of monetary policy shocks, money demand shocks, technology shocks, and the like. However, when the covariance matrix Σ has one or more infinite components, the error-term specification for the VAR model (1 and 2) is not correct. Also, the decomposition $\Sigma = A^{-1}A^{-1\prime}$ is not possible, and hence the structural model (2) cannot be obtained from the reduced-form VAR (1), once we have specified the error terms for the latter model correctly.[8] In the case of a particular VAR DGP with infinite-variance innovations, all elements of an estimate $\hat{\Sigma}$ will of course be finite, but a true finite Σ does not exist. Hence, the identification process is futile for such a DGP: there is no meaningful estimate of the structural shocks η_t and coefficients A and

[7] An instrumental-variables estimator for SVARs with long-run restrictions is presented in Shapiro and Watson (1989). Proposition 1 below applies to this case as well.

[8] Also, if more than one innovation has infinite variance, some off-diagonal entries in the variance-covariance matrix will be infinite.

B_j in the corresponding structural model (2)[9], making structural IR and FEVD analysis impossible.

A more rigorous statement of the existence problem posed for SVAR by infinite-variance innovations might be of help. One reason is that the critique proposed here might seem only to call for different estimators of A and the rest of the structural DGP that do not make use of a factorization of Σ (e.g., Shapiro and Watson 1989). In fact, though, there exists no nonsingular A that transforms the innovation vector ε_t into a vector η_t of orthogonal shocks when one or more components of ε_t has variance $\sigma^2 = \infty$. This is shown in the following proposition.

PROPOSITION 1: *Let ε_t and η_t be two random k-element vectors and let A be a k-by-k nonsingular matrix of real numbers, with $\eta_t = A\varepsilon_t$. If one or more of the elements of ε_t has infinite variance, then*

$$E(\eta_t \eta_t') \neq I$$

The proposition still holds if the identity matrix I above is replaced by any other finite k-by-k matrix W.

Proof: See appendix 1.

Thus, when at least one innovation ε_{it} has infinite variance, no suitable transformation A exists that can generate structural shocks η_{it} satisfying the crucial identifying condition of orthogonality, or for that matter having any covariance matrix called for by a structural model such as (2). It is a simple matter to show that this transformability problem arises in almost all SVAR models if one or more of the reduced-form shocks has infinite variance. These include, for example, the A, B, and AB models presented in Lütkepohl (2006: 358–368), all of which explicitly require a finite covariance matrix Σ.

[9] Another implication of infinite variance time series is that standard estimators will generally be inefficient. Robust estimation for stable models is a complex subject; see footnote 4 for some references. Moreover, bootstraps for impulse response functions can fail when the shocks have thick tails (Kilian 1998). Athreya (1987) also discusses problems with the bootstrap under infinite variance.

4. THE RESERVES VAR: DATA, ESTIMATION PROCEDURE, AND PRELIMINARY RESULTS

Our VAR was chosen to resemble closely many of those used in the monetary VAR literature, as initiated by Sims (1980) and documented in surveys such as Christiano, Eichenbaum, and Evans (1999); Leeper, Sims, and Zha (1996, 1–39); Sims (2010); Stock and Watson (2001).

The data are monthly and span the period January 1959–November 2007.[10] The VAR's variables are industrial production (IP), the consumer price index for all urban consumers (CPI-U), the producer price index (PPI) for crude materials[11], the federal funds rate (FFR), and the Federal Reserve's nonborrowed reserves (NBR) and adjusted total reserves (TR) series. All variables other than FFR were used in their officially deseasonalized forms and transformed by taking logs.[12] A constant and 12 lags of each variable appear on the right-hand side of each equation in our primary VAR. We also performed various tests using a 3-lag specification. (The 12-lag specification was selected by starting with 12 lags and testing down with a standard LR test for the omission of the last lag; the 3-lag model was selected by the AIC and the FPE criterion.) Finally, we follow the bulk of the SVAR literature in estimating our VAR in levels.[13]

The coefficients of the reduced form (1) and the corresponding innovation vectors ε_t, t = 1, 2, 3,…., T–1, T, were estimated using equation-by-equation ordinary least squares (LS). This estimator is relied upon in numerous monetary-VAR articles such as Christiano, Eichenbaum, and Evans (1996), Lanne and Lütkepohl (2008b), Bernanke and Mihov (1998a and b), and Strongin (1995), which all employ specifications somewhat similar to the ones estimated in this paper. In addition to the widespread use of the equation-by-equation LS estimator in the SVAR literature, at least two other reasons can be adduced to justify this study's reliance on this method: **1)** under the null hypothesis of i.i.d. normal shocks, equation-by-equation LS is the ML estimator for the VAR (Lütkepohl 2006, 89–90). Under the null

[10] This period does not precisely correspond to the sample period, because of the use of presamples for all VAR estimates reported in this paper. See notes below Table 3.
[11] This commodity price index is generally included in monetary VARs for the reasons discussed in Sims (1992) and elsewhere in the subsequent literature.
[12] The NBR variable, described below, fell to negative levels in January 2008, making the log transformation impossible. The decline began with a sharp fall in the previous month. A somewhat arbitrary decision was made to truncate the sample so as to omit the entire episode, rather than including one part of it but not another.
[13] Differencing all of the variables in (2) or transforming (2) to a VECM would not affect the α of a VAR error term with a stable distribution, because a linear combination of α-stable variables is α-stable (Samorodnitsky and Taqqu 1994, 2).

hypothesis, this set of regressions yields pointwise consistent estimates of the realizations ε_{i1} $,\varepsilon_{i2}$, $\varepsilon_{i3},\ldots$, $\varepsilon_{i(t-1)},\varepsilon_{it}$, which can be used for our tests; or alternatively, **2)** under a somewhat different null hypothesis of standard white-noise shocks with finite fourth moments, the coefficient estimators would be consistent and qualify as the efficient GLS estimators (Lütkepohl 2006, 73–75).

The principal empirical concern of this paper is the distribution of the innovations in the reduced-form VAR. The estimated innovations for each equation in our primary full-sample VAR are plotted in figure 2, along with dotted lines at plus and minus one standard error from the mean. Some extreme observations are quite distant from the mean. Figure 2 gives the impression that the scale of some of the shocks changes over time. Some additional results and diagnostics appear in Table 1. In general, standard regression output should be viewed as potentially misleading when one or more error terms has infinite variance, because autocorrelations and unconditional moments of order greater than 2 also do not exist in such conditions, and the corresponding sample statistics do not generally converge to constants, among many other problems (for example, see Cohen, Resnick, and Samorodnitsky 1998) . Nonetheless, Table 1 shows that each set of residuals indeed has excess kurtosis (with estimates ranging from 4.46 for IP to 123.60 for TR), and some are very skewed. The residuals tend to have very weak sample autocorrelations. Five of the six equations had R^2s that exceeded 99.7 percent, and the lowest was greater than 98.1 percent. All of the characteristic roots of the VAR lay within the unit circle in the imaginary plane, meaning that the stability condition was met. Our 3-lag estimate yielded similar regression diagnostics, some of which are shown in Table 2. In the next section, we present the results of the key formal tests in this paper.

5. DO A VAR'S REDUCED-FORM SHOCKS HAVE INFINITE VARIANCES? ESTIMATED CHARACTERISTIC EXPONENTS $\hat{\alpha}$ AND TESTS OF NORMALITY

Proposition 1 in Section 3 establishes that we cannot orthogonalize the innovations in a standard VAR model when at least one of them has infinite variance. This section investigates the estimated shocks from the VAR described in section 4 above to see if they suffer from this problem. In this section, we limit our attention to unconditional stable distributions.

Here is our 6-equation stable VAR model, which is a version of (1):

$$Y_t = C(L)Y_{t-1} + \varepsilon_t$$

$$\varepsilon_t \equiv (\varepsilon_{1t}, \varepsilon_{2t}, \varepsilon_{3t}, \varepsilon_{4t}, \varepsilon_{5t}, \varepsilon_{6t})' \sim F(\varepsilon), \quad \varepsilon \equiv (\varepsilon_1, \varepsilon_2, \varepsilon_3, \varepsilon_4, \varepsilon_5, \varepsilon_6)'$$

$$E(\varepsilon_t | \mathcal{I}_{t-1}) = E(\varepsilon_t)$$

$$F_{\varepsilon i}(\varepsilon_i) = \int_A \varepsilon dF(\varepsilon) = S(\alpha_i, \beta_i, \gamma_i, \delta_i), \quad A = \{x \in \mathbb{R}^6 : x_i = \varepsilon_i\}$$

$$\alpha_i \in (1,2], \quad i = 1, 2, 3, 4, 5, 6$$

for t = 1, 2, 3…, T–1, T

with presample $Y_{-(p-1)}, Y_{-(p-2)}, Y_{-(p-3)}, \ldots, Y_{-1}, Y_0$ given

where the subscript i denotes the VAR equation number, t indexes time, $\mathcal{I}_t$ is time t information, and S(.) is a general alpha-stable probability law, a concept that we presented in Section 2. The distribution F exists within the framework of a probability space $(\mathbb{R}^6, \mathcal{B}^6, F)$. (We drop the formal assumption that $\alpha_i > 1$ in our tests below; imposing this constraint would not change any of our estimates.) One way of specifying this model would be to let F(.) be a general multivariate stable distribution (Samorodnitsky and Taqqu 1994, 55–110), though it would not be feasible to estimate such a model, which would be infinite dimensional.
In any case, Proposition 1 in section 3 shows that it is germane to estimate the stable parameter vectors $(\alpha_i, \beta_i, \gamma_i, \delta_i)$, i = 1, 2, 3, 4, 5, 6 and test

$H_0: \alpha_i = 2$, for $i = 1, 2, 3, 4, 5, 6$

against

$H_1: \alpha_j \in (1, 2)$, for one or more j $\in \{1, 2, 3, 4, 5, 6\}$

Our feasible tests are of the form:

$H_{0i}:\ \alpha_i = 2$

against

$H_{1i}:\ \alpha_i \in (0, 2)$

Of course, these latter tests are not independent across all of the equations in a given VAR estimate.

Akgiray and Lamoureux (1989), Borak, Misiorek, and Weron (2011), Garcia, Renault, and Veredas (2010), Kogon and Williams (1998), Lombardi and Calzolari (2008), and Rachev and Mittnik (2000) discuss the relative merits of some methods for estimating stable parameters. DuMouchel (1973) shows that except for some "exceptional parameter values," including α = 2, the maximum likelihood (ML) estimates $\hat{\alpha}, \hat{\beta}, \hat{\gamma},$ and $\hat{\delta}$ are consistent and

$$n^{1/2}(\hat{\alpha}-\alpha, \hat{\beta}-\beta, \hat{\gamma}-\gamma, \hat{\delta}-\delta)$$

is asymptotically normally distributed with mean (0, 0, 0, 0) and covariance matrix $n^{-1}\mathcal{J}^{-1}$, where $\mathcal{J}$ is the Fisher information matrix.[14]

Here, we use three estimators of α, β, γ, and δ: the quantile estimator of McCulloch (1986), the characteristic-function regression estimator of Koutrouvelis (1980) and Kogon and Williams (1998), and the ML estimator (DuMouchel 1973; Nolan 2001). For the ML procedure, we use an algorithm and software developed by Nolan (2001), which are discussed, for example, in Borak, Misiorek, and Weron (2011, 7–8 and 13) and Rachev and Mittnik (2000, 119–136). Our tests of normality were discussed briefly in the first section of this paper. Our preferred test is an LR test. In effect, we use our restricted ML (least squares) coefficient estimates $\hat{C}_{MLR}$ to concentrate the log-likelihood function. This estimation procedure yields pointwise consistent estimates of the error terms of the restricted model. Then, we use the ML estimator for alpha-stable parameters, which is superconsistent under the null hypothesis (DuMouchel 1983), to conduct valid two-step tests of α = 2 for these innovations. Our LR test is similar to the one discussed in McCulloch (1997), except that we are testing VAR residuals to make inferences about the error terms, while McCulloch analyzes a test to be used on stable data.[15] Hence, we cannot use McCulloch's tabulated Monte Carlo critical values.[16]

As mentioned earlier, it is not feasible to estimate the general stable VAR model at the beginning of this section, because of its high dimensionality. Fortunately, though, our use of LS coefficient estimates does not prevent us from conducting valid tests that lead to a number of fairly conclusive results. Our estimates enable us to obtain an LR test statistic $-2LLR_{LB}$ that can be used as a lower bound on the true test statistic that we would hypothetically find if we

[14] A typical element of this latter matrix is

$$\mathcal{J}_{ij} = \int_{-\infty}^{\infty} \frac{\partial f}{\partial \theta_i} \frac{\partial f}{\partial \theta_j} \frac{1}{f} dx$$

where f is the likelihood function and θ_i is an element of the stable parameter vector θ = (α, β, γ, δ) (Nolan 2001, 384).

[15] For this use of the term "pointwise consistent," see Greene (1993, 309). The superconsistency of the ML stable-parameter estimator is covered in DuMouchel (1983). Lanne and Lütkepohl use a similar two-step "quasi-ML" procedure in the context of a similar problem (2008b, 7).

[16] We found that using the McCulloch (1997) Monte Carlo critical values for the LR test would have resulted in a large number of additional rejections of our null hypotheses, compared to our actual bootstrap LR test.

possessed full unrestricted ML estimates of the stable VAR model presented at the beginning of this section, as shown by the inequality below:

$$-2\text{LLR}_{\text{LB}} = -2\big(\ell\big(2,0,\hat{\gamma}_{MLR0},\hat{\delta}_{MLR0},\hat{C}_{MLR0};Y\big) - \ell(\hat{\alpha}_{ML},\hat{\beta}_{ML},\hat{\gamma}_{ML},\hat{\delta}_{ML},\hat{C}_{MLR0};Y)\big)$$

$$= -2(\ell\big(2,0,\hat{\gamma}_{MLR0},\hat{\delta}_{MLR0},\hat{C}_{MLR0};Y\big) - \max_{\alpha,\beta,\gamma,\delta}\ell(\alpha,\beta,\gamma,\delta,\hat{C}_{MLR0};Y))$$

$$\leq -2\big(\ell\big(2,0,\hat{\gamma}_{MLR0},\hat{\delta}_{MLR0},\hat{C}_{MLR0};Y\big) - \max_{\alpha,\beta,\gamma,\delta,C}\ell(\alpha,\beta,\gamma,\delta,C;Y)\big)$$

$$= -2\big(\ell\big(2,0,\hat{\gamma}_{MLR0},\hat{\delta}_{MLR0},\hat{C}_{MLR0};Y\big) - \ell(\hat{\alpha}_{FML},\hat{\beta}_{FML},\hat{\gamma}_{FML},\hat{\delta}_{FML},\hat{C}_{FML};Y)\big) = -2\text{LLR},$$ with probability 1

where $\ell(\cdot;Y)$ is the log-likelihood function, the ML subscript denotes our ML stable-parameter estimates, MLR0 denotes our restricted (Gaussian) estimates, FML denotes hypothetical full ML estimators, and $Y = \{Y_t\}_{t=-(p-1)}^{T}$. The inequality demonstrates that the use of -2LLR_{LB} rather than –2LLR in our LR tests is conservative, in the sense that it does not change any result from a failure to reject H_{0i} to a rejection of H_{0i}. Since the test statistic has a nonstandard distribution, we use parametric-bootstrap critical values. In addition to the LR test, we performed similar bootstrap tests of H_{0i}, using our estimators $\hat{\alpha}_{MC}$, $\hat{\alpha}_{CF}$, and $\hat{\alpha}_{ML}$ as test statistics. Our bootstrap tests are explained in appendix 2.

Our results for the full-sample VAR, with both 12-lag and 3-lag specifications, are shown in Table 3. Three estimates and four test statistics for each equation are reported on rows corresponding to each equation of the two VARs, with the 12-lag results appearing in the upper half of the table. Reporting first the results of the ML estimates and the corresponding tests, the null hypothesis of normality ($\alpha = 2$) is rejected at the .01 level for all error terms in both VARs. For the 12-lag primary specification, $\hat{\alpha}_{ML}$ ranged from 1.55 to 1.77, while the ML estimates for the error terms in our alternative 3-lag VAR ranged from 1.40 to 1.75. The quantile and characteristic function estimators yielded estimates that did not differ greatly from the ML estimates. Moreover, bootstrap tests using these latter estimates rejected H_{0i} for all i in both full-sample VARs, with all $\hat{\alpha}_{CF}$ tests achieving a .01 significance level.

For comparison, we also report results from Jarque and Bera (1987) normality tests in the right-hand column of Table 3, with bootstrap significance levels again indicated by one or two asterisks. The alternative hypothesis for this test is a nonnormal member of the Pearson family of distributions, though it has proven to be somewhat robust under a stable, non-Gaussian alternative hypothesis (Bera and McKenzie 1986; Frain 2007). For both lag-length specifications and all equations, these tests reject the null hypothesis at the .01 significance level.

Also, we provide a loose check on our work by reporting asymptotic confidence intervals in Table 3, below each ML estimate inside square brackets. These intervals are computed as $\hat{\alpha}_{ML} \mp 1.96 \times \sqrt{\hat{\sigma}^2_{ML}}$, where $\hat{\sigma}^2_{ML}$is the upper-left element in $n^{-1}\mathcal{J}^{-1}$ with $\mathcal{J}$ = the Fisher information matrix, which is computed as explained in Nolan (2001) (see also footnote 14). These confidence intervals assume exact knowledge of the coefficients C_j, j = 1, 2, 3,…, p-1, p. Hence, they cannot be used for valid tests of the error terms in our VARs. As shown in the table, they do not contradict our findings that $\alpha_i < 2\ \forall i$ in our two full-sample VARs. Finally, table 3 also reports the results of bootstrap LM tests for ARCH (Engle 1982). Both 3- and 12-lag test equations were tried. For each full-sample VAR equation, at least one of these tests rejected the null hypothesis that the shocks were homoscedastic. We return to this subject below.

6. SUBSAMPLE ANALYSIS

It is has been noted many times that structural breaks have probably occurred in DGPs for postwar U.S. macro data (McConnell and Perez-Quiros 2000; Stock and Watson 1996, 2002). Moreover, as seen earlier in Figure 2, some of our full-sample residual series appear to include some fairly lengthy periods of high volatility or low volatility. Hence, a subsample analysis is desirable as a way of increasing the number of homoscedastic residuals.

Two sample sets of sample breaks were used. First, following Bernanke and Mihov (1998a, 163), we break the sample at October 1979 and April 1988. In light of the need for a large dataset for each estimate, we use only the first and third of the three subsamples that we have created with these breaks. Also, for the same reason, we extend Bernanke and Mihov's third subperiod to the end of our sample in November 2007.

The other sample break we employ is based on analyses by such authors as Stock and Watson (2002); Christiano, Eichenbaum and Evans (1999); Frale and Veredas (2009); and Lanne and Lütkepohl (2008a, b), who find it useful to break US macro datasets into subsamples at or near the February 1984 observation. Somewhat arbitrarily, we choose a specific break date of February 1984, resulting in separate estimates for January 1959–January 1984 and February 1984–November 2007.

Our subperiod tests lead to numerous rejections of H_{0i}, as seen in Tables 4–7.[17] The vast majority of tests reject normality in all of the subperiods that we studied, except for 1966:1–1979:9, which had only T = 165 residual vectors for both specifications. The three different estimators yield estimates that tend to differ more than they do in the full sample, reducing their credibility somewhat. Nonetheless, the answers given by our five tests of normality—the tests based directly on the estimates $\hat{\alpha}_{MC}$, $\hat{\alpha}_{CF}$, and $\hat{\alpha}_{ML}$; the lower-bound LR test; and the Jarque-Bera moment-based test (Bera and McKenzie 1986; Jarque and Bera 1987; Kilian and Demiroglu 2000)—tend to coincide even for our subsample estimates. More modern tests, except for the LR test; tests on shocks from VARs with more parsimonious specifications or estimated using more recent data; and tests on longer runs of data—all of these tended to result in more rejections of H_0: $\alpha = 2$.

Next, we focus on findings from our LR test, though the tests based on $\hat{\alpha}_{ML}$ and $\hat{\alpha}_{CF}$ resulted in more rejections of the null hypothesis $\alpha = 2$. We are particularly interested in VAR equations whose error terms are homoscedastic, with $\alpha < 2$. Such findings are common in Tables 4–7, particularly for the equations corresponding to our bank reserves variables, NBR and TOTRES. In particular, for the most recent subperiods that we tried, namely 1984:2–2007:11 and 1988:4–2007:11, parametric-bootstrap LR tests on the latter two error terms all rejected normality at the .01 level of significance, while there were no signs of ARCH in these reserves equations. Among our two lag-length specifications and four subsamples, only two other error terms showed no signs of ARCH yet appeared to have $\alpha < 2$: those in the IP and NBR equations in the 12-lag VAR for the 1959:1–1984:1 subperiod. Hence, in a number of cases, the heavy tails observed in the distributions of our full-sample VAR residuals cannot be convincingly explained by structural breaks in the covariance matrix for the innovations. Moreover, in recent samples, ARCH or GARCH is not a good explanation of the pronounced excess kurtosis in NBR and TOTRES residuals.

[17] The stability condition was not met by some of our subsample VARs. Some had one or two roots just outside the unit circle: for the 3-lag specification, the 1959:1–1984:1 and 1966:1–1979:9 subperiods and for the 12-lag specifications, the 1959:1–1984:1, 1966:1–1979:9, and 1988:4–2007:11 subperiods. Standard diagnostics were satisfactory in all cases.

7. RESULTS WITH GARCH-FILTERED RESIDUALS

Suppose that the shocks in equation i of the reduced form (2) were generated by the widely used GARCH(1,1) model

$$\varepsilon_{it} = \sigma_{it}\upsilon_{it}$$

$$\upsilon_{it} \sim NIID(0,1)$$

$$\sigma_{it}^2 = q_{i0} + q_{i1}\sigma_{i(t-1)}^2 + q_{i2}\varepsilon_{i(t-1)}^2$$

$$\sigma_{i0}^2 = \bar{\sigma}_i^2$$

$$q_{ik} > 0 \text{ for } k = 0,1,2 \; ; \; i = 1,2,3,4,5,6; \;\; t = 1,2,3,\ldots,T-1,T$$

where the fourth line imposes an initial condition (Bollerslev 1986). If the ε_{it} were generated by this process, their unconditional distribution would be thick-tailed, despite the fact that the shocks υ_{it} were normally distributed. Moreover, as long as $q_{i1} + q_{i2} < 1$, we could be assured that the shock process, ε_{it}, was covariance-stationary and had finite unconditional variance (Nelson 1990).

This model has often been investigated as an alternative to an alpha-stable model for financial data (e.g., Ghose and Kroner 1995). For each equation i, a GARCH(1,1) model can be fitted to the estimated shock realizations $\hat{\varepsilon}_{it}$, t = 1, 2, 3,…., T from the VAR estimates reported in Section 4 to explore the possibility that a finite-variance, heteroscedastic model can account for the residuals' thick tails. We estimate (7) using QML, which has been shown to be rather robust to nonnormality and/or serial dependence in the υ_{it} process (Jensen and Rahbek 2004; Lee and Hansen 1994; Lumsdaine 1996).[18] The estimated GARCH(1,1) models for the innovations in the full-sample VARs are shown in Tables 8 and 9. For i = 3, 4, corresponding to the error terms from the PPI and FFR equations in both lag-length specifications,

$$\hat{q}_{i1} + \hat{q}_{i2} \approx 1$$

which suggests an IGARCH model (Engle and Bollerslev 1986). This sum is much greater than 1 for i = 5, 6 in both VARs, implying that processes generating the NBR and TOTRES shocks have infinite unconditional variances and are not covariance stationary. As we show in the next section, the GARCH models for these shocks do not appear to fit the residuals well compared to the two unconditional non-Gaussian models. The GARCH models for ε_{it}, i = 1, 2,

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[18] Some sources related to GARCH estimation in the presence of heavy tails include Linton, Pan, and Wang (2010), Hall and Yao (2003), Huang, Wang, and Yao (2008), Berkes, Horváth, and Kokoszka (2003), and Mikosch and Straumann (2006).

3, 4 were not precisely estimated, which means that we are short of information on which to base inference about whether the weak stationarity condition $q_1 + q_2 < 1$ is met in those cases. Our results for the standardized residuals $\hat{\upsilon}_{it} = \hat{\varepsilon}_{it}/\hat{\sigma}_{it}$, suggest that for all i, υ_{it} has infinite variance, given the assumption of a stable conditional distribution. This point is seen in table 10. The conditional estimates $\hat{\alpha}_{ML} \in [1.7043 \;\; 1.9165]$. Two LM tests on each series of filtered residuals fail to reject a no-ARCH null in all cases but one at the 5 percent level using standard chi-squared test cutoffs, indicating that the filters may be yielding a good signal of the conditional distributions. (These test statistics are among those shown in Table 10.) P-P plots for the ML estimates show alpha-stable fits that are roughly as good as those shown in Figures 3–8 for the unfiltered residuals. Hence, our data suggest that many of the innovations in our VARs may have both time-varying scale parameters and infinite-variance conditional distributions. A model with these properties would almost certainly imply that the corresponding error terms had unconditional distributions with infinite variances.

8. GOODNESS OF FIT VIS-À-VIS STUDENT'S t AND GARCH(1,1) MODELS

The focus of this paper has been on pitfalls resulting from infinite variance in the innovations of monetary SVARs. This focus necessitates an emphasis on finding out whether one or more shocks in a given VAR has an infinite-variance unconditional distribution. To make such tests feasible, we have made an assumption of stability that cannot be tested formally. As Nolan observes, "As with any other family of distributions, it is not possible to prove that a given data set is stable" (2001, 388).

Nonetheless, a comparative analysis of several stochastic shock models along the lines of Blattberg and Gonedes (1974), Tucker (1992), and Rachev and Mittnik (2000, 149–190) might assure us that we have relatively good stable, non-Gaussian fits. At the same time, a well-fitting alternative shock model could shed additional light on the validity of the infinite-variance critique, as long as the parameter space of the model in question could be partitioned into finite- and infinite-variance regions. Hence, for the innovation in each VAR equation, we measure the fit of a normal distribution, as well as three alternative shock models. The latter models include the GARCH(1,1) filtering model described in the previous section, the i.i.d. stable model used in section 5, and an i.i.d. Student's-t model. The t distributions are fitted to each set of residuals using the ML estimators for the t-distribution parameters μ, σ, and ν. For our stable fits, we use the ML estimates reported in Table 3.

As seen in Tables 11 and 12, we have a mixture of success stories to report in these exercises, some in support of the infinite-variance hypothesis. Three goodness-of-fit criteria are reported for each equation in both tested VAR specifications: 1) the log-likelihood of the model evaluated at the ML estimates; 2) the Anderson-Darling (AD) measure of fit; and 3) the Kolmogorov-Smirnov (KS) distance.[19]

As shown in Table 11, the standard VAR(12) model seems to have innovations that are fairly well modeled by all of the models other than the i.i.d. Gaussian shock distribution, whose fits are reported in the first three columns. Also, among the tested shock models, the NBR and TR innovations are best modeled using an i.i.d. stable non-Gaussian shock, according to almost all of the results reported in the last two rows of the table. For the other 4 shocks, the AD and KS goodness-of-fit measures mostly favored the unconditional t model. The log-likelihood criterion chose the t distribution for the residuals in the CPI equation and selected the GARCH(1,1) model for the IP, PPI, and FFR residuals.

Turning to the innovations in the standard VAR(3) model, the data in the last three rows of Table 12 show that among the three tested models, the FFR, NBR, and TR shocks seem to conform best to a stable, non-Gaussian unconditional model, according to all three criteria. On the other hand, the IP innovations are best modeled by the unconditional Student's t model, by all three criteria. The results are ambiguous for the CPI and PPI innovations, with at least one criterion selecting each of the three non-Gaussian models for these error terms. It should be pointed out that we have already mentioned the NBR and TOTRES shocks numerous times in this paper in connection with rejections of our null hypothesis, often for subsamples that appeared to be homoscedastic.

Following Michael (1983), Nolan recommends the use of variance-stabilized P-P plots[20] to determine if a dataset is consistent with an hypothesis of stability (Nolan 2001, 388). Accordingly, such plots are reported in Figures 3–7 for the residuals from our 12-lag, full sample VAR. On each of these figures, we depict lines corresponding to our Student's t and normal fits, in addition to our ML stable fits. Our P-P plots are constructed from points

[19] The Anderson-Darling and Kolmogorov-Smirnov measures of goodness-of-fit are somewhat standard. The formulas for these criteria can be found in Rachev and Mittnik (2000, 163).

[20] The formula for the abscissa in Michael's stabilized P-P plots is $t_i = (2/\pi)\arcsin(((i-.5)/n)^{.5})$, and the ordinate can be found using $s_i = (2/\pi)\arcsin(((F(x_i))^{.5})$ where x_i is the i^{th} highest observation and F(.) is the estimated cumulative distribution function (Michael 1983, 12).

representing all observations. The variance-stabilizing transformation spreads observations near the tail of the distribution, so that the variance is roughly constant along the straight line. The figures seem to confirm that the unconditional stable and t models fit all six sets of innovations in our primary specification very well indeed, while the estimated normal distributions for most of the shocks appear to fit very poorly. The pattern in all 6 normal plots is to start out below the 45-degree line on the left side of the figure, quickly rise above the line, cross the line once again at approximately the median observation, and finally rise above the 45-degree line again, remaining above the line for the observations in the right tail of the distribution. This pattern indicates that the Gaussian fit is not thick enough in either tail to fit the data, an observation consistent with the high kurtoses reported for the residuals in Table 1.

Overall, the P-P plots for the NBR and TOTRES residuals confirm the message of our goodness-of-fit and log-likelihood measures, which indicate that the alpha-stable distribution provides the best fit among our tested models for these shocks in our VAR(12) specification. For the other 4 error terms in the primary specification, the estimated t distributions seem to fit the residuals at least as well.

The tails are crucial in assessing the veracity of an hypothesis of infinite variance. Most of our stable plots seem to be relatively good in this regard. In particular, only the plots for the IP and FFR innovations seem to indicate poor stable fits for any observations at either extreme of their respective distributions. Moreover, the t distributions appear to be somewhat handicapped in fitting the asymmetries of the empirical distributions of the CPI and NBR innovations, and also perhaps that of the FFR innovation.

P-P plots for the VAR(3) specification showed stable fits that were similar in quality, and non-variance-stabilized P-P plots for the stable fits seemed to cast an even more favorable light on our stable estimates.

Finally, it should be noted in passing that one of our estimated t-distributions, namely the one for the FFR shock in the VAR(3) model, had an estimated degrees-of-freedom parameter $\hat{\nu} = 1.982 < 2$. This value, if correct, would imply an infinite-variance t distribution and hence a non-transformable VAR, by the argument in Section 3.

9. CLOSING DISCUSSION

This paper reports estimates of the characteristic exponents α of the innovations ε_{it} in a six-variable monetary VAR. The reason for seeking these estimates is that for α < 2, alpha-stable distributions have infinite variances, making it impossible to transform the reduced-form DGP into a set of structural equations with orthogonal structural shocks. Proposition 1 shows that no method of finding orthogonal disturbances can work when at least one innovation has infinite variance, because no nonsingular transformation of the innovations yields orthogonal disturbances. For our somewhat typical 6-equation monetary VARs, we have reported a great deal of empirical evidence in Sections 5, 6, 7, and 8 that infinite variance is present, especially in the full-sample estimate.

The work by Hill (2006) cited in section 1 and other, similar efforts may offer some hope for an alternative approach when standard SVAR analysis is precluded by problems with infinite-variance. The empirical generality of the findings presented here is not yet known. Hence, caution seems warranted in the use of SVAR.

Appendix 1: Proof of Proposition 1

PROPOSITION 1: *Let ε_t and η_t be two random k-element vectors and let A be a k-by-k nonsingular matrix of real numbers, with $\eta_t = A\varepsilon_t$. If one or more of the elements of ε_t has infinite variance, then*

$$E(\eta_t \eta_t') \neq I$$

The proposition still holds if the identity matrix I above is replaced by any other finite k-by-k matrix W.

Proof:

We have

$$\varepsilon_t = A^{-1}\eta_t. \tag{A1}$$

We shall assume that at least one element of ε_t has infinite variance and that, as above, $E(\eta_t\eta_t')$ = I (or = W), and proceed until we find a contradiction. Without loss of generality, assume that the first element of ε_t has infinite variance. The first equation in the system (5) can then be written

$$\varepsilon_{1t} = a_{11}\eta_{1t} + a_{12}\eta_{2t} + ... + a_{1k}\eta_{kt}$$

where the $a_{1\mathrm{t}}$ are the elements of the top row of A^{-1} and the η_{jt} are the elements of η_t. Then, the variance of ε_{1t} is

$$\begin{aligned}\text{var}(\varepsilon_{1t}) = a_{11}^2 \text{ var}(\eta_{1t}) + a_{12}^2 \text{ var}(\eta_{2t}) + ... + a_{1k}^2 \text{ var}(\eta_{kt}) + 2a_{11}a_{12} \text{ cov}(\eta_{1t}, \eta_{2t}) + \\ 2a_{11}a_{13} \text{ cov}(\eta_{1t}, \eta_{3t}) + ... + 2a_{1k}a_{1(k-2)} \text{ cov}(\eta_{kt}, \eta_{(k-2)t}) + 2a_{1k}a_{1(k-1)} \text{ cov}(\eta_{kt}, \eta_{(k-1)t})\end{aligned} \tag{A2}$$

Since by assumption the left side of (6) is infinite, at least one term on the right side must be infinite. But since $E(\eta_t\eta_t') = I$, the right-hand side of (6) equals k. This is a contradiction. The weaker assumption $E(\eta_t\eta_t')$= W, where W is some finite matrix, obviously implies a similar contradiction. Q.E.D.

Appendix 2: Bootstrap Methodology

We estimate the following reduced-form VAR, using two different lag-length specifications and a number of different sample periods:

$$Y_t = C_1Y_{t-1} + C_2Y_{t-2} + ... + C_pY_{t-p} + \varepsilon_t$$

with

$$\varepsilon_t \sim N(0,\ \Sigma)$$

The parametric bootstrap procedure is somewhat standard (see, for example, Davidson and MacKinnon 2004) and is similar to the one used in Kilian and Demiroglu (2000) for a Jarque-Bera normality test.

The simulation uses a presample ($Y_{-(p-1)}$, $Y_{-(p-2)}$, $Y_{-(p-3)}$,….,, Y_{-1}, Y_0) representing the first p data vectors in the sample, the ML estimates $\hat{C}_j$ and $\hat{\Sigma}_{ML}$ of the reduced-form coefficients C_j and the innovation covariance matrix Σ. In this case, the maximum likelihood estimator for the coefficients is the equation-by-equation least-squares estimator (Lütkepohl 2006, 87–93).

The procedure goes as follows. Suppose the sample contains T + p observations. Generate T 6×1 vectors $\varepsilon_1^{SN}, \varepsilon_2^{SN}, \varepsilon_3^{SN}, \ldots, \varepsilon_{T-1}^{SN}, \varepsilon_T^{SN}$ consisting of serially and component-wise independent draws from a standard normal distribution. Premultiply each vector by the lower triangular matrix V_L, where $\hat{\Sigma} = V_L V_U$ and V_U is the transpose of V_L. This yields the simulated shock vectors $\varepsilon_t^{SIM} = V_L \varepsilon_t^{SN}$, t = 1, 2, 3,….,T–1, T.

Next, create a bootstrap series as follows: set $Y_{-(p-1)}^{SIM}, Y_{-(p-2)}^{SIM}, Y_{-(p-3)}^{SIM}, \ldots, Y_{-1}^{SIM}, Y_0^{SIM}$ equal to $Y_{-(p-1)}, Y_{-(p-2)}, Y_{-(p-3)}, \ldots, Y_{-1}, Y_0$. Generate the rest of the series recursively, using the formula

$$Y_t^{SIM} = \hat{C}_1 Y_{t-1}^{SIM} + \hat{C}_2 Y_{t-2}^{SIM} + \ldots. + \hat{C}_p Y_{t-p}^{SIM} + \varepsilon_t^{SIM}$$

t = 1, 2, 3,…, T–1, T

Estimate C_j , j = 1, 2, 3,…, P–1, P and Σ for the simulated data using the ML estimators, as explained above. Save the bootstrap residual vectors e_t^{B1}, t = 1, 2, 3,….., T from these latter regressions. Stack these column vectors horizontally into a $6 \times T$ matrix e^{B1}. Repeat this simulation and estimation procedure 9,998 times, generating the bootstrap residual matrices $\varepsilon^{B2}, \varepsilon^{B3}, e^{B4}, \ldots, e^{B9998}$, and e^{B9999}.

Compute estimates and test statistics from each of these sets of bootstrap residual vectors. For example, the first component of the shock vector ε_t was the error term in the IP equation of the VAR. Using the first rows of $\varepsilon^{B1}, \varepsilon^{B2}, e^{B3}, \ldots, e^{B9998}$, and e^{B9999}, perform each bootstrap test for the IP residuals as follows. To begin, use McCulloch's (1986) quantile estimator to generate 9,999 estimates of α. Sort them in ascending order. Call the 99th-lowest estimate $\alpha^*_{.01}$ and the 499th-lowest estimate $\alpha^*_{.05}$. Reject the null hypothesis for the IP error term at the 1 percent significance level if the quantile estimate $\hat{\alpha}_{MC} < \alpha^*_{.01}$. If the null is not rejected in this step, then reject at the 5 percent level if $\hat{\alpha}_{MC} < \alpha^*_{.05}$. Repeating this bootstrap test procedure for rows 2 through 6 of $\varepsilon^{B1}, \varepsilon^{B2}, e^{B3}, \dots, e^{B9998}$, and e^{B9999} yields test outcomes for the 5 other innovations in the VAR.

Next, form rejection regions and perform bootstrap tests as described in the previous paragraph for the Kogon-Williams (1998) estimator $\hat{\alpha}_{CF}$ and the ML estimator $\hat{\alpha}_{ML}$. Nolan describes his ML algorithm in Nolan (1999).

The bootstrap LR test proceeds in a similar fashion. Again using each row of bootstrap residual vectors from the matrices $\varepsilon^{B1}, \varepsilon^{B2}, e^{B3}, \dots, e^{B9998}$, and e^{B9999}, test the joint hypothesis H_0: $\alpha = 2 \; and \; \beta = 0$ using the log-likelihood ratio test statistic $-2LLR_{LB}$ $=-2\left(\ell(2,0,\hat{\gamma}_{MLR0},\hat{\delta}_{MLR0},\hat{C}_{MLR0};Y) - \ell(\hat{\alpha}_{ML},\hat{\beta}_{ML},\hat{\gamma}_{ML},\hat{\delta}_{ML},\hat{C}_{MLR0};Y)\right)$ for all 6 innovations. In this case, the critical values for the test are the 99th- and 499th-highest realizations of the bootstrap test statistic. (See Section 5 for a discussion of this test and test statistic.)

Perform the bootstrap Jarque-Bera normality tests and the ARCH tests in a similar fashion to the other tests, obtaining and using different sets of critical values for each of the two lag-length specifications used in the ARCH test equations.

Repeat the entire process in the preceding paragraphs for all four subsample periods and for both lag-length specifications.

All of these procedures seemed to converge as expected.

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Figure 1. Densities of Standard Normal Distribution and Symmetric Stable with Alpha = 1.7

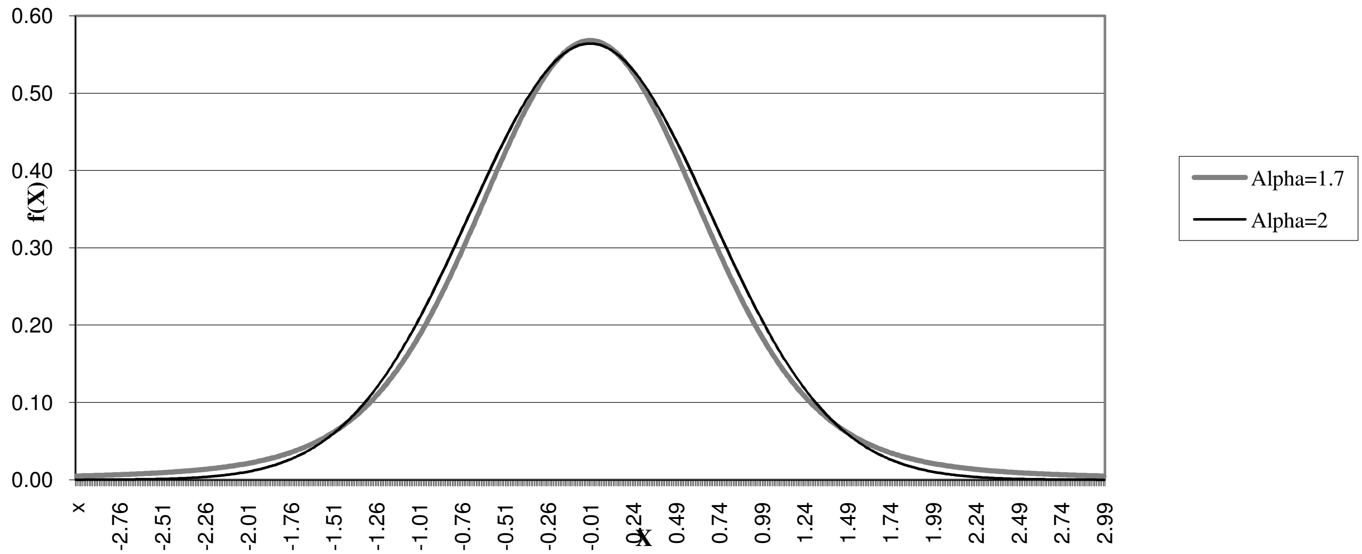


Figure 2. Reduced-Form VAR Least-Squares Residuals (Shocks) ε_t^{LS}

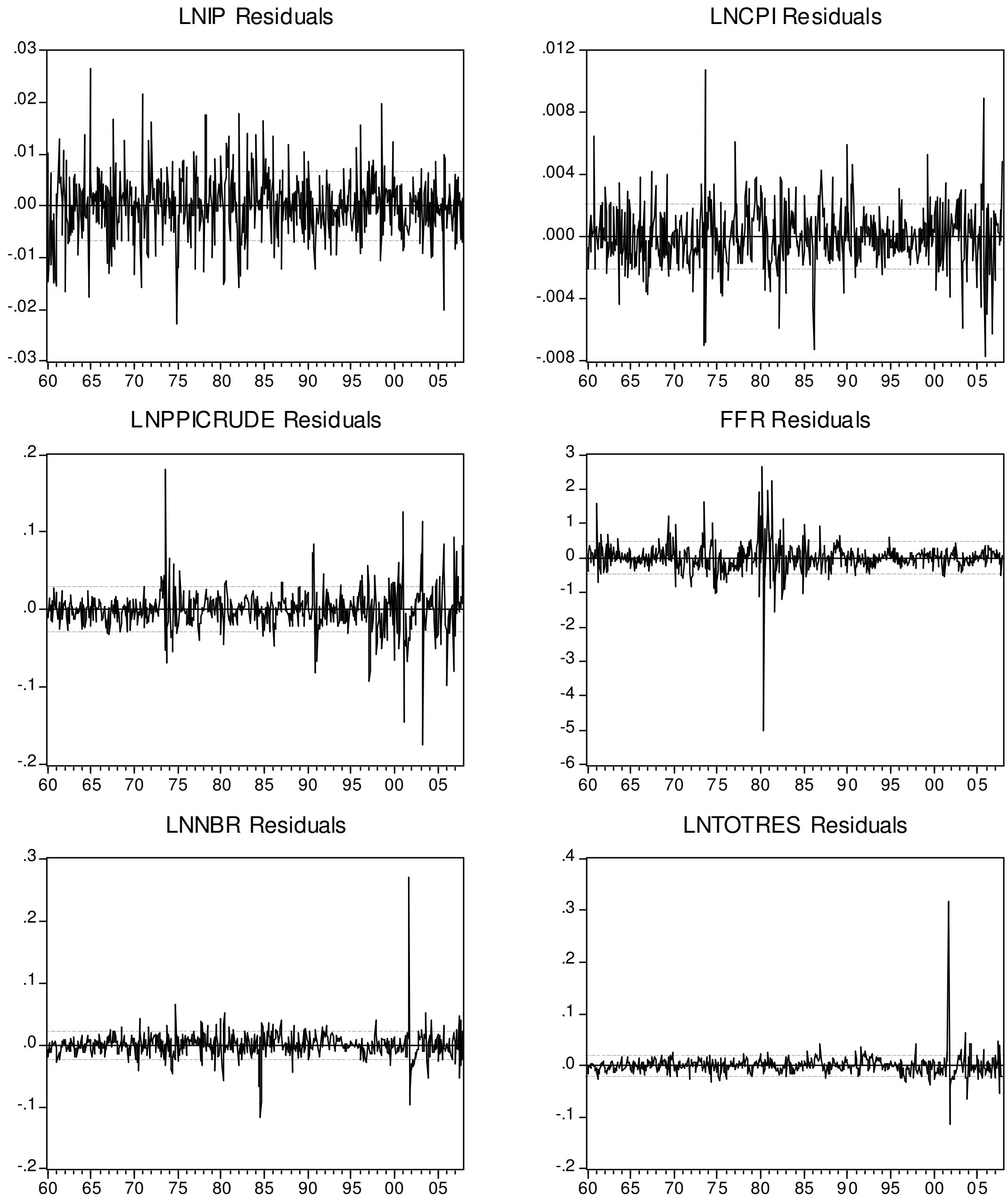
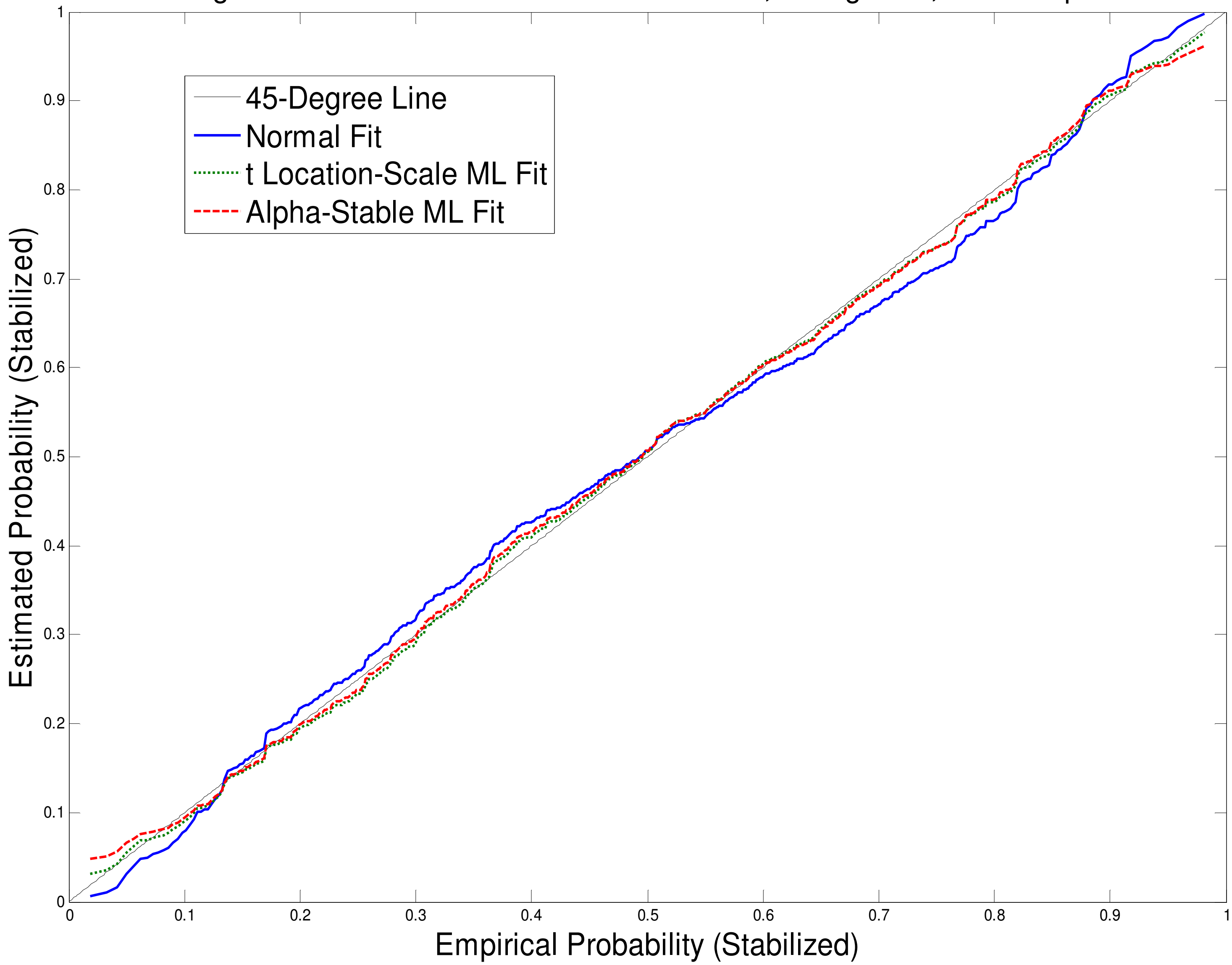


Figure 3. Stabilized PP Plot for IP Residuals, 12-lag VAR, Full Sample



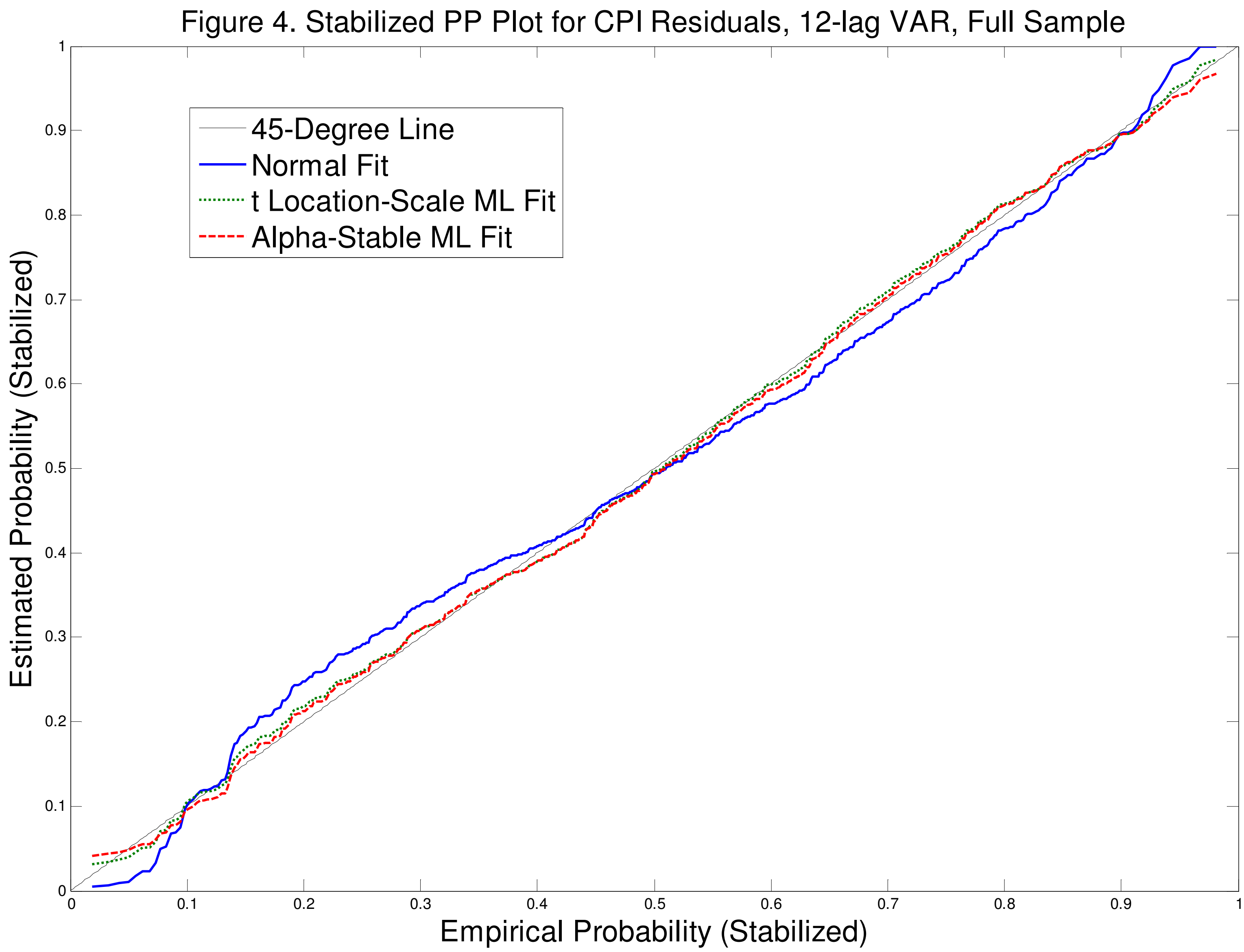
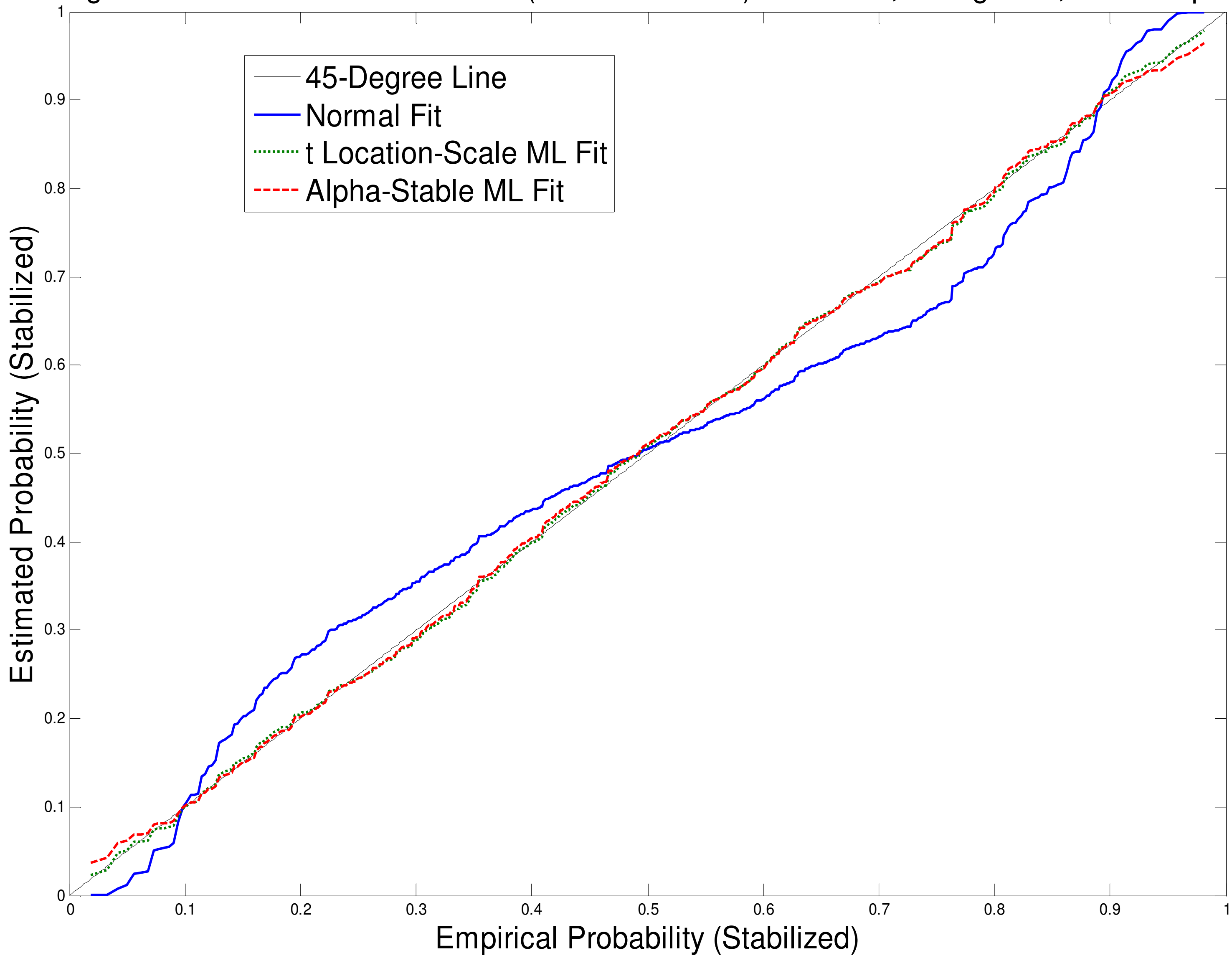


Figure 4. Stabilized PP Plot for CPI Residuals, 12-lag VAR, Full Sample
45-Degree Line
Normal Fit
t Location-Scale ML Fit
Alpha-Stable ML Fit
Estimated Probability (Stabilized)
Empirical Probability (Stabilized)
0
0.1
0.2
0.3
0.4
0.5
0.6
0.7
0.8
0.9
1

Figure 5. Stabilized PP Plot for PPI (Crude Materials) Residuals, 12-lag VAR, Full Sample



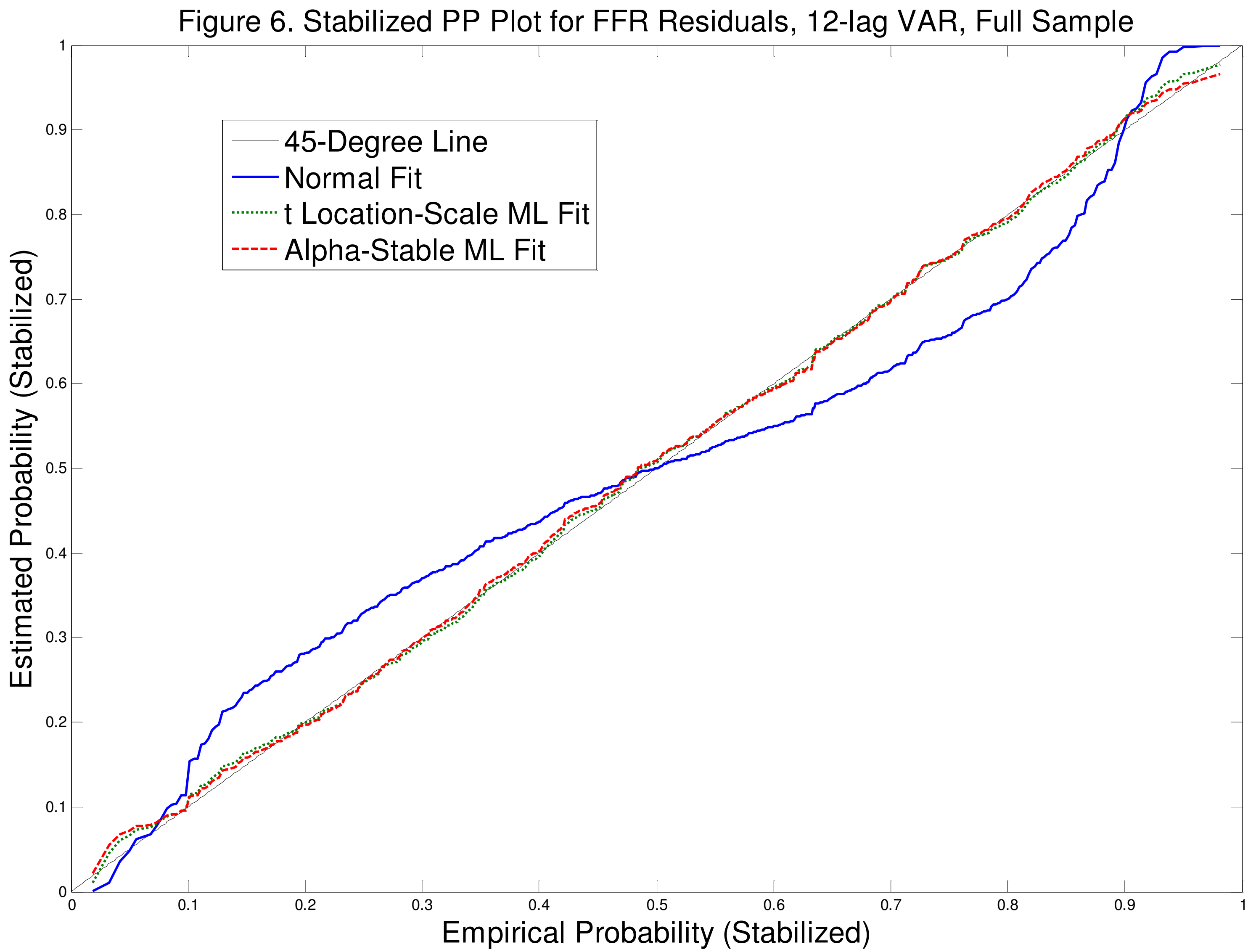
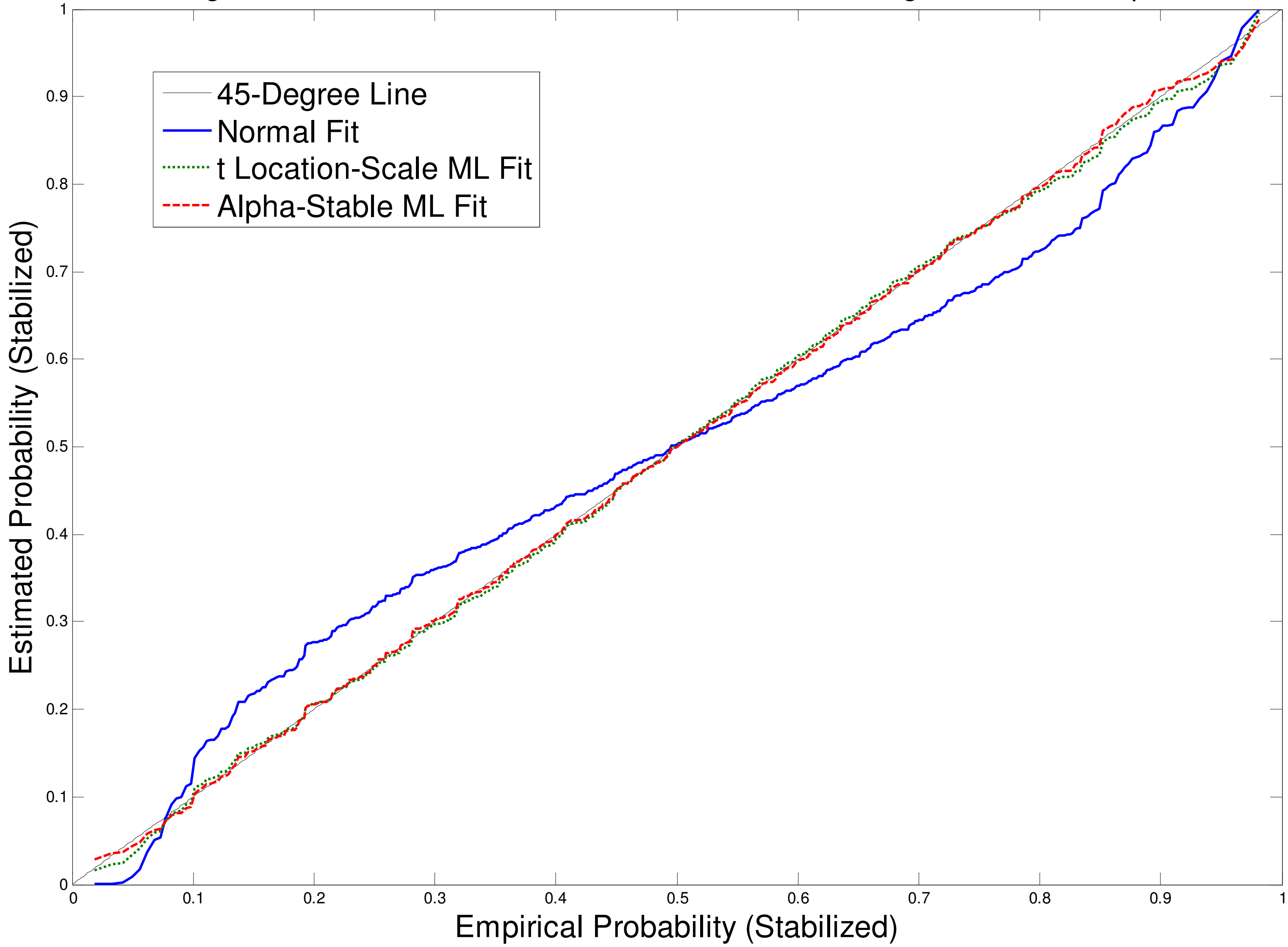


Figure 6. Stabilized PP Plot for FFR Residuals, 12-lag VAR, Full Sample
45-Degree Line
Normal Fit
t Location-Scale ML Fit
Alpha-Stable ML Fit
Estimated Probability (Stabilized)
Empirical Probability (Stabilized)
0
0.1
0.2
0.3
0.4
0.5
0.6
0.7
0.8
0.9
1

Figure 7. Stabilized PP Plot for NBR Residuals, 12-lag VAR, Full Sample



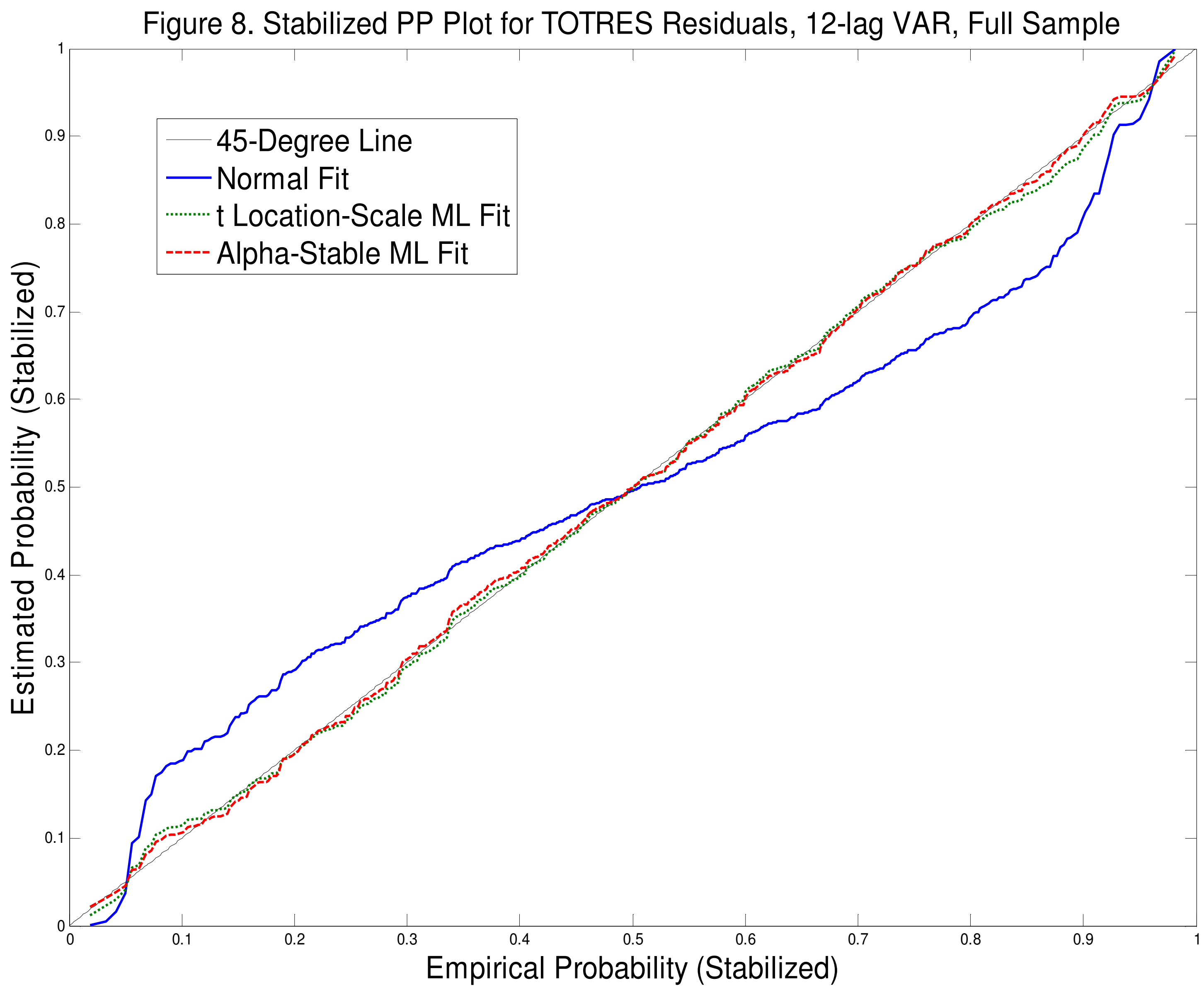


Figure 8. Stabilized PP Plot for TOTRES Residuals, 12-lag VAR, Full Sample
45-Degree Line
Normal Fit
t Location-Scale ML Fit
Alpha-Stable ML Fit
Estimated Probability (Stabilized)
Empirical Probability (Stabilized)
0
0.1
0.2
0.3
0.4
0.5
0.6
0.7
0.8
0.9
1

Table 1. Sample Statistics for Innovations $\hat{\varepsilon}_t$, 6-Variable Monetary VAR, 1959:1–2007:11, 12-lag Specification (T=575)

	IP Equation	CPI Equation	PPI Equation	FFR Equation	NBR Equation	TR Equation
R^2	.999770	.999991	.997691	.981802	.998588	.998751
Log likelihood	2102.953	2766.124	1256.021	-348.3336	1394.758	1445.107
Std. Dev.	0.006249	0.001972	0.027257	0.443849	0.021413	0.019618
Skewness	0.022306	0.175226	0.070495	-1.509032	2.794186	6.973937
Kurtosis	4.464086	6.285196	12.24979	35.74013	47.98869	123.6040
Jarque-Bera	51.40351	261.5130	2050.320	25899.56	49239.45	353142.5
Probability	0.000000	0.000000	0.000000	0.000000	0.000000	0.000000

Note: For the full-sample estimates and for our full-sample break (1959:1–1984:1 and 1984:2–2007:11), we made use of all data in both of our lag-length specifications, so as not to lose any information. In other words, the 12-lag VARs for the full sample and for 1959:1–1984:1 use data from 1959:1–1959:12 as a presample, whereas our 3-lag versions of those VARs use 1959:1–1959:3 as a presample period. For the other subperiods, the presample observations were drawn from months prior to the stated sample period. For example, for the 1966:1–1979:9 VAR, the 3-lag VAR had a presample period of 1965:10–1965:12, and the presample period for the 12-lag VAR was 1965:1–1965:12.

Table 2. Sample Statistics for the Innovations $\hat{\varepsilon}_t$, 6-Variable Monetary VAR, 3-Lag Specification, 1959:1–2007:11 (T=584)

	IP Equation	CPI Equation	PPI Equation	FFR Equation	NBR Equation	TR Equation
R^2	.999706	.999990	.997483	.977797	.998438	.998546
Log likelihood	2049.329	2762.148	1244.609	-410.0522	1380.782	1417.596
Std. Dev.	.007247	.002138	.028746	.488734	.022767	.021376
Skewness	.561245	.006509	.198247	-1.466878	3.772518	8.449394
Kurtosis	10.73691	5.995864	12.34196	35.06997	69.68863	166.8595
Jarque-Bera	1487.248	218.4007	2127.450	25235.85	109604.7	660297
Probability	.000000	.000000	.000000	.000000	.000000	.000000

Table 3. Results for Full Sample (1959:1-2007:11) VAR Innovations
Presamples:1959:1–1959:12; 1959:1–1959:3

VAR(12) equation for:	Estimates of Stable Distribution Characteristic Exponent H_0: α=2			LR Statistic ($-2LLR_{LB}$) for H_0: $\alpha = 2$ and $\beta = 0$	Engle LM Test Statistic H_0: No (G)ARCH		Jarque-Bera Test of Normality H_0: $\varepsilon_t \sim NIID(\mu, \sigma^2)$
	$\hat{\alpha}_{MC}$	$\hat{\alpha}_{CF}$	$\hat{\alpha}_{ML}$ [95% asymptotic c.i.‡]		3-lag Test Equation	12-lag Test Equation	
Industrial Prod.	1.6874**	1.8664**	1.7692** [1.65,1.89]	20.6794**	29.172**	39.973**	51.40**
CPI-U	1.7279*	1.8189**	1.7445** [1.62,1.86]	58.2219**	52.052**	61.367**	261.51**
PPI (crude materials)	1.5987**	1.6141**	1.5539** [1.42,1.68]	192.0093**	70.356**	73.753**	2,050.32**
Fed Funds Rate	1.5668**	1.5884**	1.5607** [1.44,1.69]	311.7442**	34.892**	77.131**	25,899.56**
Nonborrowed Reserves	1.7167*	1.7391**	1.7330** [1.61,1.85]	255.7571**	14.300**	14.075	49,239.45**
Total Reserves	1.6864**	1.7543**	1.7663** [1.64,1.88]	433.9345**	10.868*	10.697	353,142.50**

VAR(3) equation for:	Estimates of Stable Distribution Characteristic Exponent H_0: α=2			LR Statistic ($-2LLR_{LB}$) for H_0: $\alpha = 2$ and $\beta = 0$	Engle LM Test Statistic H_0: No (G)ARCH		Jarque-Bera Test of Normality H_0: $\varepsilon_t \sim NIID(\mu, \sigma^2)$
	$\hat{\alpha}_{MC}$	$\hat{\alpha}_{CF}$	$\hat{\alpha}_{ML}$ [95% asymptotic c.i.‡]		ARCH(3) Test Equation	ARCH(12) Test Equation	
Industrial Prod.	1.5726**	1.7540**	1.6498** [1.52,1.78]	103.5208**	30.045**	42.786**	1,487.25**
CPI-U	1.7415*	1.8201**	1.7136** [1.59,1.83]	55.1185**	78.043**	88.467**	218.40**
PPI (crude materials)	1.4275**	1.5615**	1.4949** [1.37,1.62]	213.9893**	82.045**	89.599**	2,127.45**
Fed Funds Rate	1.3438**	1.4071**	1.3979** [1.27,1.52]	439.8073**	50.361**	129.672**	25,235.85**
Nonborrowed Reserves	1.7368*	1.6804**	1.6811** [1.56,1.81]	366.0446**	19.773**	20.024	109,604.7**
Total Reserves	1.6023**	1.7376**	1.7496** [1.63,1.87]	613.9451**	17.527**	17.668	660,297.0**

Significance levels for bootstrap test statistics are * for p=.05 and ** for p=.01. (See sections 1 and 5.) All critical values for the bootstrap tests were computed using a parametric bootstrap algorithm with n = 9,999 (see appendix 2). The same set of bootstrap 9,999 runs was used for all tests reported for a given specification–sample period combination. Estimators of α were $\hat{\alpha}_{MC}$ = McCulloch (1986) quantile estimator; $\hat{\alpha}_{CF}$ = characteristic function estimator (this algorithm was first presented in Kogon and Williams 1998); $\hat{\alpha}_{ML}$ = maximum likelihood estimator. ‡ = conditional on estimates of error-term vectors ε_t, $t = 1, 2, 3, \ldots, T-1,\ T$. The 95% confidence intervals reported in square brackets beneath $\hat{\alpha}_{ML}$ are based on the Fisher information matrix (DuMouchel 1973). Strictly speaking, these intervals are not valid for inference about the error terms, unless the true VAR coefficients are known. All three estimates, as well as the confidence intervals for the MLE, were computed using John Nolan's STABLE 5.1 MATLAB® toolbox, purchased from Robust Analysis, Inc. The toolbox was run on MATLAB® R2010b and R2011a. The use of this new software accounts for some slight changes from the estimates presented in earlier versions of this paper. The (G)ARCH test statistic = T.R^2, where T = VAR sample length, and R^2 is the coefficient of determination from a least squares estimate of the test equation. This approach to testing for GARCH and ARCH was first suggested by Engle (1982) and the Jarque-Bera test statistic is from Jarque and Bera (1987). The bootstrap procedure in appendix 2 was also used to compute critical values for these latter tests.

Table 4. **Results for 1959:1-1984:1 Subsample VAR Innovations**
Presamples: 1959:1–1959:12; 1959:1–1959:3

VAR(12) equation for:	Estimates of Stable Distribution Characteristic Exponent H_0: α=2			LR Statistic ($-2LLR_{LB}$) for H_0: $\alpha = 2$ and $\beta = 0$	Engle LM Test Statistic H_0: No (G)ARCH		Jarque-Bera Test of Normality H_0: $\varepsilon_t \sim NIID(\mu, \sigma^2)$
	$\hat{\alpha}_{MC}$	$\hat{\alpha}_{CF}$	$\hat{\alpha}_{ML}$		3-lag Test Equation	12-lag Test Equation	
Industrial Prod.	1.7260	1.9424*	1.9551	3.0145*	4.694	13.918	7.36*
CPI-U	1.8244	1.9037**	1.8671**	12.8939**	38.604**	44.889**	43.20**
PPI (crude materials)	1.8005	1.8581**	1.8747**	60.8660**	18.901**	20.421	2,618.87**
Fed Funds Rate	1.5649**	1.7251**	1.7453**	101.4364**	17.931**	29.406**	4,130.54**
Nonborrowed Reserves	1.6854*	1.8536**	1.7808**	20.4521**	7.261	18.939	72.31**
Total Reserves	1.7628	2.0000	1.9999	0.0000	1.608	16.360	.41
VAR(3) equation for:	Estimates of Stable Distribution Characteristic Exponent H_0: α=2			LR Statistic ($-2LLR_{LB}$) for H_0: $\alpha = 2$ and $\beta = 0$	Engle LM Test Statistic H_0: No (G)ARCH		Jarque-Bera Test of Normality H_0: $\varepsilon_t \sim NIID(\mu, \sigma^2)$
	$\hat{\alpha}_{MC}$	$\hat{\alpha}_{CF}$	$\hat{\alpha}_{ML}$		ARCH(3) Test Equation	ARCH(12) Test Equation	
Industrial Prod.	1.5756**	1.8151**	1.7660**	46.7945**	1.430	31.723**	827.28**
CPI-U	2.0000	1.8655**	1.7654**	23.1198**	28.839**	63.131**	80.15**
PPI (crude materials)	1.6572*	1.7256**	1.6726**	95.0605**	16.984**	29.329**	4,703.58**
Fed Funds Rate	1.2872**	1.4590**	1.3945**	159.6491**	22.594**	64.394**	4,041.70**
Nonborrowed Reserves	1.6589*	1.8265**	1.7526**	27.8273**	13.032**	28.114**	106.50**
Total Reserves	2.0000	1.9430*	1.9380*	11.8520**	.711	63.723**	53.59**

See notes below Table 3.

Table 5. Results for 1966:1-1979:9 Subsample VAR Innovations
Presamples: 1965:1–1965:12; 1965:10–1965:12

VAR(12) equation for:	Estimates of Stable Distribution Characteristic Exponent H_0: α=2			LR Statistic (–2LLR$_{LB}$) for H_0: $\alpha = 2$ $and\ \beta = 0$	Engle LM Test Statistic H_0: No (G)ARCH		Jarque-Bera Test of Normality H_0: $\varepsilon_t \sim NIID(\mu, \sigma^2)$
	$\hat{\alpha}_{MC}$	$\hat{\alpha}_{CF}$	$\hat{\alpha}_{ML}$		3-lag Test Equation	12-lag Test Equation	
Industrial Prod.	1.8891	1.9626	1.9999	.0000	1.957	7.766	1.33
CPI-U	1.7979	1.8648	1.8284*	13.4315	23.830**	27.905*	45.75
PPI (crude materials)	1.5987*	1.8071**	1.7452**	17.6958	17.112**	22.208	92.73*
Fed Funds Rate	1.6319	1.8449*	1.7642**	11.0474	11.901**	20.446	36.01*
Nonborrowed Reserves	1.8935	1.9432	1.8760	2.6099	.142	6.182	5.11
Total Reserves	1.9271	1.9813	2.0000	.0000	4.157	15.480	.73

VAR(3) equation for:	Estimates of Stable Distribution Characteristic Exponent H_0: α=2			LR Statistic (–2LLR$_{LB}$) for H_0: $\alpha = 2$ $and\ \beta = 0$	Engle LM Test Statistic H_0: No (G)ARCH		Jarque-Bera Test of Normality H_0: $\varepsilon_t \sim NIID(\mu, \sigma^2)$
	$\hat{\alpha}_{MC}$	$\hat{\alpha}_{CF}$	$\hat{\alpha}_{ML}$		ARCH(3) Test Equation	ARCH(12) Test Equation	
Industrial Prod.	1.7590	1.9811	1.9823	.0012	.809	7.779	.33
CPI-U	1.5456*	1.8277**	1.7818**	16.6233**	28.526**	34.296	60.61*
PPI (crude materials)	1.6624	1.6760**	1.6369**	44.6898**	11.996**	26.913*	538.66*
Fed Funds Rate	1.6162*	1.8410**	1.7369**	13.1812	5.096	9.299	52.45*
Nonborrowed Reserves	1.5688*	1.8832*	1.7965**	2.0135	4.019	13.683	5.65
Total Reserves	1.7575	1.9761	1.9999	.0000	1.160	12.235	.85

See notes below Table 3.

Table 6. Results for 1984:2-2007:11 Subsample VAR Innovations
Presamples: 1983:2–1984:1; 1983:11–1984:1

VAR(12) equation for:	Estimates of Stable Distribution Characteristic Exponent H_0: α=2			LR Statistic (–2LLR$_{LB}$) for H_0: $\alpha = 2$ and $\beta = 0$	Engle LM Test Statistic H_0: No (G)ARCH		Jarque-Bera Test of Normality H_0: $\varepsilon_t \sim NIID(\mu, \sigma^2)$
	$\hat{\alpha}_{MC}$	$\hat{\alpha}_{CF}$	$\hat{\alpha}_{ML}$		3-lag Test Equation	12-lag Test Equation	
Industrial Prod.	1.7634	1.8914**	1.8375**	11.6034*	16.321**	20.471	33.43
CPI-U	1.6705*	1.8548**	1.7782**	26.1559*	12.886**	20.802	207.16*
PPI (crude materials)	1.5295**	1.7084**	1.5953**	45.3838**	45.886**	51.028**	171.64*
Fed Funds Rate	1.6915	1.7976**	1.7120**	36.9772**	2.514	43.219**	193.60**
Nonborrowed Reserves	1.4618**	1.6506**	1.6493**	139.5244**	3.129	2.297	11,060.19**
Total Reserves	1.6042*	1.6901**	1.6816**	180.8232**	1.639	1.676	38,934.49**
VAR(3) equation for:	Estimates of Stable Distribution Characteristic Exponent H_0: α=2			LR Statistic (–2LLR$_{LB}$) for H_0: $\alpha = 2$ and $\beta = 0$	Engle LM Test Statistic H_0: No (G)ARCH		Jarque-Bera Test of Normality H_0: $\varepsilon_t \sim NIID(\mu, \sigma^2)$
	$\hat{\alpha}_{MC}$	$\hat{\alpha}_{CF}$	$\hat{\alpha}_{ML}$		ARCH(3) Test Equation	ARCH(12) Test Equation	
Industrial Prod.	1.9644	1.9168*	1.8664**	9.6563	13.630**	18.586	26.00
CPI-U	1.5186**	1.7179**	1.5848**	43.5816**	18.128**	39.283**	176.07**
PPI (crude materials)	1.4627**	1.6611**	1.5268**	57.5632**	54.261**	60.039**	224.21*
Fed Funds Rate	1.3255**	1.6682**	1.5061**	42.7829*	24.670**	53.715**	136.05**
Nonborrowed Reserves	1.6106**	1.5202**	1.5323**	268.3360**	6.394	6.068	41,554.01**
Total Reserves	1.8037	1.6149**	1.6023**	381.6470**	4.018	4.144	145,966.8**

See notes below Table 3.

Table 7. Results for 1988:4-2007:11 Subsample VAR Innovations
Presamples: 1987:4–1988:3; 1988:1–1988:3

VAR(12) equation for:	Estimates of Stable Distribution Characteristic Exponent H_0: α=2			LR Statistic (–2LLR$_{LB}$) for H_0: $\alpha = 2$ and $\beta = 0$	Engle LM Test Statistic H_0: No (G)ARCH		Jarque-Bera Test of Normality H_0: $\varepsilon_t \sim NIID(\mu, \sigma^2)$
	$\hat{\alpha}_{MC}$	$\hat{\alpha}_{CF}$	$\hat{\alpha}_{ML}$		3-lag Test Equation	12-lag Test Equation	
Industrial Prod.	2.0000	1.9401	1.9067	8.5964	18.010**	24.603*	23.04
CPI-U	1.8266	1.8773*	1.8389*	22.0323	2.138	9.482	205.71
PPI (crude materials)	1.7713	1.8620**	1.8131**	21.5806*	18.217**	19.970	137.24*
Fed Funds Rate	1.7425	1.9431	1.9279	1.0027	1.751	4.719	3.93
Nonborrowed Reserves	1.8169	1.7341**	1.7624**	121.9855**	.125	.286	15,848.09**
Total Reserves	1.7205	1.7121**	1.7369**	146.4446**	.130	.274	24,430.07**

VAR(3) equation for:	Estimates of Stable Distribution Characteristic Exponent H_0: α=2			LR Statistic (–2LLR$_{LB}$) for H_0: $\alpha = 2$ and $\beta = 0$	Engle LM Test Statistic H_0: No (G)ARCH		Jarque-Bera Test of Normality H_0: $\varepsilon_t \sim NIID(\mu, \sigma^2)$
	$\hat{\alpha}_{MC}$	$\hat{\alpha}_{CF}$	$\hat{\alpha}_{ML}$		ARCH(3) Test Equation	ARCH(12) Test Equation	
Industrial Prod.	1.8876	1.9341*	1.8850*	7.961*	9.414*	13.072	19.49
CPI-U	1.4456**	1.7046**	1.6092**	36.2592**	7.639	27.462**	240.96**
PPI (crude materials)	1.5028**	1.7569**	1.6576**	29.7809**	32.272**	44.075**	114.28**
Fed Funds Rate	1.5699**	1.8584*	1.7740**	4.3353	6.713	14.597	10.79*
Nonborrowed Reserves	1.8100	1.6721**	1.6795**	219.2355**	1.457	1.579	63,815.98**
Total Reserves	2.0000	1.6431**	1.7441**	280.6783**	1.038	1.116	104,048.9**

See notes below Table 3.

Table 8. Estimated Coefficients for GARCH(1,1) Model (7) of Shocks from 6-Variable VAR(12), Full Sample*

Variance Equation for	Variable	QML Coef. Estimate	S.E.**
IP residual	Constant	2.22 E-05	7.11 E-06
	Resid(-1)^2	.2126	.0712
	GARCH(-1)	.2140	.1945
CPI residual	Constant	1.04 E-06	3.24 E-07
	RESID(-1)^2	.1688	.0741
	GARCH(-1)	.5621	.1145
PPI residual	Constant	2.76 E-05	1.14 E-05
	Resid(-1)^2	.2544	.0855
	GARCH(-1)	.7339	.0627
FFR residual	Constant	.0075	.0029
	RESID(-1)^2	.2754	.1198
	GARCH(-1)	.7062	.0800
NBR residual	Constant	3.07 E-05	1.39 E-05
	Resid(-1)^2	.6979	.3830
	GARCH(-1)	.5499	.0359
TR residual	Constant	1.14 E-05	1.70 E-05
	RESID(-1)^2	.7858	.5580
	GARCH(-1)	.5879	.0312

Notes: *Presample variances computed using backcasting parameter = 0.7
**S.E. = Bollerslev-Wooldridge (1992) robust standard error

Table 9. Estimated Coefficients for GARCH(1,1) Model (7) of Shocks from 6-Variable VAR(3), Full Sample*

Variance Equation for	Variable	QML Coef. Estimate	S.E.**
IP residual	Constant	2.13E-06	6.02E-07
	Resid(-1)^2	.0248	.0102
	GARCH(-1)	.9221	.0195
CPI residual	Constant	5.72E-07	1.07E-07
	RESID(-1)^2	.1391	.0246
	GARCH(-1)	.7354	.0322
PPI residual	Constant	2.08E-05	6.48E-06
	Resid(-1)^2	.3450	.0349
	GARCH(-1)	.6745	.0280
FFR residual	Constant	.0041	.0015
	RESID(-1)^2	.3549	.0350
	GARCH(-1)	.6834	.0311
NBR residual	Constant	.0001	1.24E-05
	Resid(-1)^2	1.834	.0680
	GARCH(-1)	.0852	.0154
TR residual	Constant	6.63E-05	8.86E-06
	RESID(-1)^2	2.7431	.1116
	GARCH(-1)	.0480	.0372

Notes: *Presample variances computed using backcasting parameter = 0.7
**S.E. = Bollerslev-Wooldridge (1992) robust standard error

Table 10. Full Sample (1959:1-2007:11) VAR Results for GARCH-Filtered Residuals $\hat{\varepsilon}_{it}/\hat{\sigma}_{it}$

Filter: $\sigma_{it}^2 = q_{i0} + q_{i1}\sigma_{i(t-1)}^2 + q_{i2}\varepsilon_{i(t-1)}^2$, where i = equation number in VAR model $Y_t = C(L)Y_{t-1} + \varepsilon_t$

VAR(12) equation for:	Estimates of Stable Distribution Characteristic Exponent			LR Statistic (–LLR$_{LB}$)	Engle LM Test Statistic		Jarque-Bera Test Statistic
	$\hat{\alpha}_{MC}$	$\hat{\alpha}_{CF}$	$\hat{\alpha}_{ML}$ [95% asymptotic c.i.‡]		3-lag Test Equation	12-lag Test Equation	
Industrial Prod.	1.6849	1.9027	1.8444 [1.74,1.95]	11.8895	.6118	12.66955	35.01
CPI-U	1.7762	1.8554	1.7769 [1.66,1.89]	34.9797	2.697325	11.7484	96.61
PPI (crude materials)	1.8852	1.8768	1.8153 [1.71,1.93]	34.0086	3.7352	8.20525	144.26
Fed Funds Rate	1.8864	1.8568	1.8421 [1.74,1.95]	73.2875	.443325	4.1078	1,016.56
Nonborrowed Reserves	1.6456	1.8446	1.8536 [1.75,1.96]	188.7763	.076475	0.33235	51,749.66
Total Reserves	1.5989	1.8281	1.8725 [1.78,1.97]	241.4658	.07245	0.271975	101,861.7
VAR(3) equation for:	Estimates of Stable Distribution Characteristic Exponent			LR Statistic (–LLR$_{LB}$)	Engle LM Test Statistic		Jarque-Bera Test Statistic
	$\hat{\alpha}_{MC}$	$\hat{\alpha}_{CF}$	$\hat{\alpha}_{ML}$ [95% asymptotic c.i.‡]		ARCH(3) Test Equation	ARCH(12) Test Equation	
Industrial Prod.	1.6354	1.8181	1.7043 [1.58,1.83]	42.5659	19.44428	45.75698	116.62
CPI-U	1.9707	1.8712	1.7802 [1.67,1.89]	31.2983	4.749088	11.97375	78.09
PPI (crude materials)	2.0000	1.9414	1.9165 [1.83,2.00]	18.3762	2.583032	8.243744	63.08
Fed Funds Rate	2.0000	1.8592	1.8070 [1.70,1.92]	61.6938	.43216	4.781792	476.05
Nonborrowed Reserves	1.5627	1.8002	1.7925 [1.68,1.90]	247.0763	.082928	0.270976	92,083.84
Total Reserves	1.6871	1.7853	1.7649 [1.65,1.88]	353.0423	.051392	0.251704	258,758.9

Notes: See notes below Table 3. No significance levels shown on this page because of filtering. See notes below Table 3. ‡ = conditional on estimates of error-term vectors ε_t, $t = 1, 2, 3, \ldots, T-1, T$.

	Table 11. Measures of Goodness of Fit for Error-Term Models for VAR(12), Full Sample (1959:1–2007:11)												
	Normal (i.i.d.) 2-parameter model			Student's t (i.i.d.) 3-parameter model			Alpha-Stable (i.i.d.) 4-parameter model			GARCH(1,1) 4-parameter model $\varepsilon_{it} = \sigma_{it}\upsilon_{it},\ \upsilon_{it} \sim NIID(0,1)$ $\sigma_{it}^2 = q_{i0} + q_{i1}\sigma_{i(t-1)}^2 q_{i2}\varepsilon_{i(t-1)}^2$ $\sigma_{i0}^2 = \bar{\sigma}_i^2$			
	LL	AD	KS	LL	AD	KS	LL	AD	KS	LL	AD	KS	LL(N(0,1);$\hat{\upsilon}_{it}$)
IP	2,103.0	.1462	5.0175	2,117.9	*.0662*	*2.4703*	2,113.3	.1692	3.2248	*2,119.3*	.1107	5.0970	-816.1
CPI-U	2,766.1	.1801	5.5547	*2,796.9*	*.0764*	*3.0474*	2,795.2	.1164	3.0908	2,790.5	.1377	5.1548	-815.6
PPI (Crude Materials)	1,256.0	.3157	10.8857	1,353.6	*.0675*	2.3197	1,352.0	.0820	*2.2106*	*1,388.8*	.1145	4.3711	-815.3
FFR	-348.3	.3756	11.9169	-191.9	*.0742*	*2.1579*	-192.5	.1032	2.7361	*-163.7*	.1383	5.1481	-816.1
NBR	1,394.8	.3118	9.1533	1,520.5	.0661	1.5419	*1,522.6*	*.0499*	*1.1269*	1,455.7	.2298	6.8753	-815.8
Total Reserves	1,445.1	.5152	12.5962	1,658.2	.0964	*2.0918*	*1,662.1*	*.0634*	2.5362	1,565.1	.4556	9.2725	-815.7

LL=log likelihood; AD=Anderson-Darling Measure of Fit; KS=Kolmogorov-Smirnov Distance

	Table 12. Measures of Goodness of Fit for Error-Term Models for VAR(3), Full Sample (1959:1–2007:11)												
	Normal (i.i.d.) 2-parameter model			Student's t (i.i.d.) 3-parameter model			Alpha-Stable (i.i.d.) 4-parameter model			GARCH(1,1) 4-parameter model $\varepsilon_{it} = \sigma_{it}\upsilon_{it},\quad \upsilon_{it} \sim NIID(0,1)$ $\sigma_{it}^2 = q_{i0} + q_{i1}\sigma_{i(t-1)}^2 + q_{i2}\varepsilon_{i(t-1)}^2$ $\sigma_{i0}^2 = \bar{\sigma}_i^2$			
	LL	AD	KS	LL	AD	KS	LL	AD	KS	LL	AD	KS	LL(N(0,1);$\hat{\upsilon}_{it}$)
IP	2,049.3	.1945	7.1257	*2,104.3*	*.0577*	*2.3144*	2,101.1	.1722	2.4081	2,093.4	.1553	6.2910	-833.2
CPI-U	2,762.1	.2247	6.4497	2,791.4	.1225	*3.8983*	2,789.7	*.1022*	4.5450	*2,794.8*	.1735	5.0583	-828.6
PPI (Crude Materials)	1,244.6	.3398	11.0483	1,353.7	*.0613*	1.9151	1,351.6	.1117	*1.7361*	*1,434.8*	.0924	3.1161	-828.2
FFR	-413.3	.4506	14.2341	-193.5	.0912	2.8816	*-193.4*	*.0798*	*2.0657*	-1,280.4	.1368	4.8741	-828.5
NBR	1,380.8	.4147	10.7551	1,561.5	.0738	2.7201	*1,563.8*	*.0603*	*2.5813*	1,457.7	.2829	9.0277	-828.7
Total Reserves	1,416.4	.6297	14.1476	1,719.1	.0931	*3.2457*	*1,723.4*	*.0863*	4.2136	1560.6	.3727	10.8914	-828.4

LL=log likelihood; AD=Anderson-Darling Measure of Fit; KS=Kolmogorov-Smirnov Distance

Note: In the tables above, italics are used to denote the winner of the competition corresponding to that cell in the table and all others for the same equation and measure of fit. For example, in the 12-lag specification, the shock in the PPI-for-crude-materials equation is best modeled by a stable, non-Gaussian distribution according to the Kolmogorov-Smirnov (KS) measure of distance, by a GARCH(1,1) model according to the likelihood criterion, and by a t distribution according to the Anderson-Darling (AD) measure of distance.