Quantitative Easing, Functional Finance, and the “Neutral” Interest Rate

by

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ABSTRACT

The main purpose of this study is to explore the potential expansionary effect stemming from the monetization of debt. We develop a simple macroeconomic model with Keynesian features and four sectors: creditor households, debtor households, businesses, and the public sector. We show that such expansionary effect stems mainly from a reduction in the financial cost of servicing the public debt. The efficacy of the channel that allegedly operates through the compression of the risk/term premium on securities is found to be ambiguous. Finally, we show that a country that issues its own currency can avoid becoming stuck in a structural “liquidity trap,” provided its central bank is willing to monetize the debt created by a strong enough fiscal expansion.

Keywords: Floor System; Debt Monetization; Functional Finance; Policy Coordination; Neutral Interest Rate

JEL Classifications: E10, E12, E44, E52, E58
1. INTRODUCTION

The financial crisis of 2007-2009 that subsequently led to the European sovereign debt crisis that erupted violently in Spring 2010 has brought to the fore the urgent need to resort to new macroeconomic instruments. To be sure, some OECD economies find themselves in a difficult position insofar as, in the years to come, they will not be able to rely on expansionary fiscal policies to pull their economies out of recession owing to the fact that both their public and private sector are highly leveraged. In addition, some of these countries exhibit large external imbalances. In this scenario, several central banks like the US Fed, the European Central Bank (ECB), and the Bank of England have resorted, at least since 2008, to large-scale purchases of sovereign debt to try alleviating the financial cost to their treasuries of the massive fiscal expansions that were engineered to counteract the collapse of private expenditure during the recent recession. It is well-documented by now that the Bank of Japan sporadically purchased government securities between 2001 and 2006 (Ueda 2010). Likewise, the US Fed has purchased large quantities of mortgage-backed securities in order to provide liquidity to the private sector and compress term and risk premiums.

Lenza et al. (2010) provide a detailed description of the set of "unconventional" monetary policy measures recently adopted by the ECB, the Fed, and the Bank of England in the wake of the failure of Lehman Brothers in September 2008. For instance, when evaluating the measures adopted by the ECB, they conclude that the measures (i) succeeded in narrowing the spread between the 3-month EURIBOR and the overnight interest swap rate which offers a proxy for secured rates, and (ii) helped flatten the money market yield curve. A similar study for Japan is in Ueda (2010) who concludes that the unconventional measures adopted by the Bank of Japan were effective in containing risk/term premiums in dysfunctional money markets albeit he admits that their overall effectiveness is unclear. A recent study by the Federal Reserve Bank of New York estimates that the purchases of longer-term assets by the Fed lowered the term premium by about half a percentage point (Gagnon et al. 2010). Similar evidence is in D´Amico and King (2010). However, these results are somewhat qualified in a study by Hamilton and Wu (2011) who find that $400 billion of Fed purchases of treasuries could only reduce longer-term maturities by up to 14 basis points. As for the unconventional operations pursued by the Bank of England since March 2009, Joyce et al. (2010) estimate that the former depressed gilt yields by about a hundred basis points.
Finally, Baumeister and Benati (2010) explore the impact on output in the US, the Eurozone, Japan, and the UK of a compression in the long-term bond yield spread during the period 2007-2009 and find that it exerted a powerful effect on output growth.

The main aim of this study is to explore, at a theoretical level, the potential for macroeconomic stabilization of debt monetization. For that purpose, we develop a simple macroeconomic model with Keynesian features representing a closed economy made up of four different sectors: creditor and debtor households, businesses, and the government. We obtain several results. First, we show that the potential expansion that stems from debt monetization operates mainly through a reduction in the financial cost to the treasury of servicing the public debt. Second, we find that the efficacy of a compression of the risk/term premium on different types of securities is ambiguous. Third, we show that full monetization of government debt can prevent the "neutral" real interest rate from falling into negative territory. Our main policy conclusion is that the reaping of the expansionary effects brought about by large-scale debt monetization requires a very high degree of coordination between fiscal and monetary authorities.

The study is organized as follows. The following section discusses some institutional details of a "floor system," i.e., the monetary policy arrangements that allow for the separation of the setting of short-term nominal interest rates in the inter-bank market and the purchase of large quantities of both private and public debt by the central bank (CB). Section 3 formally expounds the relation between the notion of the "liquidity trap" (LT) and the "neutral" interest rate. Section 4 displays the theoretical model. Section 5 displays the results of the steady-growth analysis. Finally, section 6 summarizes and concludes.

2. DIVORCING RESERVES POLICY FROM INTEREST RATE POLICY: THE "DECOUPLING" PRINCIPLE

Conventional economic textbooks typically present the setting of short-term interest rates by CBs as the result of the manipulation of the supply of banking system reserves and the existence of significant "liquidity effects." According to this view, by injecting or draining base money through open market operations CBs guide market interest rates towards their target. However, several scholars have recently argued that this approach is fundamentally misconceived. They note, for instance, that event studies show no
relationship between the quantity of bank reserves and the level of short-term interest rates (Friedman and Kuttner 2010). Further, they argue that CBs no longer set short-term interest rates this way and that today this process generally involves little or no variation in the supply of CB liabilities (Disyatat 2008; Friedman and Kuttne, 2010). Instead, they insist that CBs control short-term interest rates because, as the monopoly suppliers of base money, they can set both the quantity and the terms on which the latter is provided. In this approach, CBs are able to fully control the supply of reserve balances and this, in turn, allows them to set the price at which they are supplied by simply announcing the target short-term interest rate. Such “announcement” effects entail that "so long as the public knows this price and expects the central bank to take action accordingly when the price moves sufficiently far away from it, market trades will typically be anchored at or close to this price" (Disyatat 2008, p. 7). To the extent that by simply announcing their intentions CBs can easily move short-term interest rates without undertaking open market operations since the simple threat to adjust liquidity as needed to achieve the new rate is enough to make money markets coordinate on the new rate, there arises a strong disconnection between the level of short-term interest rates and the quantity of liquidity actually supplied.

Over the last two decades, one can observe two types of regimes for controlling overnight interest rates: a reserve regime and a rate corridor. A reserve regime relies mostly on the imposing of reserve requirements that must be met on an average basis over a maintenance period of several weeks. By contrast, a pure interest rate corridor regime relies on standing facilities to help achieve a target interest rate. In particular, a ceiling for inter-bank interest rates is provided by a lending facility at a penalty interest rate above the target overnight interest rate, while a floor is provided by the interest rate on reserve balances. A floor system is a variant of interest rate corridor regime with an adjustable floor whereby reserve balances are remunerated at the target overnight rate of the CB in the inter-bank market. Crucially, by adopting a floor system a CB can target independently both the amount of bank reserves and the overnight interest rate. Borio and Disyatat (2010) have labelled this feature the "decoupling" principle.¹ The rest of

¹ This stands in marked contrast to a framework in which the inter-bank nominal interest rate always lies above the rate at which excess reserves are remunerated. When this is the case, all undesired reserve balances have to be drained via bond sales so that when a budget deficit is incurred and reserves are thus created "whether the non-government sector holds additional bonds or reserve balances is unrelated to a 'financing' versus 'monetizing' decision but instead depends upon the particular method of interest rate maintenance that is in place" (Fullwiler 2007, p. 1023).
this section briefly introduces the institutional details of the floor system and discusses some of its implications.

In a floor system the CB can manipulate short-term interest rates by varying the interest rate it pays on reserves instead of varying the aggregate quantity of reserves it supplies (Goodfriend 2002; Keister et al. 2008; Borio and Disyatat 2010). Under this approach, the interest rate paid on reserves effectively becomes a floor below which the market rate cannot fall.\(^2\) By adopting a floor system, a CB actually divorces the quantity of reserves from the interest rate target and in the process it gains one degree of freedom insofar as it can now employ interest on reserves together with open market operations to target independently short-term interest rates and the quantity of reserves.\(^3\) As pointed out in Keister et al. (2008, p. 42), "the supply of reserves could therefore be increased substantially without moving the short-term interest rate away from its target."

Specifically, a CB would be able to target any aggregate quantity of reserves above the minimum required to keep the inter-bank rate at the interest-on-reserves floor. This is illustrated in Figure 1 below where we measure the aggregate quantity of reserves in the horizontal axis and the interest rate in the vertical one. The downward-sloping locus represents the demand for reserves function whereas the dotted vertical lines represent the level of required reserves and the target supply of reserves of the CB. The negative slope in the reserve demand locus reflects the fact that the demand for reserves depends inversely on the inter-bank interest rate and becomes infinitely elastic when the market interest rate hits the interest rate paid on reserves. It is clear that if the supply of reserves is large enough to cut the horizontal segment of the reserve demand locus, the CB could use open market operations to target the total quantity of reserves, and independently use interest on reserves to pursue interest rate policy.

\(^2\) Reserve balances are risk-free and, hence, dominate all other investments in terms of liquidity. Consequently, an institution should only be willing to lend out its reserve balances if the interest rate earned in the market is at least as high as the interest rate the institution would receive from the CB. Thus the quantity of reserve balances demanded by banks varies inversely with the short-term interest rate that prevails in the inter-bank market since the former represents the opportunity cost of holding balances at the CB.

\(^3\) As noted in Lavoie (2010, p. 11), the Reserve Bank of New Zealand and the Central Bank of Norway had adopted a floor system before the financial crisis got under way whereas, partly as a response to its inability to control interest rates in the aftermath of massive quantitative-easing operations, the US Fed adopted a floor system on November 6, 2008.
It has been argued that under a ‘floor system’ the interest rate paid on reserve balances may fail to become an effective floor for overnight market interest rates if some institutions trading in the relevant money markets do not have access to interest on deposits at the CB. As argued in Bowman et al. (2010, p. 2), "the effectiveness of the central bank interest rate as a floor for short-term money market rates depends on the ability and willingness of institutions with access to borrow from institutions without access and on the competitive pressures to arbitrage any differences between the rates available at the central bank rate and in the market." In particular, if institutions with access demand a spread between the interest rate they pay when they borrow from institutions without access and CB deposit rates, the difference between the two rates may not go to zero. Notwithstanding it, and after analyzing the behavior of overnight market interest rates in several economies whose CBs are currently operating a floor system, Bowman et al. (2010) conclude that, with the exception of the Bank of England and the Norges Bank, the policy rate floor has served as an impermeable floor for the overnight market interest rate even in periods when CB balances rose dramatically.\(^4\) In the case of the Norges Bank, they find that trading has occasionally occurred at rates slightly below the policy rate floor whereas, in the case of the Bank of England, they

\(^4\) The five CBs they study are: the ECB, the Bank of Japan, the Bank of England, the Bank of Canada, and the Norges Bank.
show that the permeability of the policy rate floor has been more limited since March 2009 when the former removed the limits on holdings that could be remunerated at its main policy rate.

Next, we address the crucial question why a CB may want to implement a floor system. The literature identifies several advantages to adopting a floor system among which the following two are of paramount importance: (i) an increase in broad liquidity and the resulting increase in financial asset prices (and the corresponding decrease in yields), and (ii) higher treasury revenue in the form of interest income. First, the purchase of financial assets financed by CB money should push up the prices of the types of assets actually purchased. For instance, when a financial firm sells an asset to the CB, its deposits increase. If the firm regards the extra deposits to be an imperfect substitute for the assets it has sold it will then try to rebalance its portfolio back to its desired composition by purchasing other assets. To be sure, in the process the firm just shifts the deposits to the seller of those assets but, crucially, this will bid up the prices of the assets exchanged. This is often referred to as the “portfolio balance effect” (Bank of England 2009). Finally, as the prices of the assets purchased by the sellers of the financial assets in the subsequent rounds rise, their relative yields should also fall thus encouraging the sellers to switch into other types of assets in search for a higher return, hence, pushing up and down respectively other asset prices and yields as well.

Second, Goodfriend (2002) argues that the implementation of a floor system will bring about an increase in the transfers of the CB to the treasury. The argument runs as follows. Implementing a floor system implies that the CB will pay interest on reserves. There are two effects. First, there is the effect associated to the increase in both reserve deposits and assets purchased by the CB. Second, there is the effect associated to the payment of interest on pre-existing reserve deposits. To be sure, the latter will tend to reduce transfers from the CB to the Treasury because it eliminates the tax on pre-existing reserves. Notwithstanding, Goodfriend (2002, p. 5) argues that reserve balances are relatively small and may shrink further in the future if they continue to earn no interest. He recognizes that the purchase of debt by the CB will increase its transfers to the treasury as long as there is a positive spread between the securities acquired by the CB through open market operations and the interest rate it pays on reserve balances. He

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5 See Goodfriend (2002) for a detailed discussion of these advantages.
concludes that a CB should be able to self-finance interest on the enlarged demand for reserves under a floor system and still transfer some income to the treasury.

3. THE LIQUIDITY TRAP AND THE "NEUTRAL" INTEREST RATE

An LT is usually defined as a situation in which the real interest rate (in a closed economy without a government sector), at which aggregate saving at the level of output consistent with constant inflation and investment would be equal, is negative (Krugman 1998). If we denote by \( \omega \) the minimum (ex-ante) actual real interest rate that a CB can set for a given expected rate of inflation, we may define a LT as a situation where:

\[
\omega < r^n
\]  (1)

where \( r^n \) denotes the "neutral" interest rate which we define here as the long-term real interest rate which is neutral with respect to the inflation rate and tends neither to increase it nor to decrease it in the absence of transitory supply shocks. To the extent \( r \) is a long-term real interest rate it includes a positive time-varying term premium \( \mu > 0 \) that lenders will require either to grant credit or to purchase long-dated securities. Thus, we can express \( \omega \) as the difference between \( \mu \) and the expected rate of inflation \( \pi_e \) or:

\[
\omega = \mu - \pi_e
\]  (2)

Combining (1) and (2), we say an economy is in a LT if:

\[
r^n + \pi_e < \mu
\]  (3)

Expression (3) tells us that the lower are \( r^n \) and \( \pi_e \), and the higher is \( \mu \) the more likely it is an economy exhibits a LT. Finally, if condition (3) is satisfied when the economy is in steady-growth, we say it exhibits a "structural" LT (Palacio-Vera 2010).

4. THE MODEL

In this section we display a closed economy model that we employ subsequently to analyze the macroeconomic impact of the monetization of public and private debt. We assume the economy is made up of four sectors: creditor and debtor households, businesses, and the government. The government is broken down into the treasury and the CB. Financial intermediaries channel saving flows from lenders to borrowers so that
trades of securities among the four sectors (or within them) are brokered by them. Importantly, we assume that the degree of "liquidity preference" (LP) is exogenous and reflected in the values taken by the relative proportion of debtor households and the long-run debt-to-capital ratios of debtor households and businesses.

Some have argued that debt monetization by the CB may stimulate bank lending and, hence, increase aggregate spending. The argument typically runs as follows. As the CB purchases assets from banks, the liquidity of the asset side of banks’ balance sheets increases. As this occurs, "banks should also be more willing to hold a higher stock of illiquid assets in the form of loans as they have the funds to cope with the potentially higher level of payments activity" (Bank of England 2009, p. 93). Be that as it may, Fullwiler and Wray (2010) criticize the argument that filling banks with more reserves will encourage them to lend out the excess and argue that "suggesting that more reserve balances cause banks to make more loans is functionally equivalent to suggesting more excess reserves causes banks to reduce lending standards" (Fullwiler and Wray 2010, p. 7). In a similar vein, Borio and Disyatat (2010, p. 77) argue that "the amount of credit outstanding is determined by the banks’ willingness to supply loans, based on perceived risk-return trade-offs, and by the demand for loans" rather than by the availability of reserves. As Lavoie (2010, p. 14) notes, this claim represents a succinct presentation of the endogenous money view largely endorsed by Post Keynesians. Consequently, a key assumption of this study will be that the aggregate availability of bank reserves does not constrain loan expansion and, hence, an increase in bank reserves does not per se increase banks’ willingness to grant credit.

Next, we assume that creditor households ultimately finance the debt issued by the other sectors albeit they may sell part of it to the CB. Notwithstanding, financial intermediaries treat the securities issued by different sectors as imperfect substitutes. In addition, we assume that debtor households, businesses, and the government exhibit a positive target debt-to-capital ratio. In turn, the former can be interpreted as reflecting the interaction of borrowers’ and lenders’ risk (Minsky 1975). As for businesses, we assume that investment decisions are not directly affected by the degree of leverage so they rely on adjustments of the retention rate to hit their target debt-to-capital ratio. Furthermore, we assume that all securities are perpetuities that pay coupons forever so

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6 There is some recent evidence that increases in bank liquidity positions may increase lending albeit the effect appears to be rather weak (see, for instance, Bowman et al. 2011).
that the issuer does not have to redeem them. Finally, we assume that a floor system has been set up so that financial intermediaries hold reserve balances at the CB which get remunerated at the inter-bank interest rate.

4.1. The Supply Side

Let us consider a one-sector economy with two inputs, labor and capital, and assume that (i) there is a large number of identical firms, and (ii) they all utilize the same technology. If we aggregate across all firms, we may define potential output $\bar{y}$ as:

$$\bar{y} = \tau \cdot \bar{N} \leq v \cdot k, \quad (4)$$

where $\bar{N}$ denotes the level of employment that keeps inflation constant in the absence of transitory supply shocks, $k$ denotes the aggregate (physical) capital stock expressed in real terms, and $\tau$ and $v$ denote respectively the productivity of labor and capital when the factors are fully utilized. The current rate of capacity utilization is:

$$U = \frac{y}{v \cdot k} \leq 1 \quad (5)$$

where $y$ denotes aggregate output in real terms.

We assume there is a “constant inflation employment rate” (CIER) which results from the conflicting income claims of workers and businesses so that inflation increases if the current employment rate exceeds the CIER and vice-versa. Therefore, the CIER represents de facto an "inflation barrier." Following Palacio-Vera (2009), we refer to the rate of capacity utilization when $y = \bar{y}$ as the "constant inflation capacity utilization" or CICU which we denote by $\bar{u}$ or:

$$\bar{u} = \frac{e \cdot \bar{N}}{v \cdot k} = p \cdot \frac{\tau \cdot \bar{N}}{K} = p \cdot \left(\frac{\tau}{v}\right) \cdot \left(\frac{\bar{e} \cdot L}{K}\right) \leq 1 \quad (6)$$

where $\bar{N} = \bar{e} \cdot L$, and we denote by $\bar{e}$ the CIER, by $L$ the labor force, by $p$ the price level, and by $K = p \cdot k$ the capital stock expressed in nominal terms. For the sake of simplicity, we assume that $\bar{e}$ is constant, there is no overhead labor, and businesses are fully integrated, producing all the materials required for their final output so that prime costs are made up only of labor costs. The "natural" rate of growth is:

$$g_n = n + a \quad (7)$$
where \( a \) and \( n \) are the growth rate of labour productivity \( \tau \) and \( L \) respectively. Taking logs in (6) and differentiating with respect to time we obtain the rate of change of \( \bar{u} \) in discrete time:

\[
\frac{\Delta \bar{u}_t}{\bar{u}_{t-1}} = \pi_{t-1} + g_a - g_f
\]

(8)

where \( \pi \) is the inflation rate and \( g \) is the net investment rate in nominal terms.

Next, we assume that inflation dynamics are determined in a conventional way as:

\[
\Delta \pi_t = \phi \cdot (u_{t-1} - \bar{u}_{t-1})
\]

(9)

Let us assume that businesses set a constant mark-up over prime costs. This being the case, the real (product) wage is determined by firms’ profit-maximization objectives:

\[
\frac{w}{p} = \frac{\tau}{m}
\]

(10)

where \( w \) is the average nominal wage, and \( m > 1 \) is one plus the average mark-up set by businesses over prime costs. Assuming overhead labor away, the profit share \( \varphi \) can be expressed as:

\[
\varphi = \left( \frac{m-1}{m} \right) = \left( 1 - \frac{w}{p} \right) \tau
\]

(11)

where \( \partial \varphi / \partial m = 1/m^2 > 0 \).

4.2. Consumption, Investment, and Private Sector Indebtedness

We break down total nominal consumption into nominal consumption by debtor households \( C_D \) who represent a certain proportion \( z \) of total households and nominal consumption by creditor households \( C_C \). We assume that all profit and interest income accrues to creditor households and that interest is paid in arrears so that the debt service depends on both last period’s debt and nominal interest rate. Following Palley (1994), we also assume that debtor households consume a proportion \( c_D \) of their net disposable income—equal to wage income minus the debt service—and finance additional consumption by borrowing from creditor households or:

\[
C_{D,t} = c_D (1 - \varphi)(1 - t_w) Y_{t-1} - (\mu_H + i_{t-1}) D_{H,t-1} + \Delta D_{H,t}
\]

(12)
where \( Y = p \cdot y \) is aggregate nominal output, \( \mu_H \) is the risk premium on household debt, \( i \) is the inter-bank overnight nominal interest rate, \( D_H \) is total outstanding nominal debt by debtor households, \( \Delta D_H \) represents the change in debtor households’ debt stock, \( t_w \) is the tax rate on wage income and \((\mu_H + i_{t-1})D_{H,t-1}\) denotes their debt service.\(^7\)

As for creditor households, we assume that they consume a constant proportion \( c_c < c_D \) of their income which is equal to the sum of wage, profit, and interest income:

\[
C_{ct} = c_c \left( (1 - \varphi) (1 - t_w) Y_{t-1} + (1 - \gamma_c)(1 - t_c) \varphi Y_{t-1} + CHII_t \right)
\]  

(13)

\[
CHII_t = ((\mu_H + i_{t-1})D_{H,t-1} + (\mu_F + i_{t-1})(1 - \lambda_F^M)D_{F,t-1} + (\mu_G + i_{t-1})(1 - \lambda_G^M)D_{G,t-1} + RII_t)(1 - t_c)
\]

(14)

\[
RII_t = \lambda_F^M i_{t-1} D_{F,t-1} + \lambda_G^M i_{t-1} D_{G,t-1}
\]

(15)

where \( CHII \) denotes creditor households’ after-tax total interest income, \( RII \) denotes the interest flow that stems from reserve balances created by CB open market purchases of debt, \( \gamma_c \) denotes businesses’ average retention rate, \( D_F \) and \( D_G \) are respectively the outstanding stock of businesses’ and treasury debt expressed in nominal terms, \( \mu_F \) and \( \mu_G \) are respectively the risk/term premium that businesses and the treasury have to pay to have their debt acquired by financial intermediaries, \( t_c \) is the average tax rate on capital income, and \( \lambda_F^M \) and \( \lambda_G^M \) are respectively the proportion of businesses’ and treasury debt purchased by the CB. If we assumed that \( c_D = 1 \), total household saving \( S^H_t \) would solely correspond to creditor households or:

\[
S^H_t = \Delta D_{H,t-1} + (1 - \lambda_F^M) \Delta D_{F,t} + (1 - \lambda_G^M) \Delta D_{G,t}
\]

(16)

We assume that the increase in debtor households’ nominal debt relative to the capital stock \( h_{H,t} \) is given by:

\[
h_{H,t} = \Delta D_{H,t} \theta^H = \left( \frac{\Delta Y_t}{Y_{t-1}} - \delta_H (d_{H,t-1} - \hat{d}_H) \right) \cdot d_{H,t-1}
\]

(17)

where \( \delta_H > 0 \) and \( 0 < \hat{d}_H < 1 \) denotes debtor households’ target debt-to-capital ratio.

Finally, total nominal household consumption is equal to:

\[
C_t = C_{D,t} + C_{C,t}
\]

(18)

\(^7\) If \( c_D = 1 \), then disposable income by debtor households will be fully spent.
Next, let us assume that businesses exhibit a "desired" rate of capacity utilization $u^* < 1$ so that they expand capacity when $u > u^*$ and stop expanding it when $u < u^*$. If so, we can express the gross investment rate $I/K_{t-1}$ as:

$$\psi + f_u (u_{t-1} - u^*) + \psi$$

where $\psi$ is the depreciation rate, $f_u > 0$ is inversely proportional to the length of the capital goods construction and delivery lags, and $\bar{f} > 0$ denotes businesses' expected rate of growth of demand. In what follows, we will assume that:

$$\bar{f} = \pi + g_n > 0$$

where $\pi$ denotes the inflation target of the CB.

Businesses' retained profits $RP_t$ depend positively on realized profits and the retention rate $\gamma_t$, and negatively on the debt service and the corporate tax rate, $t_p$:

$$RP_t = \gamma_t (\varphi Y_{t-1} - (\mu + i_{t-1}) D_{F,t-1})(1 - t_p)$$

Dividing (21) through by $K_{t-1}$, we get:

$$RP_t / K_{t-1} = \gamma_t (\varphi Y_{t-1} - (\mu + i_{t-1}) D_{F,t-1})(1 - t_p)$$

where $d_{F,t-1} = D_{F,t-1} / K_{t-1}$ represents businesses’ debt-to-capital ratio. Therefore, we can express the change in businesses debt-to-capital ratio $h_{F,t}$ as the difference between expressions (19) and (22) or:

$$h_{F,t} = \Delta D_{F,t} / K_{t-1} = \bar{f} + f_u (u_{t-1} - u^*) + \psi - \gamma_t (\varphi Y_{t-1} - (\mu + i_{t-1}) D_{F,t-1})(1 - t_p)$$

Lastly, we assume that $\gamma_t$ increases (decreases) relative to a benchmark level $\gamma^*$ when $d_{F,t}$ exceeds (falls short of) a threshold $\hat{d}_F$ or:

$$\gamma_t = \gamma^* + \gamma_d \cdot (d_{F,t-1} - \hat{d}_F)$$

where $0 < \hat{d}_F < 1$ denotes businesses’ target debt-to-capital ratio.

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8 This assumption could be interpreted as implying that the rate of capital accumulation adjusts in the long run to an exogenously given "natural" growth rate or, preferably, that the "natural" growth rate adjusts in the long run to an exogenously given capital accumulation rate.
4.3. The Public Sector

Let us consider the following expression for the treasury’s budget identity:

\[ G_t + (\mu_G + i_{t-1}) D_{G,t-1} = T_t + \Delta D_{G,t} + RCB_t \]  \hspace{1cm} (25)

where all variables except \( \mu_G \)—which denotes the risk premium on government bonds—are in nominal terms, \( G \) denotes government expenditures on goods, services, and transfers, \( D_G \) denotes the stock of outstanding nominal government debt, \( (\mu_G + i_{t-1}) D_{G,t-1} \) denotes the government debt service, \( \Delta D_{G,t} \) denotes new issues of interest-bearing debt by the treasury, \( T \) denotes tax revenue, and \( RCB \) denotes CB revenue. The CB also has a budget identity that links the changes in its assets and liabilities or:

\[ \Delta D_G^M + \Delta D_F^M + CBRII_t + RCB_t = (\mu_G + i_{t-1}) D_{G,t-1}^M + (\mu_F + i_{t-1}) D_{F,t-1}^M + \Delta H_t \]  \hspace{1cm} (26)

\[ CBRII_t = i_{t-1} (\lambda_G^M D_{G,t-1} + \lambda_F^M D_{F,t-1}) \]  \hspace{1cm} (27)

\[ \Delta H_t = \lambda_G^M \Delta D_G + \lambda_F^M \Delta D_F \]  \hspace{1cm} (28)

where \( CBRII \) captures the flow of interest payments from the CB to reserves holders, \( \Delta D_G^M \) and \( \Delta D_F^M \) denote respectively the change in the stock of public and private (nominal) debt in the CB balance sheet, \( D_{G,t-1}^M \) and \( D_{F,t-1}^M \) denote respectively the stock of outstanding government and businesses’ debt owned by the CB, and \( \Delta H \) denotes the increase in the monetary base. Thus, the right-hand side in (26) represents the “sources” of funds whereas the left-hand side represents the “uses” of funds. As we explained above, the monetization of debt will bring about an increase in the monetary base \( H \) as captured in (28) so that reserves will be remunerated at the target overnight interest rate.

Inserting (26), (27), and (28) into (25), dividing through by \( K \) the resulting expression, and rearranging, we get the dynamics of the stock of public debt relative to the capital stock or:

\[ h_G = \frac{\Delta D_G}{K_{t-1}} = b_t + (1 - \lambda_G^M) \mu_G d_{G,t-1} + i_{t-1} d_{G,t-1} - \mu_F \lambda_F^M d_{F,t-1} \]  \hspace{1cm} (29)

where \( b_t = (G_t - T_t) / K_{t-1} \).

Next, we assume that the value of the primary nominal budget deficit relative to the capital stock \( b_t \) is set by the government as follows:

\[ b_t = b_0 - \delta_G (d_{G,t-1} - \hat{d}_G) \hspace{1cm} \delta_G < 0 \]  \hspace{1cm} (30)
where $b_0$ denotes the stance of fiscal policy when $d_{G,t} = \hat{\alpha}_G$ and $0 < \hat{\alpha}_G < 1$ denotes the government target debt-to-capital ratio. The former means that the government will adopt a more restrictive stance when the current public debt-to-capital ratio exceeds the target level and vice-versa. The dynamics of public debt are thus determined by the interaction of expressions (29) and (30).

### 4.4. Goods Market Equilibrium

In this section we address the determination of goods market equilibrium. In the Keynesian tradition, there is goods market equilibrium when expenditure leakages from the circular income flow are exactly equal to expenditure injections. In the context of our model this implies that, following a change in autonomous expenditure, real income will adjust to guarantee that, in equilibrium, aggregate saving equals the sum of gross investment and the budget deficit or:

$$S_i = I_i + G_i - T_i$$  \hspace{1cm} (31)

Next, aggregate private saving can be broken down into total household saving $S_H^t$ and business saving $S_F^t$ where the latter is equal to retained profits $R_P^t$:

$$S_i = S_H^t + S_F^t = (Y_{it}^H - C_i) + \gamma_i (1-t_p)(\varphi Y_{t-1} - (\mu_p + i_{t-1})D_{F,t-1})$$  \hspace{1cm} (32)

where $Y_{it}^H$ denotes household income.

Inserting (32) into (31) and rearranging, we get:

\[
(1-c_b)(z(1-\varphi)(1-t_w)Y_{t-1} - (\mu_H + i_{t-1})D_{H,t-1}) - \Delta D_{H,t-1} + R_P - I_t - B_t = 0
\]

(33)

where $B_t = G_t - T_t$. Dividing (33) through by $K_{t-1}$ and rearranging, we get:

\[
(1-c_b)(z(1-\varphi)(1-t_w)\nu_{t-1} - (\mu_H + i_{t-1})d_{H,t-1}) - h_{t-1} + (R_P/K_{t-1}) - g_t - \psi - b \\
(1-c_c)(1-z)(1-\varphi)(1-t_w)\nu_{t-1} + (1-\gamma_i)(1-t_p)(\varphi Y_{t-1} - (\mu_p + i_{t-1})D_{F,t-1}) + \left(\frac{CHII}{K_{t-1}}\right) = 0
\]

(34)

Inserting (17) into (34) and rearranging we can obtain an expression for the rate of growth of output:
\[
\frac{\Delta Y_t}{Y_{t-1}} = \left(1-c_D\right) z(1-\varphi)(1-t_o)\nu u_{t-1} - (\mu_H + i_{t-1}) d_{H,t-1} + \left(\frac{R P_i}{K_{t-1}}\right) + \\
\left(1-c_c\right) (1-z)(1-\varphi)(1-t_o)\nu u_{t-1}+(1-\gamma_t)(1-t_f)\nu (\varphi' u_{t-1} - (\mu_F + i_{t-1}) d_{F,t-1}) + \left(\frac{C H I \mu}{K_{t-1}}\right) + \\
+ \left(1\right) (d_{H,t-1} - d_H) d_{H,t-1} - g_t - \psi - b_t \tag{35}
\]

Taking logs and differentiating with respect to time in (5) we have that:

\[
\frac{\Delta u_t}{u_{t-1}} = \frac{\Delta Y_t}{Y_{t-1}} - g_t \tag{36}
\]

Hence, the dynamics of \( u \) are determined by the interaction of expressions (4) through (36) above.\(^9\) Short-run stability requires that aggregate private saving be more responsive to changes in capacity utilization than the sum of aggregate investment and the primary budget deficit or:

\[
(1-\varphi)(1-t_o)\nu [(1-c_o)(1-z)] + (1-t_f)\nu [\gamma_t + (1-c_o)(1-t_c) + (1-\gamma_t)] = f_u \tag{37}
\]

where \( \gamma_t \) varies according to (24) so that short-run stability is more likely to be fulfilled when \( \gamma_t \) is relatively high than otherwise.

In order to complete the dynamics of the model we need to specify a monetary policy rule that drives the economy towards its long-run equilibrium. We assume that the CB sets the nominal interest rate according to the following Taylor-type policy rule:

\[
i_t = r^* + \pi_t + a_\pi (\pi_t - \pi^*) \tag{38}
\]

where \( a_\pi > 0 \) is the response coefficient of the policy rule to changes in the inflation gap, i.e., the difference between current and target inflation.

5. STEADY-GROWTH ANALYSIS

Steady-growth equilibrium represents a hypothetical period of sufficient length to enable all variables in the economy to settle at constant rates in the absence of new shocks. In steady-growth, the stock of debt of each sector must grow at a pace equal to

---

\(^9\) Expression (16) is not necessary to derive the short-run dynamics of the model whereas expression (38) is indispensable since it provides the dynamics of the nominal interest rate.
the sum of the CB´s inflation target and the "natural" growth rate or \( \Delta D_{t}/D_{t-1} = \pi^* + g_n \).

In addition, the steady-growth condition requires that \( d_{H,t} = \hat{d}_H \), \( d_{F} = \hat{d}_F \), and \( d_{G} = \hat{d}_G \) so that, for instance, expression (17) becomes:

\[
\Delta D_{H,t}/D_{H,t-1} = \left( \frac{\Delta Y_t}{Y_{t-1}} - \delta_{H}(d_{H,t-1} - \hat{d}_H) \right) = \pi^* + g_n
\]  

(39)

Next, we assume that businesses set \( \gamma^* \) so that the rate of change of \( D_{F,t} \) equals the rate of growth of nominal GDP in steady-growth. Hence, solving (23) for \( \gamma^* \) we get:

\[
\gamma^* = \frac{1}{\varphi(1 - t_p)\nu - (\mu + \pi^* + r^*)(1 - t_p)\hat{d}_F} \cdot \left( (1 - \hat{d}_F)(\pi^* + g_n) + \psi \right)
\]  

(40)

where \( r^* \) denotes the steady-growth "neutral" interest rate.

Likewise, we assume that the stance of fiscal policy as captured by \( b_0 \) in (30) is set so that the stock of outstanding public debt grows at \( \pi^* + g_n \) and \( d_{G,t} = \hat{d}_G \) in the long run. Let us denote by \( b_0^* \) the stance discretionary fiscal policy must adopt in steady-growth if the two former conditions are to be satisfied. Insofar as \( i^* = \pi^* + r^* \) and \( h_{G,t} = (\pi^* + g_n) \cdot \hat{d}_G \) in steady-growth, we can obtain an explicit expression for\( b_0 \) by inserting (30) into (29) and solving for \( b_0^* \) or:

\[
b_0^* = (g_n - r^*)\hat{d}_G - (1 - \lambda_G)\mu_G\hat{d}_G + \mu_F\lambda_F\hat{d}_F
\]  

(41)

The complete solution of the steady-growth equilibrium requires the successive combination of all equations presented above except equation (16). The model contains a non-linear dynamical system made up of six difference equations corresponding to expressions (8), (9), (17), (23), (29), and (36). However, several expressions need to be previously inserted into the dynamical system if the steady-growth equilibrium is to be fully specified. First, expressions (12), (13), (14), (15), (17), (18), (19), (20), and (36) need to be inserted into (35). Second, the expression for \( b_0^* \) in (40) has to be inserted into (30) and the resulting expression, together with (19) and (20) has, in turn, to be inserted into (35). Then, by inserting the resulting expression as well as expressions (19) and (20) into the difference equation for \( u \) embedded in (36), we can obtain the dynamical version of the aggregate demand block of the model. Third, the supply-side block of the system is represented by equations (8) and (9). However, the former is only complete after inserting expressions (19) and (20) into (8). Fourth, the debt dynamics
block of the model is represented by difference equations (17), (23), and (29) which capture the dynamics of the debt-to-capital ratio of debtor households, businesses, and the public sector respectively. As before, by inserting $b_0$ into the fiscal policy rule (30) and this, in turn, into (29), we obtain the equation that captures the dynamics of public debt relative to capital. Fifth, by inserting the expression for $\gamma^*$ in (40) into (24) and this, in turn, into expression (23) we obtain the dynamics of businesses’ debt-to-capital ratio. Last, the dynamics of households’ debt-to-capital ratio is determined by difference equation (17).

Setting $\Delta u = \Delta \pi = \Delta d_H = \Delta d_F = \Delta d_G = 0$ in the six difference equations referred to above yields a steady-growth equilibrium given by $u = \pi = u^*, \pi = \pi^*, g = \pi^* + g_n, d_H = d_H^*, d_F = d_F^*, d_G = d_G^*$ and $\hat{r} = \pi^* + \hat{r}^*$. The formal analysis of the stability conditions of the dynamical system associated to the model expounded above is extremely complex and, hence, it is not pursued here. Nevertheless, the results of the simulation exercise depicted in Figures 2 and 3 clearly suggest that, for plausible values of the model parameters and initial conditions reported in Table 1, the joint interaction of the fiscal and monetary policy rules drives the economy towards its steady-growth equilibrium in the long run.

Figure 2: Inflation rate
Figure 3: Capacity utilization

Table 1: Parameter values and initial conditions used in the numerical simulation exercise

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Value</th>
</tr>
</thead>
<tbody>
<tr>
<td>$c_c$</td>
<td>0.7</td>
</tr>
<tr>
<td>$f_u$</td>
<td>0.1</td>
</tr>
<tr>
<td>$\delta_H = \delta_G$</td>
<td>0.1</td>
</tr>
<tr>
<td>$\mu_H = \mu_F$</td>
<td>0.03</td>
</tr>
<tr>
<td>$\pi^*$</td>
<td>0.019107</td>
</tr>
<tr>
<td>$c_D$</td>
<td>0.95</td>
</tr>
<tr>
<td>$\nu$</td>
<td>0.5</td>
</tr>
<tr>
<td>$\gamma'_d$</td>
<td>0.1</td>
</tr>
<tr>
<td>$\mu_G$</td>
<td>0.01</td>
</tr>
<tr>
<td>$\pi_0 = 0.015 \times \pi^*$</td>
<td></td>
</tr>
<tr>
<td>$z$</td>
<td>0.4</td>
</tr>
<tr>
<td>$\psi$</td>
<td>0.01</td>
</tr>
<tr>
<td>$\delta_H = d_{H,0} = 0.6$</td>
<td></td>
</tr>
<tr>
<td>$\phi$</td>
<td>0.3</td>
</tr>
<tr>
<td>$A_0 = -0.070956$</td>
<td></td>
</tr>
<tr>
<td>$\varphi$</td>
<td>0.25</td>
</tr>
<tr>
<td>$g_n$</td>
<td>0.03</td>
</tr>
<tr>
<td>$\delta_F = d_{F,0} = 0.9$</td>
<td></td>
</tr>
<tr>
<td>$\lambda^H_G = \lambda^H_F = 0$</td>
<td></td>
</tr>
<tr>
<td>$u^* = u_0 = \pi_0 = 0.8$</td>
<td></td>
</tr>
<tr>
<td>$\pi^* = 0.02$</td>
<td></td>
</tr>
<tr>
<td>$\delta_H = d_{G,0} = 0.6$</td>
<td></td>
</tr>
<tr>
<td>$\gamma^* = 0.49599$</td>
<td></td>
</tr>
<tr>
<td>$A_\pi = 1.2228$</td>
<td></td>
</tr>
<tr>
<td>$t_w = 0.3$</td>
<td></td>
</tr>
<tr>
<td>$t_c = t_p = 0.2$</td>
<td></td>
</tr>
<tr>
<td>$a_\pi = 0.655$</td>
<td></td>
</tr>
<tr>
<td>$b_0 = 0.000535$</td>
<td></td>
</tr>
<tr>
<td>$A_\pi = 0.9012$</td>
<td></td>
</tr>
</tbody>
</table>

Our next task is to obtain an expression for $\pi^*$. We can do so by making all the substitutions referred to above when the economy is at steady-growth which yields:

$$\pi^* = \left(\frac{1}{A_\pi}\right) \left(A_0 + A_\pi \cdot \pi^* + A_\pi \cdot g_n\right)$$

where:

$$A_\pi = t_p (1-c_c)(1-t_c) \delta_F + (1-c_c)(1-t_c) - (1-c_D)(1-t_c) \delta_H + [1 + (1-c_c)(1-t_c)] \delta_G$$

$$A_\pi = 1 - (1-c_c)(1-t_c) \delta_H + \delta_G + [1 + (1-c_D)(1-t_c)] \delta_H - (1-c_c)(1-t_c) t_p \delta_F - (1-c_c)(1-t_c) \delta_G$$

$$A_\pi = 1 + \delta_H + \delta_G - (1 - \delta_F) [1 - (1-c_c)(1-t_c)] > 0$$

$$A_0 = -(1-c_c)\nu (1-c_c)(1-z)(1-\varphi) (1-t_w) u^* + [(1-c_D) - (1-c_c)(1-t_c)] \mu_H \delta_H$$

$$- (1-c_c)(1-t_c) \varphi u^* + (1-c_c)(1-t_c)(1-t_p) \mu_F \delta_F + (1-c_c)(1-t_c) \psi + \mu_F \lambda^H_F \delta_F$$

$$- (1-c_c)(1-t_c) \lambda^M_F \mu_F \delta_F - (1-c_c)(1-t_c) (1-\lambda^M_G) \mu_G \delta_G - (1-\lambda^M_G) \mu_G \delta_G$$
As we noted above, a rise in $r^*$ makes it less likely that the economy will exhibit a "structural" LT and vice-versa. Thus, the key issue as far as this study is concerned is whether changes in the proportion of businesses’ and government debt that is monetized by the CB do affect $r^*$. As can be seen in (42), the sign of all operators $A_r$ but $A_g$ is ambiguous. In the case of $A_r$, the ambiguity stems from the fact that a rise in the real interest rate redistributes income away from debtor households and towards creditor households. If $t_r > 0$, a proportion of the increase in creditor households’ interest income is subtracted in the form of taxes, so that the net effect on total household saving becomes uncertain. To be sure, a change in the real interest rate will also trigger other distribution effects albeit their sign is not ambiguous. Be that as it may, we assume that the normal scenario is one where $A_r > 0$ so that aggregate spending depends inversely on the real interest rate and, by the same token, this implies that an increase (decrease) in the former—relative to potential output—will bring about an increase (decrease) in $r^*$ in steady-growth. Next, we have that:

$$\frac{\partial r^*}{\partial \lambda^M_F} = \frac{[1 + (1 - c_r)(1 - t_r)]\mu_F d_F}{A_r} > 0 \quad \text{iff } \mu_F > 0 \quad (43)$$

$$\frac{\partial r^*}{\partial \lambda^M_G} = \frac{[1 + (1 - c_r)(1 - t_r)]\mu_G d_G}{A_r} > 0 \quad \text{iff } \mu_G > 0 \quad (44)$$

that is, the monetization of businesses’ and public debt, i.e., setting either $\lambda^M_F > 0$ or $\lambda^M_G > 0$ yields an increase in $r^*$ provided the respective risk/term premiums are positive and $A_r > 0$. Intuitively, debt monetization is expansionary on net because it redistributes income away from creditor households and towards the treasury.\(^{10}\) We explain this in turn. When the CB purchases securities from creditor households by issuing reserve balances, there is a decrease in the debt service of the government. This is because, provided $\mu_F > 0$ and $\mu_G > 0$, the market interest rate that businesses and the treasury pay on their outstanding debt exceeds the inter-bank interest rate. Under a floor system the latter equals the nominal interest rate the CB pays on reserve balances. As noted in Goodfriend (2002), the purchase of debt by the CB self-finances the interest to be paid on reserve balances in this context. Next, the increase in net interest income that accrues

\(^{10}\) This result was originally proved in Auerbach and Obstfeld (2005) who showed that there remains an argument for large-scale open market purchases of public debt as a fiscal policy tool when short-term nominal interest rates are zero.
to the CB when it purchases securities from households equals exactly the decrease in interest income that was accruing to the latter. Notwithstanding, a decrease by one unit in creditor households’ interest income decreases their saving by \((1 - c_i) (1 - t_i)\) whereas, under the crucial proviso that the government’s target debt-to-capital ratio remains constant, a decrease by one unit in the government’s debt service leads to a decrease in one full unit in public saving. In other words, the increase in CB net interest income is transferred to the treasury thus allowing the government to run a larger primary budget deficit \(b_0\) while simultaneously hitting its target debt-to-capital ratio \(\hat{d}_G\). This highlights that the impact of debt monetization hinges on the response of the fiscal authorities to an increase in the transfers from the CB; if the fiscal policy stance remains invariant, that is, if \(b_0\) does not increase or becomes more restrictive, debt monetization may not be expansionary. Similarly, the expansionary effect of debt monetization will also vanish if \(\mu_g = \mu_f = 0.11\).

Next, the sign of \(A_{\sigma}\) in (42) is also ambiguous. We may trace out the impact of changes in \(\pi^*\) on \(r^*\) with the aid of expression (45) below which captures the goods market equilibrium in steady-growth. To be sure, an increase in \(\pi^*\) causes a change in both \(RP^*/K_t\) and \(S^H/K\) so that, by virtue of changes in \(r^*\), goods market equilibrium is preserved since:

\[
\frac{S^H}{K_t} + \frac{RP^*}{K_t} = g_a + \pi^* + \psi + b^* 
\]  

(45)

We can trace out the adjustment process of \(r^*\) in the wake of a change in \(\pi^*\) by looking at the expressions for \(\gamma^*\) and \(b_0^*\) above. Expression (39) shows unambiguously that, since \(i^* = \pi^* + r^*\) in steady-growth, an increase in \(\pi^*\) leads to an increase in both the amount of funds businesses need to invest in order to keep capacity at the desired rate, and in businesses’ debt service and, hence, it causes a rise in \(\gamma^*\). Next, an increase in \(\pi^*\) also leads to a rise in debtor households’ debt service which, in turn, makes them cutback consumption. In turn, the former is coupled to a rise in creditor households’ interest income. However, while their interest income increases their profit income

11 Goodfriend (2002, p.6) notes that there may be periods when the yield curve slopes downward so that, interest on the CB portfolio could actually fail to cover interest on reserves. Thus, the existence of an expansionary effect depends on the risk/term premium being strictly positive and sufficiently large to cover the payment of interest on previously existing reserves.
decreases owing to the above-mentioned rise in $\gamma^*$ and, hence, the net impact on creditor households’ consumption of an increase in $\pi^*$ is ambiguous. In terms of (45), the rise in $\gamma^*$ pushes $r^*$ downward whereas the increase in $\pi^*$ pushes $r^*$ upward, the net effect being ambiguous.

As the studies alluded to in the introduction highlight, large-scale purchases of securities may help compress risk/term premiums across the economy and this, in turn, may exert an expansionary effect on aggregate demand. However, this is controversial. For instance, Fullwiler and Wray (2010) deny the efficacy of debt monetization through this channel and argue that:

If we consider the possible negative impact on income and spending resulting from lower interest income received on savings, a plausible case can be made that QE2 [after Quantitative Easing 2] will actually be deflationary if the policy is successful in lowering rates (Fullwiler and Wray 2010, p.10, term in brackets added).

As for the model displayed above, it predicts that a compression in the risk/term premium on debt issued by the government will have an expansionary effect albeit this will only occur if the government sets the stance of discretionary policy so as to hit its target debt-to-capital ratio. In other words, a decrease in $\mu_G$ will be expansionary if the decrease in the debt service of the Treasury leads to an increase in $b_0^*$. By contrast, the model also predicts that a compression in the risk/term premium on debt issued by both households and businesses is ambiguous. We discuss this below.

First, a decrease in $\mu_H$ brings about a decrease in both debtor households’ debt service and creditor households’ after-tax interest income. The former will lead to a rise in $S^H$ provided $c_D < 1$, whereas the latter will lead to a decrease in it. If $t_c = 0$ the latter effect will predominate whereas, if $t_c > 0$, the net effect will be ambiguous since:

$$\frac{\partial r^*}{\partial \mu_H} = \left\{ (1-c_D) - (1-c_D)(1-t_c) \right\} \frac{\partial \mu_H}{\partial A_r} < 0$$

Second, a decrease in $\mu_G$ will cause, as long as $\lambda_G^H < 1$, both an increase in $b_0^*$ and a decrease in households´ interest income. Both effects will push $r^*$ upward so that:
Lastly, a change in $\mu_F$ has an ambiguous impact on $r^*$. This is because the fall in $\mu_F$ leads to an increase and a decrease in creditor households’ profit and interest income respectively as well as to a decrease in transfers from the CB to the Treasury. The net effect on $S^H$ of a decrease in $\mu_F$ is negative whereas, if $\hat{d}_G$ remains constant, $b_0^*$ will decrease. Expression (48) thus shows that if $A^M_F$ is small (large) relative to $t_p$, a fall in $\mu_F$ brings about an increase (decrease) in $r^*$ or:

$$\frac{\partial r^*}{\partial \mu_F} = \frac{[A^M_F + (A^M_F - t_p)(1 - c_e)(1 - t_e)]\hat{d}_F}{A_r} \prec 0$$ \hspace{1cm} (48)$$

Next, as reflected in (42) above, the sign of $A_g$ is clearly positive which tells us that an increase in $g_n$ will lead to an increase in $r^*$. We can trace out the different channels of influence on $r^*$ of a change in $g_n$ by looking at (45) above. To be sure, an increase in $g_n$ engenders an increase in both $\gamma^*$ and $b_0^*$. The increase in $\gamma^*$ obeys to the fact that, as businesses increase the pace at which they expand capacity, they need to issue debt at a higher pace and this, in turn, will require an increase in $\gamma^*$ if they are to hit their target debt-to-capital ratio. As for $b_0^*$, the increase in $g_n$ implies that the government can hit its target debt-to-capital ratio even if its debt stock now grows faster in steady-growth. In turn, this will allow it to set a higher $b_0^*$. The latter leads to an increase in $r^*$ whereas the increase in $\gamma^*$ leads to a decrease in $r^*$. The net impact is, however, positive. To finish off this section, we provide some further results associated to the steady-growth equilibrium:

$$\frac{\partial r^*}{\partial z} = \frac{(1 - \varphi)(1 - t_w)v u^* (c_o - c_e)}{A_r} \succ 0$$ \hspace{1cm} (49)$$

$$\frac{\partial r^*}{\partial t_w} = \frac{(1 - \varphi)v u^* [(1 - c_o)z + (1 - c_o)(1 - z)]}{A_r} \succ 0$$ \hspace{1cm} (50)$$

Expressions (49) and (50) tell us respectively that an increase in the proportion of debtor households $z$—due to a decrease in the degree of LP—and in $t_w$ leads to an increase in $r^*$. The latter stems from the fact that a rise in $t_w$ brings about a decrease in
$S^H$ that is not offset by an increase in public saving. Apparently, this is also true in the case of changes in $t_c$ and $t_p$; an increase in either $t_c$ or $t_p$ leads to a decrease in $S^H$ that is not offset by an increase in public saving insofar as, in steady-growth, the government adjusts $G$ to changes in $T$ so as to keep $b_0^*$ constant. Yet, changes in either $t_c$ or $t_p$ also bring about changes in $\gamma^*$ so that the net effect on $r^*$ becomes uncertain as reflected in partial derivatives (55) and (56) in the appendix.

Table 2: Impact on $r^*$ of changes in some parameters

<table>
<thead>
<tr>
<th>$\Delta$</th>
<th>$\lambda_{Gr}^M, \lambda_{Fr}^M$</th>
<th>$z$</th>
<th>$\hat{d}_H \cdot \hat{d}_F, \hat{d}_G$</th>
<th>$\mu_H \cdot \mu_F, \mu_G$</th>
<th>$g_n$</th>
<th>$\pi^*$</th>
<th>$t_c \cdot t_p$</th>
<th>$t_w$</th>
<th>$c_c, c_D$</th>
</tr>
</thead>
<tbody>
<tr>
<td>$r^*$</td>
<td>+</td>
<td>+</td>
<td>?</td>
<td>?</td>
<td>-</td>
<td>+</td>
<td>?</td>
<td>?</td>
<td>+</td>
</tr>
</tbody>
</table>

Note: (+) and (-) mean that an increase in the variable listed on the upper row gives rise to an increase and a decrease respectively in the value of $r^*$ whereas (?) means that the effect is ambiguous.

Next, expression (51) below shows the impact on $r^*$ of an increase in $\hat{d}_G$. In addition, (52) shows that, when there is full monetization of public sector debt ($\lambda_{G}^w = 1$) and the nominal interest rate is at the zero lower bound ($i^* = 0$), a rise in $\hat{d}_G$ will lead to an increase in $r^*$ whenever the latter falls short of $g_n$. The reason is that, as depicted in (41) above, when $\lambda_{G}^M = 1$ (and even if $\lambda_{Fr}^M = 0$), the government may run a higher $b_0^*$ whenever $r^* < g_n$ so that:

$$ \frac{\partial r^*}{\partial \hat{d}_G} = \left(1 - c_c(1-t_c)\right)(r^* + \pi^*) + (g_n - r^*) - (1 - \lambda_{G}^M) \mu_G [1+(1-c_c)(1-t_c)] \Bigg|_{A_r} < 0 $$  \hspace{1cm} (51)

or

$$ \frac{\partial r^*}{\partial \hat{d}_G} = \left(1 - c_c(1-t_c)\right)i^* + (g_n - r^*) \quad \text{when } \lambda_{G}^w = 1 $$  \hspace{1cm} (52)

More generally, if the economy is mired in a "structural" LT so that condition (3) above holds and, hence, the notional steady-growth nominal interest rate is negative ($i^* < 0$), we have that, if there is full monetization of public debt, and since the nominal interest rate can not actually be lower than zero, an increase in $\hat{d}_G$ will yield an increase in $r^*$ whenever $r^* < g_n$.\textsuperscript{12}

\textsuperscript{12} By “notional” we mean the value that the nominal interest rate would take in steady-growth if there was not a zero lower bound on the nominal interest rate.
To sum up, expressions (51) and (52) above tell us that, under the institutional arrangements sketched above, it is the case that printing money to finance government spending as advocated in Abba Lerner’s *Functional Finance* approach (Lerner 1943) will yield an expansionary and non-inflationary effect, even in steady-growth, provided the current ex-ante real interest rate falls short of the current rate of growth of output. Specifically, the implementation of an expansionary fiscal policy ($\Delta d_c$) supported by full monetization of government debt can, when $r^* = 0$, push $r^*$ into positive territory. Whether or not this will suffice to pull the economy out of a structural LT will depend mainly on the level of the natural rate of growth, as well as on the size of the risk/term premium on different types of securities.

Finally, the sign of some of the remaining partial derivatives is ambiguous. For this reason, the algebra was relegated to the appendix and a summary of the results is available in Table 1 above which shows the impact on $r^*$ of a change in the parameters listed in the upper row. As such, Table 1 reports that a change in the target debt-to-capital ratio of debtor households and businesses, in the tax rates on corporate profits and capital income, and in the propensity to consume of debtor and creditor households yields an ambiguous impact on $r^*$. 

6. SUMMARY AND CONCLUSION

The purpose of this study was to explore at a theoretical level, and in the context of a Keynesian model, the conditions required for large-scale debt monetization to exert an expansionary influence in the economy when conventional monetary policy is no longer available. We obtained several results. First, we showed that, if the aggregate availability of reserves does not constrain the granting of loans by banks, the potential expansionary effect stemming from large-scale debt monetization operates mainly from the adoption of a more expansionary fiscal policy stance by virtue of an increase in the transfers from the central bank to the treasury. Second, we showed that the efficacy of the channel that operates through the compression of the risk/term premium on different types of securities exhibits considerable uncertainty; the compression of the risk/term

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13 It will be non-inflationary provided the CB sets nominal interest rates according to (38) above even though the monetization of large amounts of public debt will predictably result in an increase in the ratio $H/K$ in the new steady-growth equilibrium.
premium on debt issued by the treasury was found to be expansionary on net whereas the compression of the risk/term premium on debt issued by debtor households and businesses was found to have an ambiguous effect. Importantly, these results held under the crucial assumption that the government sets the stance of discretionary fiscal policy so as to hit an exogenously-given target debt-to-capital ratio and, arguably, may differ if the latter does not remain constant. Finally, we showed that, as long as a nation can issue its own currency a structural liquidity trap can, in most instances, be overcome through the (full) monetization of an expansionary fiscal policy as advocated in Lerner’s Functional Finance approach. To finish off, our main conclusion is that the efficacy of debt monetization ultimately hinges on the achievement of a very high degree of policy coordination between fiscal and monetary authorities as to the stance of fiscal policy, the inflation target, and the scale and mix of debt monetization by the central bank.
REFERENCES


APPENDIX

We show below the algebraic expressions for the partial derivatives reported in Table 2 and which are not embedded in the main text:

\[
\frac{\partial r^*}{\partial d_H} = \frac{\pi^* + g_n + [-c_{o} - c_{c}] + t_r(1 - c_{c})(\mu^* + \mu_H)}{A_r} \tag{53}
\]

\[
\frac{\partial r^*}{\partial d_F} = \frac{-t_p(1 - c_{c})(1 - t_r)(\pi^* + r^*) + \mu_F + 1 - (1 - c_{c})(1 - t_r)(\pi^* + g_n)}{A_r} \tag{54}
\]

\[
\frac{1 + (1 - c_{c})(1 - t_r)}{A_r} \mu_F \lambda_F \]

\[
\frac{\partial r^*}{\partial t_c} = \frac{-(1 - c_{c})(1 - t_{p})(\pi^* + g_n) + (1 - c_{c})(d_H + t_p d_F + d_G)(r^* + \pi^*)}{A_r} \tag{55}
\]

\[
\frac{\partial r^*}{\partial t_{p}} = \frac{-(1 - c_{c})(1 - t_{c})(\pi^* + r^*) + (1 - c_{c})(1 - t_{c})(\varphi u^* - \mu_F d_{F})}{A_r} \tag{56}
\]

\[
\frac{\partial r^*}{\partial c_{o}} = \frac{-d_H(i^* + \mu_H) + z(1 - t_{w})(1 - \varphi)v u^*}{A_r} \tag{57}
\]

\[
\frac{\partial r^*}{\partial c_{c}} = \frac{-d_H(\pi^* + g_n) + (1 - t_{c})(d_H + t_p d_F + d_G)(r^* + \pi^*)}{A_r} \tag{58}
\]