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Inside Money in a Kaldor-Kalecki-Steindl Fiscal Policy Model: The Unit of Account, Inflation, Leverage, and Financial Fragility*

by

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Abstract

We hope to model financial fragility and money in a way that captures much of what is crucial in Hyman Minsky's financial fragility hypothesis. This approach to modeling Minsky may be unique in the formal Minskyan literature. Namely, we adopt a model in which a psychological variable we call financial prudence (*P*) declines over time following a financial crash, driving a cyclical buildup of leverage in household balance sheets. High leverage or a low safe-asset ratio in turn induces high financial fragility (*FF*). In turn, the pathways of *FF* and capacity utilization (*u*) determine the probabilistic risk of a crash in any time interval. When they occur, these crashes entail discrete downward jumps in stock prices and financial sector assets and liabilities. To the endogenous government liabilities in Hannsgen (2014), we add common stock and bank loans and deposits. In two alternative versions of the wage-price module in the model (wage–Phillips curve and chartalist, respectively), the rate of wage inflation depends on either unemployment or the wage-setting policies of the government sector. At any given time *t*, goods prices also depend on endogenous markup and labor productivity variables. Goods inflation affects aggregate demand through its impact on the value of assets and debts. Bank rates depend on an endogenous markup of their own. Furthermore, in light of the limited carbon budget of humankind over a 50-year horizon, goods production in this model consumes fossil fuels and generates greenhouse gases.

The government produces at a rate given by a reaction function that pulls government activity toward levels prescribed by a fiscal policy rule. Subcategories of government spending affect the pace of technical progress and prudence in lending practices. The intended ultimate purpose of the model is to examine the effects of fiscal policy reaction functions, including one with dual unemployment rate and public production targets, testing their effects on numerically computed solution pathways. Analytical results in the penultimate section show that (1) the model has no equilibrium (steady state) for reasons related to Minsky's argument that modern capitalist economies possess a property that he called "the instability of stability," and (2) solution pathways exist and are unique, given vectors of initial conditions and parameter values and realizations of the Poisson model of financial crises.

Keywords: Chartalism; Climate Change; Consumer Debt; Debt Deflation; Demand-led Growth; Financial Fragility Hypothesis; Fiscal Policy; Margin Loans; MMT; Money; Nonequilibrium Economics; Nonlinear Dynamics; Neo-Kaleckian Growth Models; SFC Models; Stagnation; Wage Contour

JEL Classifications: E12, E31, E32, E37, O42

1. INTRODUCTION

"*In Kalecki, everything is real.*" − Bernard Saffran

This paper seeks to merge the following ingredients in a model of growth and distribution:

1) **A Kalecki-Stiendl investment function**;
2) **Classical/Kaleckian savings propensities**;
3) **Keynesian instability**;
4) **Wage-led aggregate demand**;
5) **Kaldorian (1940) nonlinear continuous-time output-adjustment dynamics**;
6) **Government deficits**;
7) **Government production of a public service from labor inputs**;
8) **A fixed tax rate on all household income**;
9) **Leontief technology for goods production using labor and the capital stock**;
10) **Chartalist state money**;
11) **A time-varying Kaleckian markup for goods with a nonlinearity in the speed of adjustment**;
12) **A fixed, policy-determined real interest rate on government securities**;
13) **A government spending rule with dual resource utilization targets**;
14) **One or more additional options for a fiscal policy rule**;
15) **Endogenous labor force growth**;
16) A Minskyan Poisson model of financial crashes;
17) (a) An unconventional nonlinear wage Phillips curve , or (b) exogenous government-sector wage inflation;
18) Endogenous equity prices and issuance;
19) Bank deposits;
20) Margin finance for equity positions;
21) Bank loans to workers;
22) A wage contour that fixes relative wages in the government and private sectors;
23) Endogenous productivity growth via Kaldor-Verdoorn effects;
24) Cash-flow ratio effects in the investment function;

25) The addition of energy inputs to the production function for goods;

26) Greenhouse gas accumulations via a physical identity;

27) Time-varying stock-flow norms for private sector indebtedness;

28) A time-varying interest markup for bank loans; and

29) Capital gains and stock-flow effects in consumption function for wealthy households.

Ingredients (1)–(15), which appear in bold typeface, are carried over from a recent article by one of the authors (Hannsgen 2014), though debts to previous efforts are obviously immense throughout this list. A cash-flow term is added to ingredient (1), the net investment function. A version of ingredient (16) appeared in a 2012 presentation and in the original working paper (Hannsgen 2012). We will be replacing the capacity utilization target with an unemployment rate target in ingredient (13). The wage-growth options in (17) replace a fixed-wage assumption in Hannsgen (2014). We have eliminated nontransferable, nonvoting government stock certificates and added ordinary dividend-paying stock for wealthy households. Ingredient (19), bank deposits, are of course the *inside money* referred to in the title of the paper, as they represent both an asset and a liability for the private sector as a whole. Lists of variables and dynamic equations forming a self-contained system sufficient to conduct simulations appear in appendices in the back of this paper. Given that we build directly on an earlier model from one of the authors, we hope the reader will forgive the numerous self-references below.

Kaleckian models often have asymptotically stable equilibria. Hannsgen (2014) assumed that the Keynesian stability condition was not met in equilibrium, adopted a nonlinear Kaldor-Kalecki-Steindl investment function (Kaldor 1940), and stabilized the dynamics in u and p using an active fiscal policy function. In the model, the government, whose balance sheet is consolidated with that of the central bank, makes expenditures by issuing interest-bearing liabilities and money, while using open-market operations to set the interest rate in the T-bill market. More complicated versions of the model (Hannsgen 2014, Section 6) included a time-varying markup and an unemployment insurance scheme for workers.[4]

[4] Hannsgen (forthcoming 2015) considers the effects of assuming that wealthy households reach a point of saturation in the consumption good at some level of disposable income.

The quote at the beginning of the paper, from a relatively obscure and distant observer, is of course not entirely fair to Kalecki, who is, in fact, known for adhering to some monetary doctrines now thought of as Post Keynesian (Sawyer 2001). But in the aftermath of the financial crisis, can we not say something about the highly nuanced role for inside money, credit, inflation, financial fragility in the work of the American Post Keynesian school of yesterday and today—in the framework of a model that, after all, is not Keynes's?

Moreover, various authors (e.g., Bernardo and Campiglio 2013), generally from insurgent points of view, have noted that the role of private debt creation, banks, and the like seem to have disappeared for a time from mainstream macroeconomics with the emergence of the so-called new classical school. A few of the major approaches to endogenous private credit creation in various heterodox schools of thought are represented in the following list: Charpe et al. (2011), Robinson (1958, 225–236), Davidson (2007), Delli Gatti et al. (2003), Godley and Lavoie (2012), and Minsky ([1986] 2008). Currently, much work (e.g., Isaac and Kim 2013) is being done to study the roles played by distributional shifts and excessive household debt in the recent economic turmoil in the US and elsewhere.

Hannsgen (2014) stated rates of return in real terms and omitted real wealth and real debt effects from behavioral equations, permitting the model to be solved entirely in terms of Keynesian wage units (Keynes 1936). Below, we model inflation in the wage unit, keeping a markup-dynamics model similar to the one in the earlier paper. All financial assets are now denominated in the unit of account, instead of being "real." The rate of inflation affects the value of financial assets other than equities held over time. This move enables us to structure the model so that inflation-adjusted stock/flow ratios have effects on business investment and financial fragility. With this move toward a truly monetary economy, we hope to be more in keeping with Keynes (1936, Ch. 19) and his Post-Keynesian followers (e.g., Davidson 2007; Minsky [1986] 2008; Weintraub 1963). In this respect, the nonlinear Keynesian models of Asada, Chiarella, Flaschel, Mouakil, Proaño, and Semmler (2010), Chiarella and Flaschel (2000), Charpe et al. (2011), and Ryoo (2010, 2013) have similarities to the one below, though unlike those works, we retain the nonfinancial sector markup as a state variable, rather than using an equation for goods prices.

As in Taylor and Foley (2014), extraction and use of raw materials generates a greenhouse gas, which accumulates over time. This helps us address another issue of the day pertinent to a debt-financed consumer society.

Within the tradition of Post-Keynesian models of growth and distribution, Minskyan models are developed within the formal Minskyan literature (FML).[5] One key element in an authentic mathematical rendition of the financial fragility hypothesis is time. In essence, the financial system becomes more fragile over time as the memory of the most recent crisis fades, encouraging risky forms of finance and high debt. Moreover, fragility and leverage build from one financial cycle to the next in Minsky's cyclical theory, leading to the possibility of cycles within a cycle on a larger scale (Bernard, Gevorkyan, Palley, and Semmler 2014) or the threat of a big crash that might end the financial cycle once and for all (Morin 2013).

A key element of Minsky's theory to include in an analytical model is the role of time in allowing a financial fragility variable to build up. Keen (2000) models debt accumulation through time, adding a characteristically Minskyan role for fiscal stabilization policy. Fiscal policy takes the form of a public production/spending rule in the model below. The early effort in Taylor (1985) makes a "fragility" variable increase in proportion to the difference between actual and normal interest rates. Skott (1994) used the concepts of tranquility and fragility; tranquility is operationalized in his article in a leverage variable that increases gradually. In the model in Datta (2014), fragility is operationalized in the form of a financial gearing ratio.

Along somewhat similar lines, the model below uses a prudence variable P, which is conceptually related to Skott's tranquility variable. Prudence is a psychological and institutional variable that tends to *fall* with the passage of time after a crash, leading to *increased* leverage and fragility, which is a factor, along with weak aggregate demand that directly increases the risk of financial crisis. Working in the opposite direction, the steady accumulation over time of safe assets in the form of government liabilities has a tendency to *reduce* financial fragility and the rate at which crises may be expected to occur, other things equal.

Other approaches have endogenous animal-spirits variables changing endogenously over time, leading to cyclical downturns (e.g., Ryoo 2010; Le Heron 2011). Perhaps befitting a Keynesian model in this computational day and age, Fazzari et al. (2008, p. 562) model expectations, using a simple extrapolative rule. By referring to psychological states, such Keynesian approaches sacrifice some Kaleckian concreteness in investment functions, though they model the psychological variable in terms of other endogenous variables. Nonetheless,

[5] Dos Santos (2005) has usefully capsulized and critiqued much of this literature prior to the writing of his paper. A more up-to-date account appears in Dos Santos and Macedo e Silva (2009).

these approaches help fulfill the goal set by Asimakopulos (1991, especially pp. 120–137), who noted that the state of long-term expectation was one of a number of variables in Keynes (1936) that needed to be endogenized in the effort to extend Keynes's theory to obtain a Keynesian long-run growth model. An "animal spirits" variable true to his thinking would not remain constant forever, as other variables changed.

To cite another example of the modeling of psychological and institutional forces, Kappeller and Schütz (2014) use an approach avowedly in the Minskyan and Kaleckian traditions in which private debt and sentiments are drivers, yet they drive their dynamics with exogenous changes to parameters that are assumed to occur at somewhat arbitrary points in time to generate cyclical behavior.[6] There is no model of the historical process that leads to the sudden parametric move in the authors' account.

Going a bit further in a subjective direction, we will seek a model in which an animal spirits-type shock sometimes acts as an unmoved mover, as envisioned by the Keynes of radical uncertainty (1936, Ch. 18; 1937) and in the *Keynesian Kaleidics* of G.L.S. Shackle (1974).[7] As Cantillo (2015) notes in a recent account of Shackle's approach, these shifts in expectations break a deterministic link between the past and the infinite future. These shifts in animal spirits cause jumps in financial variables at points in time that are determined largely by chance. Yet the *risk* of one of these crashes occurring increases systematically with financial fragility and is decreasing in capacity utilization. The jumps simultaneously impact bank balance sheets and stock market psychology.

Some approaches have used fragility as an objective financial "crunch factor" that reduces aggregate demand as leverage builds, but lacked a "crash factor"—a variable that might somehow model a discontinuity in financial markets or the financial sector. The usual continuous effect is made possible by making investment or the interest rate a function of some leverage or fragility variable. For example, Keen's pioneering dynamical simulation model (1995) modelled the interest rate as an increasing function of leverage. On the other hand, the

[6] Their approach is nonetheless consistent with a heterodox ("old" or "original") institutionalist view in which historical accident, particular institutions—and even conspicuous consumption—affect the path of output over the medium and long runs.

[7] Semmler and Semmler (2013) is another example of a model in which a stochastic process is used to generate exogenous, animal-spirits effects in financial markets, in turn driving the dynamics of the economy. In contrast to our conditional discrete-jump model, they use a Brownian motion. The national debt also plays a key role in their model. However, in addition, they use a regime-switching model to model financial fragility. Their stated intention is to model a eurozone economy, rather than one with chartal (unpegged national fiat) money.

resulting fragility process built over time to financial crisis only in the narrow sense of a period of elevated interest rates with attendant effects on output and growth. In the Minsky Model in Keen (2013, 2014), private debt hurts growth for the sole reason that high debt burdens add to firms' costs, reducing the profit rate, which determines net investment. Another continuous, deterministic approach to modeling the financial fragility hypothesis is to simply augment the investment function with a financial conditions variable, a tack that is taken in Asada (2006), Charles (2008), Datta (2012), Fazzari et al. (2001, 2008), Fisher and López (2014), Gallegati and Gardini (1991), Palley (2010), Patriarca and Sardoni (2011), Ryoo (2013), Schoder (2014) and many others.

Kapeller and Schütz (2014) features constraints on consumer loans that change over time with financial conditions. These models are in tune with Post-Keynesian credit rationing (e.g., Wolfson 1996b) in the sense that financial fragility entails tight credit rationing or constraints, and not just high interest rates. On the other hand, to sum up, these models and most others in the FML (leaving aside the promising heterogeneous interacting agents literature, e.g., Delli Gatti et al. 2003) feature no Minsky moment, no Minskyan singularity, only (1) the continuous phenomena of rising interest rates and/or declining investment, (2) switches between Minskyan financing regimes, or (3) imposed (exogenous) shifts in animal spirits variables.

This is all the more remarkable in light of recent history. In 2008 and the years that followed, newspapers and journals reported on the occurrence of a "Minsky moment" in US financial markets.

More generally, consider actual financial "events," such as any of the three types of financial "crashes" visible in time series data, as described by Rosser et al. (2012). These authors describe jumps of varying magnitudes and degrees of suddenness in time series data. Studies of financial time series continue to note these phenomena and have taken varying approaches to developing forecasting models for data in which large moves are relatively common, and fat-tailed distributions of various kinds seem to help account for observed patterns in return and price-change data (e.g., Hartz and Paolella 2011; Phillips and Shen 2014).

Mirowski (2010) points out that the potential for failure of a complex system can arise with innovation of modern trading algorithms, leading to possible crashes. Minsky's hypothesis is linked to such events in financial markets. Wolfson (1996a) ties the US stock market crash of 1987 into Fisher's theory of debt deflation, as well as Minsky's financial fragility hypothesis, arguing that such an equity-price crash—driven in part by modern algorithms and margin calls—might have helped to bring on a depression, if circumstances had been worse.

Moreover, the monetary aspects of existing models of financial Keynesianism lack realism or even fall prey to loanable funds fallacies. Thus, the approach in Keen (1995) does not allow for the Post-Keynesian notion of an exogenous policy-determined interest rate, even for safe government paper. In contrast, Keen's current Minsky model (2014), like Yoshida and Asada (2007),[8] has an exogenous interest rate, but this feature horns out the leverage-determined private loan rate one finds in Keen (1995). Similarly, the Minskyan model in Datta (2014) has only a policy-determined rate, i.e., no distinct bank loan rate that reflects the effects of fragility, and Fisher and López's (2014) supply-and-demand-determined bond rate is the only interest rate in their model. Asada (2006) features a policy-determined money growth rate, together with a stable money-demand function, allowing the interest rate to be determined in orthodox fashion. *Both* exogenous (i.e., policy-determined) interest rates and time-varying bank loan rates and financial conditions seem relevant to monetary industrial economies. Tauheed and Wray (2006) is an example of a model in the fiscal policy literature that tracks distinct interest rates for bank loans and government debt, but it, like the real model that appears in Hannsgen (2014), omits the effects of inflation and deflation on real asset returns.

Here, for the government bill rate, we use a constant-real-rate policy rule, as advocated by Smithin (2007) and implemented by Rochon and Setterfield (2012) in the context of a Kaleckian growth model. The overall approach to government money and debt issuance in this paper is strongly influenced by (1) the view, held by neo-Ricardians and adherents to the monetary theory of production, that real policy rates are exogenous distributional variables (see Hein 2008); (2) the chartalist/modern monetary theory (MMT) account of sovereign money creation in the process of spending (Wray 2003); (3) the functional finance view that fiscal policy can and should be solely an instrument in an effort to stabilize output and employment (see Forstater 2003 for an account and Kelton 2014, who argues the case that this view applies

[8] Apologetically noting this dilemma in their own contribution, Yoshida and Asada point out that "Although our postulate of the constant nominal rate of interest and the endogenous money supply conforms to the vision of some Post Keynesian economists such as Moore, it is true that our postulate separates the goods market from the financial market. If we consider that the essence of the Keynesian economics is the analysis of the interaction of the financial and the goods markets, our postulate may not be satisfactory" (Yoshida and Asada 2007, p. 445). To us, this indicates a false incompatibility between horizontalist endogenous money and Minsky's financial fragility view, in the absence of a more detailed fiscal policy model that combines the two.

to the post-crisis US situation); (4) the Minskyan view that government acts as a stabilizer of an inherently unstable private sector as expressed in, say, Ferri (1992), Ferri and Minsky (1991), or Palley (2011); (5) the use of output-stabilizing fiscal policy rules in a dynamical system as in Goodwin (1990, pp. 102–110), Keen (1995, 2000), and Yoshida and Asada (2007); and (6) the notion that policy interest rates should be held constant or fairly stable (Rochon and Setterfield 2007), in order to avoid various destabilizing effects of stop-go stimulus, as demonstrated in the models in Hannsgen (2005), Datta (2014), Martin (2008), and others.

For our bank rates, we adopt a markup on the cost of funds similar to that of Rousseas (1985), which has been employed by Lima and Meirelles (2007) and others in models of growth and distribution. Levy and Mántey (2006) explicate this view, which is a version of "horizontalism," or broadly, the belief that money and credit supply curves are flat. They provide some time series evidence for the Mexican case. The desired markup on bank loans varies with the financial fragility variable, which in turn depends on cash-flow and safe-asset ratios. Moreover, actual financial shocks, when they occur, cause discrete jumps in the entries in bank balance sheets, leading to changes in this desired markup, via more immediate effects on financial fragility and the dividend per unit of capital. In contrast, the interest rate on government securities is determined by policymakers and is not affected by changing financial conditions. The shifting expectations that figure in this Minskyan, stock-flow consistent (SFC) model are not views on the uncertain future returns to stocks or bonds in "household" portfolios, as in IS-LM or Keynes (1936), but instead encompass psychological factors driving wealthy households' conventional views on the likelihood of default on bank loans, which affect holders of large, liquid, uninsured deposits. Yet the jumps change not only bank balance sheets but also the state of expectation in the stock market, which is linked to the financial sector via a commonly held set of conventional views on an uncertain future. The stage is set for a jump (crash) by a slow deterioration of "prudence," which loosens psychological and institutional constraints on normalized levels of household borrowing.[9]

Moreover, as in many other recent Minskyan analyses (e.g., Ferri 2010), consumer debt plays a role in the model below. In this sense, we depart from the many models that, perhaps in keeping with the emphasis of Minsky's original theory, focus on the financial fragility of the

[9] Our model of fragility and crisis also differs in various respects from other papers, including Yasuhara (2013), that attempt to merge Kaleckian growth theory with the Minskyan hypothesis in various ways.

corporate banking sector. Of course, this emphasis on household borrowing fits in with developments in many countries that have made it far easier for ordinary households to obtain and use credit. Workers consume what they get or borrow, as in the related models of Charpe et al. (2009), Dutt (2006, 2011), Isaac and Kim (2012), and Palley (2010). The debt/disposable income ratio varies with the "prudence" variable *P*. Wealth-holding households consume only a fraction of their after-tax income, which is derived entirely from asset ownership in this model.

In the sector containing wealthy households, levels of margin borrowing help to determine the price of shares. Change in this variable leads to self-reinforcing feedbacks in the financial markets. As in Bhaduri (2011a, 2011b) and Bhaduri et al. (2015), unrealized stock market gains lead to increased consumption by this sector. Both forms of household debt exert real-debt-burden effects (Kalecki 1944; Fisher 1933; Tobin 1980; Keen 2000; Ferri 2010; Chiarella et al. 2001; Mason and Jayadev 2014; Dutt 2006, 2011; Zezza 2008). These effects include higher financial fragility when the disposable income after interest payments of either class of household declines in relation to its debt.

We have mentioned mechanisms through which a distinct event or state occurs, which is defined as a crash. One existing approach in the literature is built upon stochastic regime-switching models in which crisis and noncrisis states alternate (e.g., Ferri 2011, chapter 13). Like Nishi (2012) and many others, this approach makes use of the natural thresholds offered by Minsky's trifold partition of borrowers into hedge, speculative, and Ponzi categories.

Another way of formalizing discontinuous Minskyan dynamics is provided by the theory of catastrophe, or discontinuous movements that are induced in a variable *y* when a variable *x* moves continuously in state space (Skott 1994). Gallegati and Gardini (1991) use complex dynamics generated in part by a nonlinear relationship between the profit rate and the retention rate. Sau (2013) argues generally for complex self-reinforcing dynamics as an interpretation of financial boom-bust cycles and a rationale for rejecting the efficient market hypothesis. He sees chaos and other complex phenomena as arising from models such as Minsky's, in which a tendency exists for financial markets to endogenously become unstable following a period of tranquility. The model below also incorporates the Minskyan characteristic of the endogenous emergence of fragility but mathematically takes a probabilistic approach to modeling the financial crashes themselves.

To fill in the details of our Shacklean approach, we use a Poisson conditional probability model of rare financial jumps, as was done in Hannsgen (2012, Section 4.2), generating rare shocks whose rate of occurrence depends on the aforementioned endogenous financial fragility and capacity utilization variables.[10] While we do not include a qualitative partition of financial agents based on Minsky's tripartite indebtedness scheme, a cash-flow ratio is one of determinants of proneness to a financial crash—a reasonable approach given that we do not model individual economic units, as in the agent-based approach of, say, Ferri (2011). As pointed out in Hannsgen (2013), one-time expectational and animal spirits shocks have been employed effectively in macroeconomic simulations by such authors as Godley (2012, pp. 111–114) and Kapeller and Schütz (2014, pp. 796–806), but these authors impose shocks exogenously at an arbitrary point in the simulation.[11] While this approach is a step forward from the Hicksian approach of using the *ceteris paribus* assumption to keep expectations out of the analysis, it fails to take into account feedbacks from expectations to economic variables and vice-versa.

As P. Sweezy put it, "For some problems it may be legitimate and necessary to regard them [expectations] as largely given, though so long as we are analyzing a process in time, they are never wholly given. On the other hand, there are problems for which we must extend the scope of our inquiry so much as to include nearly all of expectations and uncertainty among the variables" (quoted in McCann 1994, p. 9).

We use a shock s whose size possesses a heavy-tailed distribution.[12] On impact, the rare shocks move an animal-spirits-type variable α_{dep} via a difference equation that almost always takes on the value zero. Jumps in this variable gradually fade over time, as modeled in a differential equation for the same variable. The pathways for α_{dep} generated by this mixed process of both jumps and continuous adjustment will be right-continuous. The value taken on by this variable at any time t determines the relative demands for bank deposits and state money. Separately, the jump variable shifts the stock-price index pr_S.

[10] See Ross (1997, pp. 235–303).

[11] Sometimes micro models of default probabilities are used in finance (Tasche 2013). For example, Delli Gatti et al. (2003) uses a stochastic process to model the stock valuation ratio. We do not discuss this strand of Minskyan macro models in this paper, as it constitutes a largely distinct literature. Of course, they do not employ a conditional Poisson model in their model. Generally, many articles in the heterogeneous, interacting agent literature employ a different form of probabilistic model—from statistical mechanics—in which agents pass randomly from one type or state to another and equilibrium consists in some average distribution of the agents themselves among various types or states (Carvalho and DiGuilmi 2014).

[12] The case for the use of fat-tailed distributions was argued in my account of heterodox shocks in Hannsgen (2013).

In telling this story about changing household asset demands, we will use the somewhat abstract concepts of "deposits" and "government money," keeping institutional and historical specificity to a minimum. The idea is to include in household demands (1) one government liability ("money") whose face value in the unit of account is stable; (2) an F-sector near-money that is subject to default risk and runs, much as in the cases of commercial paper and large negotiable certificates of deposit (CDs), which Minsky recounted in his work (1986); and (3) equities with market-determined prices.

The probabilistic approach is problematic within both mainstream and heterodox macro traditions, as in the case of Marglin's well-known treatise on growth, distribution, and prices (1984), which mentions the possibility of an expected probability distribution of future profit rates, as opposed to an expected profit rate. Marglin argues somewhat apologetically that one can still model investment as a function of the expected profit rate only on the grounds that under some conditions (say, a normal distribution of future profit rates) the results would be certainty equivalent (80).

With rare jumps added to the model, agents attempting somehow to foresee profit rates, capacity utilization rates, and other key variables years into the future would not be able to forecast well with standard econometric methods, and, given a set of initial conditions, the model will generate time paths that are nonergodic or might as well be nonergodic for all practical purposes for realistic sample lengths (Davidson 1982, 2007). That is, realizations of the model over even 200-year simulation periods would not likely be susceptible to convergent or otherwise successful econometric estimates. Thus, these simulated time paths are subject to "epistemic"[13] uncertainty, meaning they imply a lack of knowledge of the future on the part of agents in the economy. They also imply "aleatory" risk, meaning that the processes generating events themselves are subject to some form of chance. These difficulties are in addition to forecasting problems that arise with the possibility of chaos in a model of this type (Hannsgen 2012). The unlearnability of the model by the firms and households that inhabit it should allow us to steer clear of the fallacy of conflating risk and uncertainty.[14] Moreover, our approach

[13] See McCann (1994, pp. 52–53) on Lawson's concepts of aleatory and epistemic uncertainty and risk.
[14] O'Donnell (1991) describes Keynes's ideas on probability itself, arguing that his arguments about radical uncertainty coexisted with an understanding of "determinate" probabilities of various types in his work on probability theory. The more radical version was applicable in the case of expectations that affected business investment. See also O'Donnell (2015).

avoids assuming a Gaussian distribution of financial shocks, which are a major empirical weakness of most economic models, as witnessed by studies such as Hartz and Paolella (2011).

Moreover, though, our conditional probability approach has the benefit of offering a mathematical expression for something akin to Minsky's financial fragility hypothesis, to add to the discipline's not-insubstantial modeling efforts, mostly deterministic, in works such as those cited above in our discussion of the FML. In Section 10, we present a mathematical statement and informal proof that the model has a tendency to leave any postulated equilibrium point—Minsky's "instability of stability." We further set our contribution within the context of the formal Minskyan literature in the section below on the financial part of the model.

The stock-flow macroeconomics popularized by Godley and Lavoie (2012) and recently surveyed by Caverzasi and Godin (2015) have elucidated the implications of policies continued over the medium and long terms within the framework of Post-Keynesian theory. Specifically, flows of income and expenditure for each sector imply changes in financial positions. The real-debt-burden effect is only one of several channels through which financial assets and debts affect real variables in the model below. Much work has been done to reconcile the insights from the SFC modeling literature with those of MMT and its chartalist antecedents, and in fact the SFC and MMT approaches are not regarded as inconsistent by neochartalist authors such as Wray (2012).

The implications for medium- and long-run sustainability of fiscal policy rules are tracked below within an SFC model. These include the salutary effects of liquid government securities—safer by far than securities subject to default risk— that accumulate on private balance sheets during extended periods of economic tranquility (Minsky [1986] 2008). While Hannsgen's (2012, 2014, forthcoming 2015) previous efforts to model output-stabilizing fiscal policy within a Kaldor-Kalecki-Steindl growth model with sovereign money and a variable markup attempted to carefully account for all financial stocks and flows, these earlier papers relied on assumptions that did not allow implied stocks of debts and assets to affect investment and consumption behavior, except indirectly through the rate of occurrence of financial crises in an early working paper (Hannsgen 2012). Hence, the models in question were influenced by stock-flow macroeconomics, but lacked the emphasis on stock-flow norms and medium-term sustainability that characterize models such as those in Godley and Lavoie (2012). Earlier brave efforts to use Post-Keynesian ideas in an SFC model with bank money and lending include, e.g., Zezza (2008). Here, balance sheet variables influence financial fragility, consumption expenditures, and investment.

Also, however, note that we model asset prices and returns without relying upon hydraulics involving a system of purportedly stable asset supply and demand curves—as in, say, Tobin (1982),[15] Godley and Lavoie ([2007] 2012), Asada, Chiarella, Flaschel, Mouakil, Proaño (2010), or Godley and Lavoie's growth model prototype (2012, pp. 378–444)—a weakness that characterizes some SFC approaches to the markets for financial assets. In contrast, three examples of departures from a static Walrasian system would be (1) the system in Godley and Lavoie (2012, pp. 131–167), which can move owing to errors in expectations of wealth in the next time period; (2) the fundamentalist-chartalist models of stock-price expectations in Asada, Chiarella, Mouakil, Proaño, and Semmler (2010), and (3) Semmler and Semmler's (2013) use of a Brownian motion for animal spirits in their 3-equation EU model. How would an SFC model with static asset demand curves ever generate a move similar to a typical stock market crash or empirically account for observed shifts in the velocity of monetary aggregates? One would suppose that such large moves are likely to have long-run effects on solution pathways in a model with multiple equilibria; hence it may be very important to include them. The crashes generated by the Poisson process allow for endogenous dynamics in these markets.

Foley (2014) recently decried the state of post-Keynesian fixed-markup models, as well as "good old Keynesian" macro models (IS-LM, etc.), which often assume fixed nominal wages. These are certainly weaknesses but surmountable within the framework of Keynesian and Kaleckian growth models. On the other hand, some of the dynamical models mentioned above in the Goodwinian tradition (e.g., Goodwin 1967; Rose 1967) featured changes in distribution as a central and highly topical theme, in spite of the aforementioned shortcomings of these landmark papers in their financial and monetary detail.

Post-Keynesian Sidney Weintraub (1963) was among the first economists to look into the behavior of wage shares over time in models in which the behavior of the markup was one of the key determinants of functional income distribution and inflation. He published a series

[15] Tobin argues, "[T]he perceptions, conventions, and habits that underlie asset demand functions do not change suddenly. It is sufficient for our immediate purpose that the functions are stable over the medium-term horizons of economic fluctuations and stabilization policies" (Tobin 1982, p. 186). The model presented here takes a more long-term approach, incorporating technological progress, etc. Moreover, clearly the assumption of stability would not comport well with Minsky's theory of financial fragility.

which appeared to show that the wage share hovered around a fairly stable average for largely unexplained reasons. He dubbed this lack of trend “short-period,” or “year-to-year” constancy or, in one hyphenated phrase, “near-constancy” (p. 60). In his analysis, he called attention to the functional distribution of income as a potentially important force in the determination of inflation. On the other hand, as one of the pioneers of Post-Keynesian economics, Weintraub influentially argued partly on these grounds for the use of a constant markup model (Kregel 1989). Weintraub’s series trends upward beginning in about 1940 (1963, p. 60), though Weintraub was reluctant to draw the conclusion that a structural break or new trend was underway. Today, it is widely recognized that a sea change in the opposite direction is underway, powerfully shaping economic performance in the US and the rest of the world (Hein 2012). It seems doubtful that Weintraub would oppose the use of variable markup models today, were he alive and still active. Post Keynesians such as Tcherneva (2014) point out that distributional headwinds are central to the insufficiency of fiscal policy responses to the recessions attending the global financial crisis in most countries.

The role of a changing markup as a way of generating needed internal funds is an integral part of the Post-Keynesian theory of the firm as proposed in the 1970s by Eichner and Kregel (1975) and others. Given that money and finance become crucial in this paper, we use a different model of the dynamics of the markup, adding a cash-flow motive to the Kaleckian approach in the earlier model. In the model below, when industrial firms are short of funds because payment commitments exceed net revenues, they rapidly seek to raise the markup, though doing so tends in the aggregate to undercut aggregate demand. This imperative is a source of a nonlinearity in the differential equations for the markups for both business sectors, which are specified so that the time derivative of these distributional variables rises sharply as funds begin to fall below needed levels.

It is important to draw upon such ancient lessons from the American Post-Keynesian literature in order to improve growth-and-distribution models in other heterodox traditions. All earlier versions of the model below (Hannsgen 2014, forthcoming 2015) assumed fixed nominal wages. Imposing the constant wage assumption in those papers was equivalent to using the wage as a numéraire, as the nominal variables did not appear as arguments in any of the behavioral equations, a property of some growth models known sometimes as *nominal-wage neutrality* (Skott 1989, pp. 127–129). It was noted that it was reasonable to assume that as a first approximation, nominal quantities would not matter in an environment of low inflation and interest rates such as the one that has prevailed in much of the world since the financial crisis (Hannsgen 2014, footnote 15, p. 11). But with substantial inflation or deflation over the medium or long run, a macro model should, if possible, take into account price and nominal-wage effects, especially in the context of an SFC model. Hence, in this paper, we still allow for a changing markup, but this time we augment the model with inflation/deflation in the wage unit, as in chapter 19 of Keynes's *General Theory* (1936, pp. 257−279; see also Charpe et al. 2011; Chiarella and Flaschel 2000; Keen 2000; Hein 2008, pp. 131–176; Skott 1989, pp. 114–140). It has been debated whether the relationship between broad inflation and employment is a direct or inverse one. According to Chick's authoritative summary, Keynes himself in the *General Theory* debunked the notion that deflation would *necessarily* move an economy toward full employment, but could not pin down definite results, given his assumptions: "…Chapter 19 shows that when wages do not alter instantaneously and with perfect information all round but alter through time and in uncertainty, both Z and D [curves] shift, with ambiguous results" (Chick 1983, p. 249).

It is well known that Kalecki (1944) argued against Pigou that a falling general wage and price level was most likely to have a *depressing* influence on the economy, as it would raise the real burden of existing debts. Keynes cited this argument in chapter 19 of the *General Theory*, which addressed the effects of changes in money wages, referring to the aggravating effects of deflation arising from increases in the inflation-adjusted burden of debt as an "embarrassment of…entrepreneurs" (1936, p. 264). In the model below, we allow inflation and deflation to influence real financial stocks through accounting identities that relate stocks to flows. The resulting changes in the inflation-adjusted stocks in turn affect spending decisions and financial risks in various ways. As SFC modelers and Minskyans have been at great pains to tell modelers in other schools of thought, there is no more crucial determinant of spending in the financial crisis and its aftermath than leverage (Zezza 2008).

As we have outlined above, this paper combines ideas that have appeared before in many articles on growth and distribution. In combining fiscal policy, bank and chartal monies, general inflation and deflation, endogenous technological change, greenhouse gas accumulation, and our model of financial crises, the model's most similar competitors in the modern literature cited here and below might include, for example, Asada, Chiarella, Flaschel, Mouakil, Proaño, and Semmler (2010); Datta (2014); Godley and Lavoie (2012, 378–444); Hein (2012); Isaac and Kim (2013); Keen (2000, 2013); Le Heron (2011); Palacio-Vera (2012); and Ryoo and Skott

(2013). Although we can hardly claim that our effort stands on a par with these works, none combines all of the assumptions that form the basis for this model. Moreover, in this paper, we take strong exception to various claims and arguments in some of those precursors.

Among the individual elements of the model herein, the main innovation consists in the model of financial crises outlined in Section 5, though versions of this model were set forth in working papers by one of the authors (Hannsgen 2012, 2013). We believe we have achieved a more satisfactory statement of that model than the one in those papers. As far as we know, the fiscal policy rule presented in Section 4 was first published in Hannsgen (2014). In addition, none of the closely related precursors listed above includes an atmospheric greenhouse gas variable.[16]

2. FISCAL TARGETING RULES: THE TASK IN THIS PAPER AND RELATED LITERATURE

This approach may do justice to a world in which the distribution of income can change. Setting prices free to move may help us create a model that can serve as a realistic testing ground for alternative fiscal policies. Earlier work with a simpler "real" version of the model below took the first step in the direction of looking at alternative fiscal policy rules in a Kaldor-Kalecki-Steindl model with chartal money and an endogenous markup (Hannsgen 2014). (See Commendatore et al. 2009; Keen 1995, 2000; Bortz and Storm 2010 for related models in the Kaleckian tradition with some form of fiscal policy. Day and Yang [2011] use exogenous fiscal policy parameters in an elegant dynamic Keynesian growth model.) In the calibrated 2D (constant-markup) version of that model, a fiscal policy rule with capacity utilization and public production targets generated stable limit-cycle dynamics (Hannsgen 2014, Section 3). The larger 3D model with a Kaleckian anticyclical markup could not be stabilized using this rule and yielded pathways that displayed intermittency. A 5D model with a growing labor force and an unemployment insurance program was developed in the penultimate section of the paper, but there was no result for the 5D model that reliably and generally tamed chaos or provided conditions that assured the existence of equilibrium in this case.

In contrast to the mixed results obtained with the joint targeting rule, a balanced-budget targeting rule for fiscal policy produced unstable, saddle-point-like dynamics in the 2D and 3D

[16] Godin (2012) is an SFC model that tracks greenhouse gas emissions.

cases. (See Hannsgen [forthcoming 2015] for additional discussion and results with a nonlinear capitalist consumption function.)

With this effort to introduce bank money and loans, we add the possibility of wage-unit inflation to our macroeconomic stabilization task. Recently, Greenwood-Nimmo (2014) has looked at the effectiveness of inflation-targeting policy rules using a two-instrument approach in an open-economy setting with a constant markup, conflicting claims inflation, and Keynes/Godley consumption and investment functions. Greenwood-Nimmo's policy function shares an orientation toward cyclical stabilization with our own preferred fiscal policy functions, contrasting with austerity rules that target levels of the government deficit or debt. This sort of fiscal policy rule dominates an official list of sorts of those now in force in various countries around the world (Schaechter et al. 2012).

Moreover, much academic work to incorporate fiscal policy into Kaleckian and Keyensian growth models has focused on ensuring the existence of a stable equilibrium in the medium or long runs. For example, one approach is to set fiscal policy so that government debt will be constant as a proportion of the capital stock in a long-run growth path (e.g., Asada 2006; Palley 2013; Panico 1997).

Hannsgen examines and rejects a rule of this type in the context of a real growth model in Hannsgen (2014) and (forthcoming 2015), finding that it leads to instability in capacity utilization and public spending. Others who reject "austerity" fiscal policy rules include Palley (2012), who demonstrates some ill effects of a deficit cap. Crossing such a threshold results in a discontinuous fiscal policy function switch in a scenario developed in Hannsgen (2014, Section 4.2). Somewhat similarly, Godley and Lavoie (2012, 160–165) present a model in which autonomous public spending changes discontinuously whenever the fiscal deficit reaches upper and lower bounds.

In contrast, somewhat more along the lines of Yoshida and Asada (2007) and similar papers, we use a model that has cyclical dynamics and use our fiscal rule to moderate these cycles, or in larger versions of the model, to attempt to counter more general forms of nonlinear macroeconomic fluctuations, including irregular cycles, Poisson jumps, and even chaos.

The approach used here contrasts with Greenwood-Nimmo's paper in a number of regards: (1) we use a Kalecki-Steindl investment function augmented in this new paper with cash-flow effects; (2) we use a time-varying markup; (3) we make use of a Kaleckian consumption function with differing savings propensities for the two household sectors; (4) we use a constant real-interest-rate rule; (5) we use a point target, rather than a target *band*, which Greenwood-Nimmo deems more practical (see below); and (6) the targets in the model below are for the unemployment rate and public production, rather than inflation or another country's interest rate. In the context of this model the need for an unemployment target, rather than a target for capacity utilization as in Hannsgen (2014), is related to the abandonment of the assumption of a given ratio of output to labor hours employed. Our model can be interpreted as a closed-economy model or a world model of sorts, which is consistent with our assumption that policymakers can choose some constant real interest rate without regard to exchange rates or foreign real interest rates.

Some might gather that the closed-economy assumption would greatly restrict the applicability of the lessons learned using the model in the paper. Within neochartalist models more generally, the policy space to choose an interest rate can always be obtained by assuming a flexible exchange rate, capital controls, or a closed economy. A recent article noted that large numbers of countries have exchange rate regimes that lie close to the floating-rate end of the spectrum (Ghosh et al.). While the unfortunate events in eurozone countries following the crises that began in 2010 or so have made fiscal policy in currency union countries an urgent matter commanding world attention. Examples of sovereign-currency countries include Canada, Mexico, Russia, Sweden, Japan, the United States, South Africa, and Brazil.

Certainly, insight would be gained relative to this paper in a model with an external sector and floating exchange rates or capital controls. See Bortz and Storm (2010) for an open-economy, SFC Kaleckian model with fiscal policy, though, like Nimmo-Greenwood, Bortz does not specify nonlinearities or discrete jumps that would generate endogenous cycles or other dynamics to be tamed by a fiscal policy rule. While these paradigm cases in SFC modeling rely upon an equilibrium model, they use a simulation methodology with imposed exogenous shocks to examine stabilizing properties of fiscal policy in their financially detailed settings.

In the realm of current ideas about interest rate policy, our approach can be compared to proposals by mainstream policymakers for a formal nominal GDP target, the Fed's dual inflation and unemployment objectives, and Palley's proposal (2006) to target the "minimum-unemployment rate of inflation," notable for its unconventional Phillips curve.

Looking at things from a different angle, parallels in the world of monetary policy rules exist in articles by Datta (2014) and Duménil and Lévy (1999) and Allain (2014); the last two of these articles introduce endogenous policy largely in an investigation of ways to iron out long-run inconsistencies and tendencies to diverge from balanced growth—clearly in stark contrast to a Minskyan or Post Keynesian approach.

Yoshida and Asada (2007) assume a fiscal policy function involving capacity utilization and nominal output in another nonlinear model, though they differ in their monetary detail. The need for an inflation or nominal GDP target is obviated to some extent in the model below by the existence of a wide range of unemployment rates in our nonlinear wage Phillips curve over which the nominal wage remains roughly constant. This curve will not generate sharply accelerating inflation as policy is loosened near an equilibrium point. The model abstracts from the "bad luck" that can occur when a broad range of world commodity prices exogenously and sharply rises. For these reasons, a moderately low unemployment rate target will be consistent with price stability, perhaps obviating the need for a separate inflation target in the policy function. On the other hand, both the markup and the nominal wage grow increasingly unstable at very low or very high values of capacity utilization *u*, adding a complication to the policy problem.

Greenwood-Nimmo notes, commenting on the fiscal policy rule in Godley and Lavoie (2007), that "it seems unlikely that the form of fiscal intervention advocated by Godley and Lavoie…could be fine-tuned to the degree required to achieve a point target in practice, as activating and deactivating public works projects, for example, is likely to generate a somewhat lumpy path of government spending [i.e., one that moved in big steps rather than smoothly]. For this reason, the use of a band target [a range, rather than a specific number] for fiscal policy seems more appropriate."

Specifically, Godley and Lavoie's preferred rule (2007) called for a level of government spending that would *immediately* fill the gap between actual and potential output—and hopefully keep unemployment low. To remedy this problem, Greenwood-Nimmo (2014) advocates a rule with discrete jumps in spending in specific amounts that go into force abruptly once inflation exceeds or drops below certain upper and lower bounds or thresholds. To his credit, he advances the goal of rehabilitating fiscal policy relative to New Keynesian approaches in which only monetary policy mattered.

To justify lumpiness in their policy function, Greenwood-Nimmo (2014) argue that it would be hard to continuously increase and decrease spending on employment-generating projects as required to hit the precise spending/production target specified by a fiscal policy rule. We abstract from this practical and important issue in our model, but wish to note (1) that proposals have been made to draw up a list of infrastructure projects in advance for use during a crisis (e.g., Shubik 2009); (2) much work has been done to flesh out appropriate administrative details in the context of proposals for a government employer-of-last-resort (ELR) program[17]; and (3) our use of a dual target rule with slow adjustment toward both capacity utilization and public production targets models a slow fiscal policy adjustment process, which in principle makes some allowance for legislative lags, implementation lags, etc.[18]

One of the authors of this paper showed in Hannsgen (2014) that the model generated a stable equilibrium or orbital stability (a stable limit cycle) in 2D state space. A limit cycle is born at a critical of the value of a bifurcation parameter. It is also true that in theory—given our calibrated parameter values—manipulating one parameter in the fiscal policy rule can gradually narrow the amplitude of the cycle in the 2D model to a stable equilibrium at the public production target and at equilibrium capacity utilization. In principle, stability can always be achieved as long as Assumptions 1–3 in Hannsgen (2014) hold[19] simply by reducing the weight on the public production goal in the policy function to or beyond the critical value. A stable limit cycle with a period of nearly 4 years was found for a calibrated version of the model with a realistic cyclical amplitude (Hannsgen 2014, 18–20)—a bit short by most reckonings for the US.

The discussion in Caverzasi and Godin (2015) distinguishes between analytical and numerical solutions of SFC models and among discursive and simulation resolutions of theoretical SFC models. This paper is less analytical than Hannsgen (2014) in that the earlier article examined the dynamic properties of the model for all cases in which the assumptions of wage-led AD and Keynesian instability in output adjustment held. That model could be analyzed near the equilibrium point, while in this case we lack a steady state (see Section 10).

[17] See Charpe et al. (2015) for a model of labor market dynamics under such a program.
[18] On lags in fiscal policy, see also Yoshida and Asada (2007), who use time-dependent public expenditure adjustment, while the model in this paper relies upon state-dependent adjustment of actual to desired spending. Martin (2008) extends the work in Godley and Lavoie (2007) to active monetary policy, which is shown to require a debt-stabilizing rule in the context of Keynesian stability. A model with a countercyclical interest rate otherwise generates exploding debt ratios. In contrast, this paper and its predecessor have concentrated on a case with Keynesian instability and a Kaldorian nonlinearity in the investment function. An earlier paper by Hannsgen (2005) showed via simulations that given that investment was determined by the change in the interest rate from the previous period, strongly reactive monetary policy generated explosive behavior. Indeed, Minsky tended to oppose reliance on countercyclical rates, emphasizing lender-of-last-resort activity (e.g., Minsky 1986). On the other hand VAR evidence of the type cited in the earlier paper is subject to an infinite variance critique, among others (Hannsgen 2012).
[19] These assumptions were (1) Keynesian instability; (2) a condition on one policy coefficient, which the author called "policy instrumentalism"; and (3) wage-led aggregate demand (AD). See Hannsgen (2014) for precise mathematical statements of these assumptions.

We now rely much more on simulation for given initial conditions and parameter values than in Hannsgen (2014), though no simulations have been completed for the full model. And in this paper, we prove the existence of solutions under some weak conditions (see Section 10). Appendix 1 provides a list of variables and Appendix 2 contains a complete list of dynamic equations (19 differential and integro-differential equations, 3 difference equations, a behavioral equation involving time, and an inhomogeneous Poisson process) with which the model can, in principal, be simulated. We apologize for not describing in more detail the substitutions needed to reduce the model to the compact form presented there, which constitutes the only form of "analytical solution" we can offer for the full model.

3. SOME PRELIMINARY MODEL DETAILS

The model contains 5 sectors: (1) the K-sector, which includes nonfinancial firms; (2) K-sector households, the members of which hold wealth and receive only asset-related income; (3) the W-sector, which contains workers who own no assets and receive wage income; (4) the P-sector, which combines the central bank and the government; and (5) the F-sector, which makes loans and accepts deposits. The balance sheets of these sectors are as follows.

Balance Sheets (in Nominal Terms)

Hannsgen (2014) featured a consolidated K-sector, which included both firms and wealthy households, though the consumption and investment decisions of this sector were modeled with independent equations. Here we wish to model the market for shares, which confer ownership of firms on a separate household sector. Also, we include both nonfinancial and financial sectors, which we call the K- and F-sectors. The three aforementioned sectors' balance sheets are shown below. Wealthy households hold bank deposits *Dep* and the total stock of N_S shares with each share priced at pr_S. Their only liabilities are margin loans *ML*. Nonfinancial firms own the stock of capital goods, which is worth $k.pr_U$ in nominal terms, and hold all energy reserves, an assumption made for the sake of simplicity. The physical amount of these reserves is denoted *ER*es and their purely notional per-unit price is pr_F. K-sector firms are entirely financed with equity. In nominal terms, the F-sector holds reserves M_F, bills *B*, consumer loans D_W, and margin loans *ML*, against deposits of *Dep*. They are assumed to have no net worth, distributing all net profits to K-sector households.

Table 1 Balance Sheet for Sectors with Assets

Wealthy households		**K-sector firms**		**F-sector**	
Assets	Liabilities	Assets	Liabilities	Assets	Liabilities
Dep $N_S pr_S$ M_h	ML	$k.pr_U$ $ERes.pr_F$	$N_S pr_S$	M_F B D_W ML	Dep

In the public sector balance sheet below, the central bank and the government are consolidated (See Wray [2003] for an explanation of this consolidation as it appears in the chartalist/MMT tradition; previous examples of the use of this consolidated balance sheet approach in dynamical models with fiscal policy include Asada [2006] and Hannsgen [2014].) Hence, we omit central bank interest income. We assume that this sector carries no financial assets on its books. It uses electronic means to keep track of reserves, netting out receipts. Similarly, it destroys any t-bills purchased in the open market, rather than holding them as financial claims against itself. To spend, it creates new central bank deposits, and it prints new t-bills to meet F-sector open-market demand and continuously maintain the targeted P-sector interest rate. We neglect variations in public sector employment and output that might arise from these central banking operations. We assume that there are no P-sector entities or local governments lacking access to funds from the central bank.

Table 2 Balance Sheet for Public Sector

P (public)-sector	**Assets**	**Liabilities**
	None	M B

The final balance sheet is that of the workers, who make up the W-sector. They hold no assets, only consumer debt D_W. We will calibrate the model so as to avoid solution pathways in which the F-sector is in debt to the workers. This assumption allows us to assume that all disposable income is spent in the same instant that it is earned, resulting in a complete lack of assets, which helps to define this class in our simplified model.

Table 3 Balance Sheet for Working Sector

W (worker)-sector	**Assets**	**Liabilities**
	None	D_W (consumer loans)

The transactions flow matrix below shows money flows and, in the bottom row, the resulting changes in stocks. The stock of government debt and the interest rate affects income through the payment of interest to wealthy households, an effect emphasized in chartalist accounts such as Tauheed and Wray (2006) and Hannsgen (2014) and noted in the Kaleckian context at least as early as You and Dutt (1996).

Table 4 Transactions Flow Matrix

	W-sector	**K-sector Firms**	**K-sector households**	**P-sector**	**F-sector**
wage payments	wages	–k-sector wages		–p-sector wages	
Goods sales	–w-sector consumption	revenues from sales of consumption goods	–K-sector household consumption		
Income on assets	–interest on consumer debt	–K-sector dividends	interest on bank deposits + F-sector dividends + K-sector dividends – interest on margin loans	–interest on treasury bills	–interest on deposits – F- sector dividends + interest on margin and consumer loans and treasury bills
taxes	–income taxes		–income taxes	income tax receipts	
Changes in stocks	Net, inflation-adjusted accumulation of consumer debt	Net accumulation of capital – value of new equity issues	net, inflation-adjusted deposits + value of new equity issues + inflation-adjusted capital gains on stock holdings – net, inflation-adjusted accumulation of margin loan debt +inflation-adjusted accumulation of currency	–inflation-adjusted change in P-sector liabilities (reserves, currency, and bills)	inflation adjusted change in margin and consumer loans + inflation-adjusted change in government bills+ inflation-adjusted reserves – inflation-adjusted change in deposits ≡ 0

Notation

A key to the variables in the model appears in the first appendix. The following mathematical notation is used below in regard to the endogenous, dynamic variables.

$$\dot{x} \equiv \frac{dx}{dt}$$

$$\hat{x} \equiv \dot{x}/x$$

Δx is a finite jump in $x(t)$ at a point in time t, leading to a right-differentiable pathway.

α_x is a positive adjustment coefficient specified by subscript x.

Functions designated by f_i are C^1 (continuously differentiable) on any compact domain in the state space, where i designates how the function appears in the model. Stronger smoothness conditions can be imposed to ensure the validity of approximations used in simulating the model in continuous time; no essential properties of the functions are lost in doing so. In part to avoid singular points that violate our boundedness conditions, we will assume that we are not dealing with any pathways that involve negative zero or negative debts for households, capital stock equal to zero, etc., to avoid division by zero. Also, in considering particular solutions, it is often helpful to specify that a bounded variable x_i in the state vector $\mathbf{x}$ will stop moving at least temporarily when it reaches a boundary ∂ by using a formulation such as

$$\dot{x}_i = \min[f(\mathbf{x}), 0] \; for \; \mathbf{x} \in \partial$$

which stops movement in the positive direction at an upper bound. It is also helpful to use a similar device to temporarily stop downward motion at lower boundaries. As explained below, the C^1 property of the function is lost, along with the differentiability of the solution pathway at a point, but with no severe consequences. Otherwise, if one is content with a result about sustainability over only a subinterval of the intended simulation period, one can simply truncate the solution pathway at the point in time when the boundary is reached. We will not repeat such provisos below.

All variables are functions of time. For financial stocks, capital letters indicate nominal values, while small letters are used to indicate nominal quantities that are normalized by the price of goods pr_U times the capital stock k, except in the case of state money, which is expressed by the script letter $\mathcal{M}$ in the normalized case, e.g., $\mathcal{M} \equiv \frac{\sigma_K M}{k.pr_U}$. An exception is equities, for which there is a separate price variable pr_S. For "real" quantities, a capital letter indicates units of output, while the corresponding small letter is used for these amounts, expressed in units of capacity, e.g., $c \equiv \frac{\sigma_K C}{k}$. Later, to keep things simple, we will assume that per-unit capital requirements σ_K= 1. An important and significant exception is net investment, which is signified by NI in "real" terms and g when normalized by the stock of capital, i.e., $g \equiv NI/k$. Normalized units of the public service are defined in the following section.

4. FISCAL AND MONETARY POLICY BLOCK

The government sector hires workers and produces an output that we will call "services." It prints its own money and short-term debt, which we will call "bills." We adopt the consolidated view of the government balance sheet called for by chartalists, in which the central bank and the government proper are treated as a single entity (see, e.g., Wray 2003, for an exposition). This institutional arrangement will be possible in many countries; it is effectively in use in many countries. The latter would be countries with their own unpegged currencies and central banks that operate domestic monetary policy by setting a policy interest rate. Some examples of countries that at least formally have sovereign currencies include the US, Canada, the United Kingdom, Japan, China, and Argentina. We do not mean to include examples such as the US in roughly the period 1979–84, in which central banks are attempting to stay within a target cone for the paths of reserve aggregates, $M1$, or some other quantitative target, and do not announce a short-term interest rate target in advance. We omit the role of longer-term government securities, though in principle interest rates on notes, bonds, etc. could be set in a similar way in our assumed institutional context.

Policy is set in the following ways: (1) we assume that policymakers choose a desired nominal interest rate on bills such that the corresponding desired inflation-adjusted rate $i^d = \bar{\imath}$, where the latter is a fixed target, as advocated by Smithin (2007), and adopted by Rochon and Setterfield (2012) and Hein (2008, pp. 87–127) for use in Kaleckian models; (2) the government sector continuously keeps the inflation-adjusted policy interest rate equal to its desired level ($i = i_d$) through open market sales and purchases of bills; (3) the government sector purchases bills and pays interest and wages with its own currency and accepts this currency as a means of payment; (4) if necessary, the government conducts the aforementioned transactions through private banks by crediting these institutions with central bank deposits, which can be exchanged for currency at the central bank as necessary; (5) the government determines the nominal wage that it pays its own workers ω_P at all times t. We discuss two ways that it can select that wage in a section on wages and labor below; and (6) the government sets fractions α_{ℓ_U} and α_F of public production devoted to services that increase the rate of technical progress and services that increase the soundness of the financial system.

An implication of these assumptions in our closed-economy model is that total government liabilities *M* + *B* change at a rate equal to the government deficit *DF* at all times *t*. In the equations that follow, we normalize nominal variables by the nominal value of the stock of capital goods *k*, so that in the case of the stocks mentioned above, $\mathcal{M} \equiv {}^{M}\!/_{k \cdot pr_U}$; $b \equiv {}^{B}\!/_{k \cdot pr_U}$; $df \equiv {}^{DF}\!/_{k \cdot pr_U}$, where for simplicity, we assume that the ratio of the required capital stock to output equals 1.

Using these normalized variables, we get the stock-flow identity for the combined central bank and government. First, differentiate the definition of normalized total government liabilities $l \equiv \mathcal{M} + b \equiv \frac{L}{pr_U K}$ to obtain

$$\dot{l} \equiv \left(\frac{1}{pr_U K}\right)\left(\dot{L} - \widehat{pr}_U - g\right).$$

Substituting for $\dot{L}$ from the nominal accounting identity below and simplifying yields

$$\dot{l} \equiv p + (\widehat{pr}_U + \bar{\imath}\widehat{pr}_U - 1)b - \tau\left(\frac{y_K + y_W}{1-\tau}\right) - \left(\frac{\widehat{pr}_U + g}{pr_U k}\right).$$

The equation says that net *P*-sector liability issuance is equal to the deficit, inclusive of interest payments, and adjusted for capital accumulation and goods inflation. This balance sheet identity is an alternative to the government no-Ponzi constraint that appears in most neoclassical infinite-horizon, representative agent models, giving them the property of "Ricardian equivalence."[20] Demand then determines the *relative* amounts of the two government liabilities—bills and money—which are related to *l* through the identity

$$l \equiv \mathcal{M} + b.$$

In the section on the *F*-sector below, we provide asset demand functions and balance sheet identities sufficient to determine the demand for bills, *b*.

In macro modeling, the above institutional setup for government bills and money is similar, for example, to the one in Hannsgen (2014) and the SFC approach of Lavoie and Godley (2012, chapter 4, pp. 99–108), though the latter does not consolidate the central bank

[20] Benassy (2007) presents the non-Ricardian alternative within the infinite-horizon, representative agent literature.

and the government sector. Our chartalist formulation has the advantage that it reveals similarities between (1) “helicopter drops” of spending and other “unconventional” stimulus policies and (2) more usual policies in which the government itself cannot print money, but the central bank nonetheless buys securities in sufficient quantities to hit a given interest rate target. For example, the US government itself could not print money to finance its 2009 stimulus plan, but its central bank purchased enough debt securities to cause a $3 trillion increase in the amount of reserves on commercial bank balance sheets from 2009 to present (Phillips 2014, p. 237). Zezza (2008), Godley and Lavoie (2012, chapter 4), and many others do not adopt the chartalist/MMT convention of consolidating the government and the central bank balance sheets, but are equivalent in all other respects in their treatment of central bank operations, liability issuance, and interest rate determination. Asada, Chiarella, Mouakil, Proaño, and Semmler (2010) use a consolidated balance sheet, albeit one with exogenous government money. For detailed analyses of the institutional links between the treasury and the central bank in the case of the US, see Tymoigne (2014) and Phillips (2014).

On the other hand, our consolidation of balance sheets in a mathematical model is not meant to gainsay the serious difficulties that can arise for governments when they must deal with a central bank that uses its interest-rate setting powers as a way of discouraging high government deficits. In keeping with some of the Kaleckian arguments in Rugitsky (2013) and Moudud and Martinez-Hernandez (2014), we use targets for government spending/production and capacity utilization that are far lower than those that could be used in a closer-to-ideal world. On the other hand, the Kaleckian model can suggest policies that would make both capitalists and workers better off, even in something far short of an ideal world. Bluntly put, *bad* policy doctrines—not just policies that favor elite groups—prevail in the world depicted in the model, and partly as a result, fiscal policy in most countries is biased toward subpar capacity utilization and employment. This situation can only leave room for improvement in fiscal and monetary policy.

In contrast to work by Panico (1997) and subsequent work by him and his coauthors, most applied Keynesian fiscal policy models are vulnerable to the charge in our court of assuming either an assumed exogeneity of the stock of money (Asada 2006, Asada, Chiarella, Mouakil, Proaño, and Semmler 2010; Chiarella and Flaschel 2000; Charpe et al. 2011), assumed balanced budgets (e.g., Allain 2013), an outright omission of monetary government liabilities (e.g., You and Dutt 1993, Dutt 2013, Ryoo and Skott 2013, a supply-determined composition of government liabilities (e.g., Asada 2006, Chiarella and Flaschel 2000, Palacio-Vera 2012), and/or an assumption that deficits either are zero or take on equilibrium-preserving values at all times (Palley 2013).

The various non-SFC approaches lead to various undesirable model properties. First, some models feature financial crowding out, which seems implausible given the coexistence of high deficits and low sovereign-debt interest rates in many cases. Second, some studies do not show that the outstanding amounts of government liabilities are willingly held by households or financial institutions at the going interest rate—in mathematical terms, an issue of inconsistency. Third, as pragmatic policy guides, rules for liability issuance that do not respond to fluctuations in asset demands would probably lead to an extreme volatility of asset prices and interest rates (Goodhart and Tsomocos 2012). See the argument in Godley and Lavoie (2012, chapter 4, appendix) against these non-MMT approaches, and Day and Yang (2011) and Hannsgen (2012) for non-optimizing approaches that *do* permit arbitrary deficits without adverse effects on the government's cost of borrowing or other portfolio effects on aggregate demand.

Many orthodox approaches require some form of long-run balance condition. Day and Yang (2011) provide an argument that a model with balanced budgets or an infinite-horizon "government no Ponzi game" (NPG) assumption clearly does not hold in the numerous countries with sovereign currencies that appear to be able to run deficits almost every year. Michl (2013) explores the implications of an optimizing model with classical/Marxian properties, both with and without the "Ricardian" NPG assumption.

On the other hand, in Fisher and López's recent working paper (2014), the central bank chooses a flow rate of helicopter drops in the form of high-powered money in an attempt to steer the rate on government securities toward a desired level $\bar{r}$, but ultimately, in their model the actual rate r is determined jointly by the amount of government debt outstanding and a stable asset-demand curve. This use of a reserves instrument in the model does not recognize the largely defensive nature of open market operations, which are in reality mostly used to offset endogenous flows of various kinds, maintaining a constant rate between policy changes. In the setting of Fisher and López's political-economic model, such an approach is intended to capture the realpolitik of a tug-of-war in rate setting between policymakers and the interests of the bond markets, but it leads to a partial crowding-out effect that operates on a day-to-day timescale. Here, the interest rate on government debt securities changes only when the central bank changes its target rate. In an applied policy setting, a central bank that followed Fisher and Lopez's model or one like it too closely might induce the kind of financial market instability highlighted in the stochastic model by Goodhart and Tsomocos (2012).

Outside of vastly different approaches within the mainstream such as the textbook New Keynesian approach of Romer (2000), relatively few models in monetary macrodynamics seem to have been constructed on the basis of endogenous (demand-determined) monetary aggregates and an exogenous (policy-determined) interest rate. Within the non-shock-driven macro category, the exceptions with nontrivial monetary policy dynamics include sections in Asada, Chiarella, Flaschel, and Franke (2010, pp. 88–96), parts of the text by Charpe et al. (2011, 124–179), Michl (2013), and an essay in Hannsgen (2002, pp. 85–125), all of which use differential equations that adjust a central bank rate in response to misses of policy targets.[21] In this paper, the central bank uses a nominal rate instrument to maintain a fixed inflation-adjusted interest rate (see below). Some have argued that "park-it" rules in general are the most stabilizing kind of interest rate rules; the government can do no better than to hold either the nominal policy rate or the real policy rate constant (Rochon and Setterfield 2007). In contrast to this traditionally Keynesian point of view, it is important to realize that most contributions to the policy aspects of the post-2008 FML (Charpe et al. 2009) still put the entire burden of stabilization on monetary policy, contrary to the gist of the rehabilitation of fiscal policy referred to above. Hence, the majority of these models with some form of countercyclical policy use a monetary aggregate or interest rate as the policy instrument, rather than a government spending variable or tax rate. To varying degrees, they argue in favor of this division of responsibilities as a matter of policy, not just on grounds of institutional realism.

In previous papers on the real version of this model (2014a, 2015), Hannsgen considered two candidate fiscal policy functions—an output-stabilizing function and a balanced-budget targeting rule. Both were used to determine the desired level of public production p in the institutional context described above. One of the alternatives featured dual targets of public

[21] Hannsgen (2002, essay 2) provides further citations to the "neoclassical horizontalist" approach in dynamic macro, which appeared in the literature beginning in the late 1980s, as well as some discussion. It is largely a critique of policy approaches that use a monetary aggregate as an instrument or intermediate target in an effort to stabilize the economy. Asada (2011) provides further discussion of work based on Taylor-style interest rate rules. Asada, Chiarella, Flaschel, Mouakil, Proaño, and Semmler (2010) analyzes the consequences of a financial-stability-oriented interest-rate rule.

production and capacity utilization. The other rule raised or lowered public production depending on whether the budget deficit was negative or positive, respectively.

This time we use the symbol p^T for public production target and un^T for the targeted unemployment rate, with the latter replacing the capacity utilization target u^T in Hannsgen (2014). As explained in Hannsgen (2014; forthcoming 2015, appendix), in the "real" versions of this model, policymakers chose the values of the targets p^T and u^T so as to be consistent with an equilibrium in u and p. One target's value was chosen by policymakers and the other followed via an output multiplier-type relation plus an equilibrium selection rule, with the latter rule going into effect if more than one capacity utilization rate was consistent with a desired public production target p^T in a "p-first" approach (Hannsgen forthcoming 2015 appendix).

However achieved, the target-consistency property of the output-stabilizing fiscal policy rule helps to prevent some well-recognized lack-of-long-run-equilibrium ("instability") problems that can arise in the presence of a natural or desired rate of capacity utilization, requiring some form of reconciliation (see, for example, Allain [2013, pp. 6–7] for a discussion). Another stabilizing behavioral property of the model is our assumed nonlinearities in the differential equations for the nominal wage and the markup (see Section 7), which imply that these variables remain constant or nearly constant within a nontrivial range of capacity utilization rates. Option 1 for the wage-growth function, in which the chosen rate of inflation of the P-sector wage drives broad wage inflation, is in tune with chartalist strands of post-Keynesianism (see below). The nonlinear wage Phillips curve used in Option 2 is a Post Keynesian feature of the model in that this tradition rejects standard Phillips relationships of all kinds.

Other recent stock-flow consistent attempts at the use of public spending rules aimed at stabilizing cyclical macro variables in Keynesian or Kaleckian models include Allain (2014), Godley ([2007] 2012), Martin (2008), Greenwood-Nimmo (2014), and Le Heron (2011, 2012). Greenwood-Nimmo (2014) proposes an inflation targeting approach. That paper, as well as Bortz and Storm (2010), experiments with fiscal policy rules in the context of a two-country model in which policy coordination issues are relevant.

The models mentioned in the previous paragraph test the ability of fiscal rules to stabilize a system with stable roots following imposed shocks. Studies closer to the realm of applied nonlinear dynamics and endogenous fluctuations include Asada (1987), Asada, Chiarella, Flaschel, Proaño, Mouakil, and Semmler (2010), Day and Yang (2011), Goodwin (1990, pp. 102–110), Keen (1995, 2000), and Yoshida and Asada (2007).

Like the present effort, Commendatore et al. (2011) and Commendatore and Pinto (2011/12) use endogenous cyclical dynamics á la Kaldor (1940) but focus on the cost-side effects of different forms of government spending. Accordingly, they use a "passive" spending rule, rather than an anti-cyclical one.[22] Many of the aforementioned studies, like the current one, seek to codify the "rehabilitation" of active (anticyclical) fiscal policy that occurred in many developed countries as a matter of pragmatism during the financial crises of the late 2000s (Arestis 2010).

The small model of Day and Yang (2011) assumes given levels of government investment, government consumption, and a given tax rate. Thus, as in Commendatore et al. (2011), Commendatore and Pinto (2011/12), and this paper, government spending impacts conditions on the supply side. They use a growth model, as in much of the Kaleckian literature cited above. Day and Yang use Keynesian private consumption and investment functions and analyze the long-term dynamic properties of their model, although as they point out (p. 221), their study is not an attempt at stock-flow consistent modeling. They find their computed equilibrium fiscal stance unrealistic.

A recent paper by Fisher and López (2014) uses an approach that emphasizes the political economy of spending decisions, and so includes the interest rate, the wage share, and the type of government in their fiscal policy function. On the other hand, methodologically, Fisher and López use a computational approach similar to the one contemplated here to work with a dynamical system similar to the ones analyzed in the papers of the previous paragraph, and in Hannsgen (2014, forthcoming 2015). Their positive approach puts them in the role of political economists. This paper finds itself in this role as well when it selects policy targets that are in general far from the ideal of zero unemployment in order to generate artificial macroeconomic data series, though it also emphasizes persistent irrationality in operative macro policy doctrines. On the other hand, there is no reason to insist on a hard-and-fast line between positive and normative models, which might discourage experimentation with policy rules that do not describe the current economic situation.

[22] Also, from a mathematical standpoint, these papers use 1D maps rather than vector fields. Of course, there are deep homologies between these two types of dynamical models—discrete-time and continuous-time—e.g., Guckenheimer and Holmes (1983, p. 179). Day and Yang (2011) also use a discrete-time framework.

For an approach to fiscal policy that makes use of case-by-case judgment rather than policy rules (within an SFC framework), the broad approach used in the Levy Institute Macro Model serves as an example (e.g., Zezza 2009). This scenario-based approach uses assumed paths in time for the fiscal policy stance in generating projections for periods of perhaps 4 to 5 years. It imposes shocks on parameter values to generate movement in a stable, (quasi-) linear model.

In contrast, the present dynamical approach results in a system that generates motion on its own, without the impetus of imposed shocks. Broadly speaking, modeling macrodynamics as the joint product of inherent financial instability and stabilizing government policy was the approach of Ferri (1992) and Ferri and Minsky (1991) (see Palley 2011). Few of these models are fully fleshed out. Currently, most dynamical models inspired by the Minskyan tradition probably fall within the heterogeneous interacting agents' literature (e.g., Delli Gatti et al. 2003). Keynesian fiscal policy functions and their stabilizing properties were first studied in detail in Phillips (1954), and are discussed at length in Section 2.

The definition of a unit of public services in this block of the model could be used to arrive at a measure of overall net output $y = p + g + c$. Here, we have used the arbitrary convention that equal amounts of labor produce equal use value. Nothing in the model depends on the validity of such a calculation. Our use of a constant labor productivity coefficient in the production function for government services p contrasts with our story about the K-sector, which implies that productivity grows faster with a higher growth rate or higher R&D expenditures in relation to the capital stock. There is some evidence that indeed industries such as health care and education, which are largely within the public sector, have stagnant productivity in comparison to manufacturing—a matter of great policy importance (Baumol 2012).

New fiscal-policy function: $\dot{p} = -\left[\alpha_{p,p}(p - p^T) + \alpha_{p,un}(un - un^T)\right], \quad p^T > 0, \quad 0 \leq un^T \leq 1$

Definition of target variable: $un \equiv \frac{N - L_p - L_u}{N}$

Budget rule 1 **Productivity-increasing public production:** $p_{\ell_U} = \alpha_{\ell_U} p$

Budget rule 2 **Financial regulation and supervision:** $p_F = \alpha_F p$

Budgeting identity and inequalities: $\alpha_{\ell_U} > 0 \qquad \alpha_F > 0 \qquad \alpha_{\ell_U} + \alpha_F < 1$

Constant-real-rate monetary policy rule: $i_P = \bar{\iota}\,,\ \bar{\iota} \geq 0$

Definition of P-sector deficit: $df \equiv p + i_P b - \left(\frac{\tau}{1-\tau}\right)(y_K + y_W)$

Definition of government liabilities: $l \equiv b + \mathcal{M}$

Definition of sovereign money: $\mathcal{M} \equiv \mathcal{M}_h + \mathcal{M}_F$

P-sector stock-flow identity in nominal terms: $\dot{L} \equiv P + (\widehat{pr}_U + \bar{\iota}\widehat{pr}_U - 1)\, B - \frac{\tau}{(1-\tau)}(Y_W + Y_K)$

P-sector stock-flow identity: $\dot{l} \equiv p + (\widehat{pr}_U + \bar{\iota}\widehat{pr}_U - 1)b - \tau\left(\frac{y_K + y_W}{1-\tau}\right) - \left(\frac{\widehat{pr}_U + g}{pr_U k}\right)$

P-sector liability equilibrium condition: $\mathcal{M} = \mathcal{M}_s = \mathcal{M}_d; b = b_s = b_d$

Definition of a unit of public services: $p = \omega_p L_p / [\omega_u \ell_p k(1 + m_{NF})]$

P-sector production function: $p = {}^{L_P}/_{\ell_P}$

5. FINANCIAL-SECTOR BLOCK

On the other hand, with Minsky, we assume that bankers view their level of risk in a way that varies over time, depending upon institutional constraints, entrepreneurial expectations, animal spirits, etc. Hence, there are periods in which lending expands beyond all bounds, and others in which it is nearly impossible for workers to obtain unsecured loans, or even mortgages. Moreover, this sense of risk is heightened across all financial markets.

Our approach to modeling financial crises focuses on movements over time driven by changing financial sentiments. These necessarily entail motion along the time axis, and not just the existence at a point in time of a rising loan supply or interest-rate-markup curve, as in the horizontalist account of monetary endogeneity (e.g., Lavoie 2014; Levy and Mántey 2006). Robinson criticized the *ceteris paribus* assumption in Marshallian supply-demand analysis and in the IS-LM model, and Palley's account of a Minskyan supercycle (2011) has emphasized the dynamic and historical aspects of Minsky's theory. This version of events has not been modeled in detail or included in typical Keynesian growth models.

In the model below, we use endogenously created bank money and reserves, with the loan rate varying to reflect the degree of indebtedness, rather than the supply and demand for loanable funds. This contrasts with some of the work in the field by some of the most sophisticated macro models with financial sectors available within, for example, the opus of what might be dubbed the "nonlinear bounded fluctuations school," which, as we saw earlier, usually assumes exogenous state money as well in models that contain it. For example, in the text by Charpe et al. (2011, p. 72), bank loans are determined as a residual after banks choose how to adjust their reserve levels, given a limited supply of loanable funds.[23] In parallel to this school's handling of state money, this approach to bank money contrasts with the SFC standard set by Godley and Lavoie (2012, p. 222) and a handful of others.

In this model, asset holdings are determined in the following way. The F-sector, which comprises financial companies, issues shares at a rate equal to net investment divided by capital. The value of these shares is determined on the stock market. The rest of the wealth of wealthy households is held in the form of money and deposits, with the proportion in deposits determined by their portfolio demand functions. In this sense, deposits and household currency holdings are endogenous. Specifically, a variable proportion α_{dep} is held in deposits. Almost always, this proportion drifts upward, reflecting increasing confidence. However, it is subject to rare Poisson shocks that cause it to jump downward instantaneously.

The story of a financial crisis in the model proceeds according to the following story, which is a slightly modified version of the one implied by the model of financial crashes in Hannsgen (2012). Following a crash, financial prudence gradually falls over time, and also depends (admittedly in a somewhat crude fashion) on the size of the regulatory and supervisory agencies for the sector, which, in the model, is a constant fraction of all government activity. (This fraction could be used as a shift parameter. A one-time downward shock to its value might be used in a simulation of "financialization," a topic that has led to much discussion in recent years, [e.g., Hein 2012].) In other words, there is a tendency, especially when regulation is lax,

[23] In contrast, Semmler and Semmler's (2013) 3-equation model of a currency union economy (2013) uses an exogenous, fixed interest rate for a Markov state corresponding to periods of financial tranquility. The banking system is assumed not to be able to stabilize the rate during crises.

for banks to ease lending standards (Palley 2011). Specifically, when prudence has reached relatively low levels, banks loosen credit limits for the household sector, allowing households to carry higher ratios of debt to disposable income. Also, required margin ratios for equity investors fall, encouraging stock purchases, which have a tendency to push P-E (price-earnings) ratios upward. These trends toward higher leverage increase the financial fragility variable *FF* for given values of the other variables. The ratio of safe assets (government bills and money) to capital in the capitalist household sector also influences *FF*, so that a history of high deficits can help to stabilize the financial sector—a Minskyan fiscal policy effect (Minsky [1986] 2008, 33–37) that some may find paradoxical. High *FF* levels lead to a high rate of occurrence of financial crashes via a Poisson-type model of rare events. Weak capacity utilization, which threatens the ability of borrowers to meet payment commitments, also increases the probability of a crisis. This model makes the exact date *t* of a crash a random occurrence, whose probability in a particular time interval and simulation depends on the pathways of *FF* and *u*.[24]

In the model, the size of a rare shock is determined by a draw from a power-law distribution. Famously, some found the financial crisis to be a black swan—a move in key variables that is so large as to be very unusual and poorly recalled from past disruptions (Taleb 2007); here we draw that insight into Minsky's theory, in which the proneness to crisis at any given time is explained by a model of financial fragility. As mentioned before, the infrequency or rarity of crises is one reason that agents in the model cannot use the time series generated by a simulation of the model economy to forecast income or demand variables. Evidence of fat tails in financial time series data (even after accounting for changing volatility) can be found in Hartz and Paolella (2011) and Kelly (2014); one example of an up-to-date statistical model would be Samorodnitsky (2010). Delli Gatti et al. (2003) make use of a heavy-tailed Lévy-stable distribution in a macro model with large numbers of heterogeneous interacting agents. Kelly (2014) is one study adducing evidence that the tails of distributions of asset price changes follow a power law and that the tail exponent, which governs the shape of the distribution, remains fairly stable over time. He finds that the tail exponent is nonetheless correlated with

[24] Tasche (2013) is an example of a probability model of a financial "event" though it is used in this study simply as a way of gauging the proper rating of debt securities. It is similar in that it is a probabilistic model of a discrete event, used in a financial context. Also, a strand of the univariate time series econometrics literature seeks to model and predict the emergence and collapse of bubbles in asset prices. For example, Phillips and Shen (2014) develop a statistical test for use as a predictor of changes of phase in a financial cycle.

business cycle variables. Some empirical time series evidence is consistent with nonlinear dynamical behavior in price series combined with stochastic shocks (Chen 1996). The statistical properties observed by Chen in asset price series "severely restrict our predictability of future price trends," a finding consistent in his view with a Keynesian perspective on financial markets and the economy as a whole.

How does the shock change the path of the economy when it occurs during a simulation? Once a shock hits the model below, the immediate balance sheet effect falls on the demand for deposits in commercial banks, on the loan assets of those institutions, and the price of shares (see Table below). Specifically, (1) a fraction of all deposits are withdrawn, increasing the share of household wealth made up of government money; (2) This change reflects a corresponding loss of performing loans in an equal amount. That is, it is assumed a certain fraction of margin loans and consumer loans become worthless at the same time that the deposits are lost, resulting in no change to the net worth of the sector; (3) the shocks also adversely and directly affect the animal spirits of participants in the stock market, where valuations P_S of course fall in any case because of the reduction in the use of leverage to finance positions. Some of the effects of the shocks appear in the equations discussed and reported in Section 9 below, which covers the household sector of the model.

In more concrete terms, this shock scenario best reflects a crisis that begins with the revelation of a large amount of bad loans on bank books or a bankruptcy of a large borrower that is foreshadowed by a long buildup of fragility. The shock to assets prompts a contemporaneous withdrawal of funds by uninsured depositors, such as holders of large certificates of deposit or holding company commercial paper. In Minskyan parlance, this is known as a "silent run."

Table 5 Balance Sheet Changes for Sectors with Assets at Time of Financial Crisis

K-sector Households		**K-sector Firms**		**F-sector**	
Assets	**Liabilities**	**Assets**	**Liabilities**	**Assets**	**Liabilities**
$-\Delta$Dep$-$ $\Delta Pr_S N_S$	ΔML		$-\Delta Pr_S N_S$	ΔML	$-\Delta$Dep

There is no particular reason for using this exact scenario in the financial sector, as there are many types of shocks that can hit a financial sector. For example, another possibility would be a self-fulfilling prophecy, in which depositors undermine liquidity by anticipating the effects of their own run, so to speak. A third scenario might involve a loss of performing assets that was not declared for a long period of time, as in the situation with banks in Japan thought for many years to be holding large amounts of unresolved bad debt, hindering their ability to make new loans. In this latter case, the shock might not necessarily involve a withdrawal of deposits. The adverse effects of shrinking the F-sector balance sheet would include a fall in the dividend paid by the sector, and an increase in the proportion of wealth held in the form of state money, which bears no interest.

F-sector dividend identity: $div_F \equiv i_W d_W + i_K ml + i_G b - i_{dep} dep$

Desired F-sector markup: $m_F^d = f_{m_F^d}(m_{NF}, FF, div_F - div^T)$

$$div^T > 0$$

Law of motion for F-sector markup: $\dot{m}_F = -\alpha_{m_F}\left(m_F - m_F^d\right)$

Consumer Loan Markup equation: $i_W = (1 + m_F) \cdot i_P \cdot consumerprem, \quad consumerprem > 1$

Deposit rate markdown equation: $i_{dep} = DepComp(1 + m_F)i_P, \; 0 < DepComp < 1$

Margin-loan interest rate markup equation: $i_{ML} = (1 + m_F) \cdot i_P$

Margin-loan supply: $ml = ml^d$

Demand for bank reserves: $\mathcal{M}_F^d = rr \cdot dep, \; 0 < rr < 1$

Walras's Law for bank asset demands: $b^d \equiv dep - \mathcal{M}_F^d - ml - d_W$

Equilibrium identities for financial assets: $\mathcal{M}_h \equiv \mathcal{M}_h^d; \mathcal{M}_F \equiv \mathcal{M}_F^d; b \equiv b^d$

Bank balance sheet identity: $dep \equiv \mathcal{M}_F + b + ml + d_W$

Financial Fragility function: $FF = f_{FF}\left(\overbrace{(y_W - i_w d_W)/d_W}^{-}, \overbrace{(y_K - i_{ml} ml)/_{ml}}^{-}, \overbrace{l}^{-}\right)$

Financial-crash-frequency function: $\lambda = f_\lambda(u, FF) \;\; (rareshock\; frequency)$

Conditional probability of shocks during a time interval (implied by Poisson Law for nonhomogeneous processes [Ross 1997, 276] and equation above)[1]:

$$P(0\; financial\; shocks\; in\; (t_1, t_1 + \Delta t) | shock\; at\; t = t_1)$$
$$= exp\left\{-\int_{t_1}^{t_1 + \Delta t} \lambda(y) dy\right\} \; for\; any\; t_1, \Delta t > 0$$

Financial-prudence function $P = f_P\left(\overbrace{T}^{-}, \overbrace{p_F}^{+}, \overbrace{u}^{-}\right), \; T \equiv \int_{\tau_{-1}}^{t} 1\, dt \quad (financial\; prudence)$, where τ_{-1} is the date of the most recent rare financial shock.

Distribution of size of rare financial shocks: $rareshock \equiv \frac{1}{2}(ArcTan(s) + 1), \; s \sim F(s) = c_1 s^{-\alpha}\; for\; s \geq 0$

6. K-SECTOR BLOCK

On the role of the public sector in the determination of productivity, Commendatore et al. (2009), Day and Yang (2011), and Dutt (2013) are previous examples in related formal work. The first of these three papers contrasts the roles of public spending that increases the current productivity of private capital with spending that does not have this effect, in the setting of a dynamical Kaleckian model. In Day and Yang (2011), one category of government spending adds to the total stock of private capital, though the latter is chronically underutilized. The paper by Dutt (2013) merges the endogenous productivity growth rates that are characteristic of neoclassical endogenous growth models with a Kaldorian framework by introducing public investment as an argument in the private investment function.

The production function features fixed productivity coefficients. We assume that the K-sector obtains only as much energy and labor as it needs at any given time *t*, while by definition this sector holds the stock of capital goods, which depreciates at a constant rate. It also acts as rentier for the stock of energy. For simplicity, we assume that unit costs of extraction, etc., for energy are zero; we get to the environmental account in Section 8 of this paper. This yields a differential equation for the goods price p_U below that is relatively manageable.

In the present model, the *growth rate* of the labor productivity coefficient is increasing in productivity-oriented public production and the growth rate of the capital stock. Unlike the approach in Commendatore et al. (2009), the direct effect of the relevant public expenditure variable on the productivity of private-sector labor here is permanent, rather than purely contemporaneous.

The other argument in the productivity growth function is the growth rate of the capital stock. This represents a Kaldorian effect (1957) which was used in Dutt (1994, pp. 98–99), for example. In essence, (1) firms learn with experience, or (2) the installation of new equipment enables firms to modernize.[25] A faster-growing economy tends to be able to achieve a faster rate of technological progress. Other Kaleckian growth and distribution models that have included endogenous technological progress in the private sector include Dutt (1986), Rezai (2012), Sasaki (2011), and many others. A recent paper by Schoder (2014) examines the effects of an

[25] If the latter interpretation is adopted, then one could use gross investment over the capital stock, instead of net investment over capital, as an argument in the technical progress function.

endogenous *capital* productivity coefficient on the dynamics of a Kaleckian growth and distribution model. We assume that the latter parameter is constant. As we shall see in a subsequent section, another demand-led-growth feature of the model is that the labor-force growth rate is endogenous.

The K-sector desires to make net investment expenditures according to a standard Kalecki-Steindl investment function (e.g., Dutt 1986), but with (1) the addition of a net-revenues-to-capital term, which is meant to capture the importance of cash flow. As pointed out in the introduction, financial variables are often found in investment functions in the FML, e.g., Charles (2008); (2) the use of the deviation of capacity utilization from its normal level, rather than a simple capacity variable. This approach is one way of fixing well-known flaws found in more-standard Kaleckian specifications, which lack an adjustment process that would adjust capacity during periods of prolonged over- or under-utilization (e.g., Skott 2012; Duménil and Lévy 2014). Of course, in a nonlinear model, this formulation by no means guarantees that the utilization will asymptotically approach the desired/normal rate; and (3) a Kaldorian nonlinearity (Kaldor 1940; see also Flaschel et al. 1997, pp. 73–81, and the references therein).

An equation in the model ensures that the sector grants new issues to existing stockholders in proportion to net investment, with each share representing a unit of capital stock. In this way, the share price model below will account for the market price of a stock certificate representing a fixed amount of capital at all times *t*.

Leontief production function:

$$U = min\left[{}^{k}/_{\sigma_K}, {}^{L_u}/_{\ell_u}, {}^{E}/_{\sigma_E}\right], \sigma_K > 0, \sigma_E > 0, \ell_u > 0$$

Goods output definition:

$U \equiv C + NI + \Delta$ $(output\ equals\ consumption + net\ investment + depreciation)$

Definition of capacity utilization: $u \equiv \frac{U\sigma_K}{k}; c \equiv \frac{C\sigma_K}{k}; g \equiv \frac{NI}{k}; \delta \equiv \frac{\Delta\,\sigma_K}{k}$

Dividend equation: $div_{NF} = \theta(1-\tau)\pi$

Labor productivity growth rate: $\hat{\ell}_u \equiv -g_u^{\ell} = -f_u^{\ell}(g, p_{\ell_U})$

K-sector Identities and Definitions

Definition of net profit rate: $\pi \equiv {}^{m_{NF}u}/_{(1+m_{NF})} - \delta$

Dividend for K-sector shareholders: $div_K = \theta(1-\tau)\pi$

Net Revenues (*nr*) of nonfinancial corporations:

$$nr \equiv \frac{net\ profit - dividends - net\ investment}{capital\ stock} \equiv (1-\theta)(1-\tau)\pi - g$$

Desired net investment function: $g^d = \varphi ArcTan\big(\gamma(u - u_{EQ}) + \kappa(\pi - \pi^T) + \varsigma nr\big)$

Net investment adjustment function: $\dot{g} = -\alpha_g\big(g - g^d\big)$

Definition of g: $\hat{k} \equiv g$

K-sector new equity issues: $\dot{N}_S = gN_S$

7. WAGE-AND-PRICE-SETTING BLOCK: MARKUP AND WAGE-SETTING CURVES

The American Post Keynesian school of economists has emphasized that large US corporations fund most investment with internal funds, rather than by issuing new securities or borrowing from banks (Eichner and Kregel 1975). They argue that large corporations, which dominate modern monetary economies, adjust their markups largely as a way of regulating cash flow in response to needs for cash. Some have argued that the markup in the industrial sector is a function of the planned growth rate of the capital stock. In our SFC context, we can make the markup a function of net revenue. This nonlinear function will have thresholds near points in the domain corresponding to zero net operating revenue and possibly levels corresponding to a target rate of profit.[26]

This distinctively Post Keynesian formulation will hopefully complement the Kaleckian term in the markup-dynamics equation, which is drawn from the earlier versions of the model below (Hannsgen 2014, forthcoming 2015). The latter part of the markup model below is additively separable and makes the rate of change of the K-sector markup m_{NF} a nonlinear, nonincreasing function of capacity utilization to determine the rate of change of firms' markup at any given time. Recently, Branston et al. (2014) have provided empirical evidence that the industrial markup is countercyclical in the US and UK, owing to increased collusion during depressed periods such as the recent crisis. Dutt (e.g., 2012) has argued in favor of the countercyclical specification in models with an endogenous markup.

As noted in the introduction, our formulation adds endogenous nominal wages to the model of Hannsgen (2014, forthcoming 2015), which included a variable markup but assumed a constant Keynesian wage unit. There have been attempts to model wages and prices or markups separately in the context of Kaleckian growth models. Among previous efforts to endogenize both nominal wages and prices in a model of growth and distribution are Asada, Chiarella, Flaschel, and Franke (2010), Rose (1967), Rezai (2012), Tavani et al. (2011), Barbosa-Filho (2014), and Charpe et al. (2011, pp. 111–118). As in the model below, Chiarella and Flaschel (2000) combine wage inflation determined by aggregate demand with price inflation determined by cost-push forces. In the context of the Kalecki-Steindl growth model, wages and prices are most often endogenized along the lines of the conflicting claims model, in which workers and capitalists have differing real wage targets (e.g., Hein 2008). In a recent version of his Minsky model, Keen (2013) uses a convex price inflation function that is relatively flat for low levels of the employment rate.

Recent work of this type is reminiscent of the early nonlinear price Phillips curve fit by Eisner (2003), which featured a downward-sloping portion at moderate unemployment rates

[26] Charles's Minskyan model (2008) links cash flow needs to the retention rate, rather than the markup.

(see also Palley 2006) as well as Rose's (1967) nonlinear wage-growth curve. Eisner's simultaneous estimate includes a nonlinear relationship between the *employment* rate and nominal wage growth which is upward sloping over most of its range (2003, p. 525). Rose (1967) mooted a functional form that suggests relatively stable nominal wages for a range of employment rates near the center of the curve. Here we use a similar shape, which features a long, flat range around middling levels of unemployment. This functional form is consistent with micro data showing large numbers of firms leaving wages constant in a given period. Most important for our multisector Keynesian model, there is a wide range over which macro policymakers do not have to worry about unemployment reaching levels that are "too low" for price stability. The purportedly elusive spot on the Phillips curve with low inflation and reasonable unemployment rates seems at times to have been found for a decade, including the US in much of the 1990s.

While the Phillips option for the differential equation for the wage does not incorporate price expectations, one option for the wage-price block in the model uses an integral formulation of the wage Phillips curve, which adds a similar lagging effect.

Relative wages are set according to a "wage contour" that fixes the ratio of the wages in the public and private sectors. The concept of a wage contour was developed by Dunlop (1957) and introduced to dynamic macro models in the Post Keynesian tradition by Eichner (1987, Chapter 8), and others in the 1970s.[27]

Alternatively, as also set out below, one could construct a variant of the model in which the government sets the private sector wage and its rate of growth and dispose of the Phillips curve. The wage contour would then ensure that the private sector wage rises as fast as the wage paid by the government and no faster. The role of the government in using its wage as a "nominal anchor" is a traditional part of MMT (chartalism) (Wray 2003, p. 91). The pricing equation in the K-sector leads to an inflation equation that is similar in nature to those found in structuralist macro (e.g., Barbosa-Filho 2014, p. 351) and is obtained by differentiating by time.

[27] This broad approach to the Post Keynesian theory of wage and price determination in an industrial economy with mass unemployment and oligopolistic goods markets—and its roots in heterodox ("old") Institutionalist labor economics and structuralist macro—is outlined in Piore (1978) and Appelbaum (1979). Charpe et al. (2015) use a model with segmented labor markets and Goodwinian dynamics to simulate the effects of an employer of last-resort program.

Wage contour: $\omega_P = \bar{c}\omega_U, \bar{c} > 0$

Wage-unit Inflation Equations

Option 1 P-sector led: $\widehat{\omega}_P = \bar{g}_\omega \geq 0$

Option 2 "wage Phillips curve": $\widehat{\omega}_U = -\alpha_\omega \left[n^{-1} \int_{t-n}^{t} un \cdot e^{s-t}\, ds\right]^5$

Note: This option requires boundary values *un*(*t*) over the interval *t* ∈ [-n, 0].

Pricing Equation: $pr_U = (1 + m_{NF})\ell_u \omega_u$

Differentiated Pricing Equation: $\dot{pr}_U = \dot{m}_{NF}\omega_U \ell_u + (1 + m_{NF})(\dot{\omega}_U \ell_U + \dot{\ell}_U \omega_U)$

Note: In the equation above, we have assumed that the purely notional price pr_E for transfers of energy within the K-sector is constant.

Nonfinancial Markup-dynamics Equation:

$$\dot{m}_{NF} = -\alpha_{NF}\left[m_{NF} - m_{NF}^d\right]$$

$$m_{NF}^d = f_{mNF}\left(\overbrace{u - u_{EQ}}^{-}, \overbrace{nr_K}^{-}\right).$$

Threshold value in desired nonfinancial markup function: $nr^{*1} = 0\ (zero\ net\ revenue)$.

Note: As in Hannsgen (2014), we intend to use a specification that is a nonincreasing quintic in $u - u_{EQ}$. The dependence on nr_K would then be additively separable. As noted in the text, another possible threshold for a nonlinearity in the function f_{mNF} might be a value associated with a target rate of profit, say the 15 percent (25 percent for privately-held firms) norm reported by Morin (2013, 44–46).

8. ATMOSPHERIC BLOCK

According to a recent study, an emissions cap aimed at limiting the probability of a 2-degree temperature rise to 20 percent would limit total carbon dioxide emissions to less than 900 metric gigatons (gt) for 2013–2049 (Wolf 2014). This budget would allow an additional 75 gt between 2050 and 2099 to keep the probability of exceeding the two-degree increase threshold at 20 percent. Burning all known fossil fuel reserves would easily lead to a rise of 3 degrees or more.

Atmospheric CO_2 law of motion ("Kaya identity")

$$\dot{G} = \theta E - \mu(mitig)U - \zeta G$$

mitig > 0

G atmospheric carbon concentration; *mitig* = mitigation effort, $\mu(\cdot)$=mitigation effectiveness function, ζ atmospheric decay rate. See Taylor (2014) for a growth and distribution model that also incorporates this relationship, which derives from physics.

9. HOUSEHOLD BLOCK

In the household equations of the model, we make use of an endogenous labor force. Endogeneity could reflect migration flows or labor force participation decisions that respond to economic conditions (Thirlwall 2002, pp. 79–96; Setterfield 2013). It is well-known that the US workforce participation rate fell during the years of high unemployment that began with the Great Recession.

We imagine, along with Kalecki, a simplified world in which a working class holds no assets and earns rather low real wages. Members of this class cannot borrow easily to, say, start a new firm, in normal times. The loans they can obtain have rather high interest rates, helping to make it rather difficult to enter the most progressive and high-markup industries as entrepreneurs. Kalecki argued that it was a reasonable approximation to assume that they consumed 100 percent of their disposable incomes. Hence, it seems likely that they would always be net borrowers, given a chance. We posit that the ratio of their debt to their after-tax disposable income at any given time adjusts toward a positive amount determined by the interest rate on consumer loans and by the level of financial prudence *P*, which we have discussed above.

Palley (2010, pp. 299–300) uses a limit on the ratio of debt service payments to household income in a Kaleckian framework with household bond finance, but drops his credit limit and imposes a simpler equilibrium borrowing condition in his bank-lending model with endogenous money (pp. 300–301). Dutt (2006, 2011) and Palacio-Vera (2012) use borrowing rules that causes consumer debt to adjust toward levels dictated by norms for the ratio of consumer debt to disposable income net of interest payments and to the capital stock, respectively. Dutt (2009), Charpe et al. (2009), and Godley and Lavoie's prototype growth model (2012, pp. 378–444) use a norm for the ratio of the flow of *new* debt to income. Hein (2012) also uses credit constraints, but the values of credit limits in his model are driven by norms for the *banking sector's own* balance sheet, rather than that of the households themselves. The borrowing function below uses a constraint that reflects the W-sector's ability to repay a loan, but varying with the endogenous financial prudence variable *P*—animal spirits for bankers, as it were.[28]

Otherwise, other things equal, changes in interest rates affect consumption spending solely through their impact on disposable income, as in Godley and Lavoie (2012, pp. 99–130) and Hannsgen (2014, forthcoming 2015).

We add consumer borrowing with a credit limit to the usual Kaleckian assumption that workers spend what they get. The latter has been introduced into Kaleckian and Keynesian growth models by Dutt (2006, 2011), Palley (2010), Zezza (2008), Isaac and Kim (2013), and others. The differential equation for *c* uses the normalized W-sector stock-flow identity

$$c_W \equiv y_W + pr_U \dot{d}_W + d_W(\widehat{pr}_U + g)$$

which takes into account the effects of price changes on the real stock of debt. The derivation of this identity is similar to that of the stock-flow identity for the P-sector described in Section 4, i.e., one differentiates the definition of normalized liabilities by time and then substitutes, using the nominal version of the stock-flow identity.

The capitalist consumption function introduces an (unrealized) capital gains effect into the determination of consumption, as in Bhaduri (2011a and b) and Bhaduri et al. (2015). Godley ([2007] 2012) and Martin (2008) analyze such an effect in the context of a fiscal policy rule. The dependence of capitalist consumption on financial asset appreciation is one avenue by which nominal quantities enter into the determination of aggregate demand in the model. The consumption function for K-sector households also contains a term that pulls the ratio of assets to capital toward a long-term norm.

[28] Wolfson (1996b) provides a model of Post Keynesian credit rationing. In his simple diagrammatic exposition, credit is curtailed when the state of expectation regarding future ability to pay falls.

In a model with commercial banks, asset demand is determined in part by the availability of margin loans. Demand for this sort of credit by K-sector households depends upon prudence P and the real interest rate on such loans i_K. In turn, margin loans over capital *ml* and the growth rate determine the price-earnings ratio, PE_RATIO. The share price is determined by after-tax earnings and this ratio and is subject to the rare financial shock.

The SFC structure of the model implies an identity linking the balances of the worker households, the firms, and the government.

W-sector desired borrowing: $d_W^d = f_{BW}(P, i_W)(y_W - i_W d_W)$

W-sector consumer debt: $\dot{D}_W = \alpha_D(D_W^d - D_W)$

W-sector consumption-expenditure function: $C_W = Y_W + \dot{D}_W$

Differentiated, normalized consumption-expenditure function:

$$c_W = y_W + pr_U \dot{d}_W + d_W(\widehat{pr}_U + g)$$

K-sector household asset identity (normalized by capital stock): $w = dep + pr_S + \mathcal{M}_h$

K-sector consumption-expenditure function: $c_K = \chi_1(y_K + \dot{pr}_S + pr_S g) + \chi_2\left(\frac{w}{w^T} - 1\right)$, $w^T > 0$

Consumption expenditure adjustment function: $\dot{c} = -\alpha_c(c - c_K - c_W)$

K-sector disposable income definition: $y_K \equiv (1 - \tau)(i_{dep} dep + div_F + div_{NF})$

W-sector disposable income definition: $y_W \equiv (1 - \tau)\frac{(\ell_U(c+g+\delta)+\ell_P \bar{c} p)}{(1+m_{NF})\ell_u}$

K-sector household deposit demand: $dep_h^d = \alpha_{dep}\left(dep_h^d + \mathcal{M}_h^d\right)$

$0 < \alpha_{dep} < 1$

Law of motion 1 for deposit demand parameter: $\dot{\alpha}_{dep} = -\alpha_{dep,dep}(\alpha_{dep} - \alpha_{dep}^*)$

$0 < \alpha_{dep}^* < 1$

Law of motion 2 for deposit demand parameter: $\Delta\alpha_{dep} = [1 - \alpha_{dep} - rareshock]^-$

where $[x]^-$ denotes the nonpositive part of x. That is, when the argument inside the brackets takes on a positive value, the whole expression equals zero. Otherwise, the whole expression equals x.

Law of motion for labor supply: $\dot{N} = Nf_n(un)$

$$f_n' < 0$$

Valuation of shares: $pr_S^* = (1-\tau)\pi \cdot PE\_RATIO$

P−E (price-earnings) ratio: $PE\_RATIO = f_{PE}(ml, g)$

$$\partial f_{PE}/\partial ml > 0; \partial f_{PE}/\partial g > 0$$

Law of Motion 1 for stock prices: $\dot{pr_S} = -\alpha_{pr_S}(pr_S - pr_S^*)$

Law of Motion 2 for stock prices: $\Delta\, ln(pr_S) = s$

where *ln* is the natural logarithm.

K-sector household margin loan demand function: $ml^d = wf_{ML}(i_{ml}, P)$

$\partial f_{ML}/\partial i_K < 0; \partial f_{ML}/\partial P < 0$

Law of motion 1 for K-sector household margin loans: $\dot{ml} = \alpha_{ml}(ml^d - ml)$

Law of motion 2 for K-sector household margin loans: $\Delta \text{ml} = \Delta\alpha_{dep}(dep_h^d + \mathcal{M}_h^d)$

10. NONEQUILIBRIUM RESULT AND THE EXISTENCE OF SOLUTION PATHWAYS

The deterministic part of the model solves for independent differential, integral, or integro-differential equations for the following 20 dynamically endogenous variables: P, α_{dep}, p, c, g, m_F, m_{NF}, ω_P, ω_U, G, N, N_S, ℓ_U, d_W, l, ml, p_S, k, pr_U, and L_P. This set of equations constitutes a mathematically complete system in that, along with a set of initial conditions, it is self-contained.

The integro-differential equation referred to in the paragraph above is the wage-growth (wage-Phillips) equation, which governs the motion of ω_U when we adopt the traditional approach (Option 2) to wage-unit inflation. Given the inherited values of government liabilities l, consumer debt d_W, and margin loans ml, and the values $N_S=g$ and pr_S, the household demand parameter α_{dep}, the F-, P-sector, and K-sector household balance sheet identities, and the reserve ratio rr together determine $\mathcal{M}$ and b, the two government liabilities variables at any time t. (See the appendices for a key to the variable names and dynamic equations.) The model also contains a time-dependent definition of the variable P. Of course, finding solution pathways requires initial conditions for each of the 20 dynamically endogenous variables (the 19 variables with (integro-)differential equations plus *P*).

In addition, the model contains three difference equations which model the magnitude of jumps in α_{dep} , *ml*, and pr_S that occur at all times *t* in which a shock is felt.

After inverting the P- and K-sector production functions to solve for the inputs and setting asset demands equal to asset supplies, the model also contains:

1. behavioral equations for
   i_G, i_K , i_W, PE_RATIO, λ, *FF*, pr_U, pr_S^*, g^d, c_W^d, c_K^d , $\mathcal{M}$, div_{NF}, *E*, L_U, d_W^d;
2. a statistical distribution function for the random variables *s* and rareshock and definitional equations for y_W , y_K, *un*, π, *u*;
3. The model also contains non-differential-equation accounting identities for nr_K, b^d, *w*, *df*, div_F , l; and
4. equilibrium identities for the variables $\mathcal{M}$, *b*

Appendix 1 contains a list of variables. Appendix 2 states the model compactly in terms of a set of variables and equations sufficient to conduct a simulation that yields solution pathways that imply solution pathways for all other variables in the model, given vectors of initial values, parameter values, and realizations of random variables. We have omitted some identities defining variables normalized by the stock of capital goods.

Does the model described above possess a steady state? For example, in the chartalist-wage-growth case, can we obtain a solution for all variables by setting the 19 differential equations equal to zero? The 3D models in Hannsgen (2014) and the one in Hannsgen (forthcoming 2015) were subjected to an analysis of the dynamics around equilibrium. The Minskyan property of the "instability of stability" makes that task impossible here, as the following discussion shows. One way to make this point is to assume that crises interrupt any posited equilibrium with nonzero probability, as the following assumption and proposition demonstrate.

Assumption: Stochastic Instability

For any reasonable candidate equilibrium x', λ(u', *FF*') ≥ λ_{LB} > 0, where *u*' and *FF*' are the elements of *x*' corresponding to the arguments *u* and *FF* in the function λ(·, ·). We define "reasonable" candidates relative to the assumptions used in the theory drawn upon above (e.g., targeted unemployment rates are typical for central banks in the developed world, the K-sector operates with substantial excess capacity, regulatory oversight is such that the F-sector is permitted to innovate, etc.)

Proposition 1: Minskyan Nonequilibrium

Given stochastic instability, the model possesses no equilibria, i.e., values for the dynamically endogenous variables that make all 20 state variables motionless for all time.

Proof: The assumption ensures that even at some x' where all time derivatives equal zero, the rate of occurrence of financial crises λ is nonzero. In this case, the probability of a crisis hitting between time t and time $t + n$ approaches 1 as $n \to \infty$. This event would cause α_{dep} and other variables to jump away eventually from any conjectured equilibrium point. ■

Moreover, presumably, a good calibration for, say, the United States, would imply constant motion and more frequent crises than the foregoing minimalist argument might appear to suggest. Hence, the quantitative results of this model have to come in the form of computed solution pathways for given initial conditions, rather than analysis of the dynamics around equilibria.

There are other reasons for nonequilibrium that are also deeply related to the model's Minskyan and stock-flow consistent properties and to our model of wages and prices. Note that even given that $u = u_{EQ}$, the right-hand derivative of prudence P as a function of time is negative, owing to the steady rise of T, the time elapsed since the last crisis. In any postulated steady state this movement alone would increase the use of leverage, causing motion in the system that violated the equilibrium conditions. Moreover, note that we do not impose conditions assuring that, $\dot{l} = 0$ (a constant government liability-over-capital ratio) even in the long run. Even with other variables at rest, this movement in turn causes motion in financial fragility *FF*, bank deposits *Dep*, etc., and hence changes in other variables. In particular, a stagnationist, SFC system of this type is likely to be characterized by a bias toward fiscal deficits over the long term (Hannsgen 2014). This tendency implies long-run private-sector surpluses, which increase the assets of wealthy households and hence their consumption spending over the "long run" in the model presented here. Moreover, in the chartalist case, the

K-sector wage ω_U rises at a constant rate, so that this variable would have to be transformed to a constant or omitted altogether to obtain a system with a steady state.

Proposition 2: Existence of Solution Pathways to the Full Model

Can we be sure that this model is sufficient to determine unique pathways for some chosen time interval $[0, t_{END}]$, given (1) initial values x_0, (2) an initial interval for the lagged values of the unemployment rate in option 2 for the wage-growth curve, (3) a vector of parameter values, and (4) the Poisson process? We can do so for any arbitrary compact domain within the state space.

Proof: First, consider the deterministic differential equations in the system in Appendix 2 as a separate system, ignoring for now the shocks from the Poisson model.

It may be helpful to

- First, substitute for all time derivatives $\dot{x}$ and growth rates $\hat{x}$ that appear on the right-hand side of the system, in particular $\widehat{pr}_U, \hat{\omega}_U,$ and $\hat{\omega}_P$.
- Second, evaluate the integral $T \equiv \int_{\tau_{-1}}^{t} 1\, dt = t - \tau_{-1}$ for elapsed time that appears in the equation for *P*, and substitute for this variable where it appears on the right-hand side of the system[29]
- Third, if option 2 is preferred for the wage-growth curve, set the integral in the equation for $\dot{\omega}_U$ equal to a new state variable *z*, adding an equation for its time derivative $\dot{z}$ to the system. This transformation makes it clear that the lagged values of unemployment that appear in the wage-Phillips curve do not pose any special problems for the existence of solutions.

Of course, locally, a system of nonautonomous ordinary differential equations has a unique solution pathway for any set of initial conditions under standard conditions by a fundamental existence and uniqueness theorem (e.g., Hirsch and Smale 1974, p. 297). All that is required is Lipchitz continuity, which is guaranteed by our assumption that all of the unspecified functions in the behavioral equations have continuous partial derivatives with respect to each of the state variables. The right-hand sides of the differential equations in the system are then mere

[29] See, for example, Wiggins (2003, p. 94) for methods that can be applied to obtain the actual numerical solutions of specific nonautonomous deterministic systems of this type. In particular, one can treat time as an additional state variable.

compositions of such functions, and hence the vector field formed by the system is also C^1, as required by the fundamental theorem. Though we lose continuity of the partial derivatives at boundaries if we are using differential equations that stop motion of some state variable in a particular direction whenever a relevant boundary is reached (see Section 3), the Lipchitz condition nonetheless still holds.

Moreover, our requirement in the statement of the proposition of some compact domain within the state space in which to establish global existence ensures that local solutions can be continued throughout any time interval $[0, t_1]$ as long as the solution remains within that domain (Wiggins 2003, p. 91). Here we are helped by our assumption that all nonlinear functions f_i on the right-hand side of the system are bounded on any given compact domain (see Section 3). So that these conditions hold, we can choose to consider pathways within some compact subset of the state space with reasonable bounds on the markup m and other variables that are not inherently bounded, in contrast to, e.g., capacity utilization $u \leq 1$.

Second, this solution to the boundary value problem is combined with the Poisson model $\lambda = f_\lambda(u, FF)$ and the formula for the fragility variable FF to prove that we have a unique solution piecewise for some finite interval $[0, t_{END}]$ piecewise in the following way. The solution $x(t; x_0)$ implies a trajectory for the Poisson intensity variable $\lambda(t; x_0) \equiv \lambda[u(t; x_0), FF(t; x_0)]$, for all $t \in [0, t_{END}]$. The first jump date can be randomly generated from a standard nonhomogeneous Poisson process (Ross 1997, pp. 235−303) with $\lambda(t; x_0)$ as the intensity variable. The portion of the solution pathway starting at the first crisis date $t = \tau_1$ is discarded. The independence criterion in the definition of a Poisson process holds within the truncated realization defined on the interval $[0, \tau_1)$. A random draw from the distribution of s then implies the size of the other random variable *rareshock* and, in turn, the discrete jumps Δs, $\Delta\alpha_{dep}$ and Δpr_S which are given by the equations for these finite differences (see Appendix 2). These jumps, occurring at $t = \tau_1$, then imply a postcrisis boundary condition $x(\tau_1; x_1) = x_1$. One then repeats the entire process above as many times as necessary to obtain the entire solution for a desired time interval $[0, t_{END}]$ piecewise, starting by obtaining the solution $x(t; x_1)$ and then using the implied pathway $\lambda(t; x_1)$ to draw a second random stopping point τ_2. Repetitions of this procedure enable one to construct a solution for the desired closed time interval with left-discontinuities at t_i, where i indexes the finite set of jump dates.■

11. FUTURE WORK

The ultimate purpose of the project discussed in this paper is to propose an approach to modeling the effects of fiscal policy in a dynamical setting using the model described above. Simulation work will be important as a way of exploring the comparative dynamics of the model. We must apologize for not being specific in this paper as to the detailed functional forms that one might use for some of the potentially nonlinear functions f_i. A preliminary, deterministic approach would simply track the intensity parameter λ as another state variable, much as the regime-switching literature often reports simulated pathways for the probability of a regime change. Inventories of goods are not modeled here and it might be useful to explicitly model the stock of consumer and investment goods carried over from previous periods as in the Keynes-Metzler-Goodwin model in Chiarella et al. (2005). Some tinkering with the new parts in this model may turn out to be necessary to gain satisfactory (empirically plausible) dynamical results. For example, the particular financial indicator used in the investment function might be varied if the results obtained did not seem to mimic the broad properties of actual macro time series. It would be easy to implement a variety of taxes, including wealth taxes, taxes that differ across types of income, inflation taxes, etc., by including appropriate parameters in the model and changing the accounting identities as needed. Policy exercises can be conducted using CDFs.

We have modeled crises using a scheme with Poisson shocks only. These shocks almost always take on the value zero. An alternative tack would be to model the stochastic process for α_{dep} or for the price-earnings ratio using some form of the Lévy-stable model, whose increments have heavy-tailed distributions.[30] Such a process would incorporate both small and large changes in financial confidence and have been implemented in models of heterogeneous, interacting agents (e.g., Delli Gatti et al. 2003). In any case, we can estimate either process as embedded in their broader deterministic context using Monte Carlo methods and the Euler-Maruyama method (Korn et al. 2010) or the asymptotic expansion method (Brouste et al. 2014).

[30] See, for example, Embrechts and Maejima (2002, pp. 9–11). Early uses of stochastic processes in growth models (e.g., Semmler and Semmler [2013]) have employed the Gaussian case [$\alpha = 2$; $\beta = 0$ in the zero parameterization] almost exclusively.

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APPENDIX 1: PARTIAL LIST OF VARIABLES

Flows and Ratios

c consumption goods produced

c_K^d desired consumption expenditures by K-sector households

c_W^d desired consumption expenditures by W-sector households

div_{NF} nonfinancial sector dividends

div_F financial sector dividends

E energy use

p public production

p_{ℓ_U} productivity-increasing government production

p_F government production of financial supervision and regulation

NI net "real" investment

g net investment divided by capital stock, i.e., NI/k

g^d desired net investment divided by the capital stock

m_{NF} markup on private sector good

m_{NF}^d desired markup on private sector good

m_F interest rate markup

m_F^d desired interest rate markup

mitig Greenhouse gas mitigation expenditures (assumed constant for simplicity)

nr net revenue of goods producers

u capacity utilization

L_U labor hours employed in K-sector

L_P labor hours employed in P-sector

un unemployment rate

ℓ_u labor requirements per unit of output produced

g_{ℓ_U} growth rate of private-sector labor productivity

λ instantaneous rate of occurrence of financial crashes (expressed in number per year)

y_K after-tax wealthy household income

y_W after-tax worker income

τ_{-1} date of most recent crisis at some time *t* along a solution path

Interest rates (adjusted for goods inflation), i.e., $(1+i_x) \equiv \frac{(1+\text{nominal rate x})}{\widehat{pr}_U}$

i_G interest rate on treasury bills

i_K interest rate on margin loans

i_W interest rate for consumer loans

i_{dep} interest rate on deposits

Wages and Prices

pr_u price of goods

pr_S price of shares

pr_S^* "fundamental" price of shares

pr_F price of fossil fuels (notional price only for internal transfers within K-sector)

ω_u nominal wage in private industry

ω_P nominal wage in government

Stocks (Mostly Normalized by the Capital Stock, *k*)

G atmospheric greenhouse gas concentrations (not normalized)

b treasury bills

b^d treasury bill demand

d_W consumer debt

l total government liabilities

ml margin loans to K-sector households

dep bank deposits

dep_h^d household bank deposit demand

$\mathcal{M}_h^d$ household currency demand

$\mathcal{M}_F^d$ F-sector reserves demand

$\mathcal{M}$ outside (government) money (used as bank reserves)

N_S number of shares outstanding

w wealth of asset-owning households

N labor force size in time units

Intangible Variables

P Financial Prudence

FF Financial fragility

α_{dep} Wealthy household deposit demand parameter

Random Variables

s primitive shock

rareshock function of *s*

APPENDIX 2: DYNAMIC EQUATIONS LIST

This list states the model in the paper in the form of 19 ordinary differential equations, 3 difference equations, 1 definition involving an integral, and the conditional Poisson model, which is made up of two random variables and an equation for the intensity parameter λ. Time appears on the right hand side of some equations; in other words, the system is nonautonomous. Given initial conditions for all variables and values for the parameters, the equations of the model determine pathways for all times t in a simulation period $[t_a, t_b]$, which also depend on realizations of the Poisson process, which determines the crisis dates, and on random draws of the sizes of the shocks. Note the definitions of y_K and y_W in Section (A), which are used in the differential equations.

(A) The model contains differential equations for the following variables: α_{dep}, p, c, g, k, m_F, m_{NF}, ω_P, ω_U, G, N, N_S, ℓ_U, d_W, l, L_P, ml, pr_U, pr_S. A list follows. Please note that we present two alternative models of wage inflation. In the case of the second wage inflation model, one of the 19 differential equations is technically an integro-differential equation.

$$\dot{p} = -\left[\alpha_{p,p}(p - p^T) + \alpha_{p,un}\left(\frac{N - L_p - \ell_U(g + c + \delta)k}{N} - un^T\right)\right], \qquad p^T > 0, \qquad 0 \leq un^T \leq 1$$

$$\dot{L}_P = \dot{p}/\ell_P$$

$$\dot{m}_F =$$

$$-\alpha_{m_F}\left(m_F - \right.$$

$$f_{m_F^d}\left(m_{NF}, f_{FF}\left(\overbrace{(y_W - (1 + m_F) \cdot consumerprem \cdot \bar{\imath} d_W) \big/ d_W}^{-}, \overbrace{\left(y_K - \left((1 + m_F) \cdot \bar{\imath}\right) ml\right) \big/ ml}^{-}, \overset{-}{\bar{l}} \right), \bar{\imath}\Big[consumerprem(1 + \right.$$

$$\left. m_F)d_W + (1 + m_F)ml + [(1 - rr)\alpha_{dep}(\ell + d_W + ml) - d_W - ml] - DepComp \cdot \alpha_{dep}(\ell + d_W + ml)\Big] - div^T \right)\Bigg)\text{, where}$$

$$y_W \equiv (1 - \tau)\frac{\ell_U(c + g) + \bar{c}\ell_P p}{(1 + m_{NF})\ell_U}$$

and

$$y_K = (1-\tau)\left(\bar{\imath}\{consumerprem(1+m_F)d_W + (1+m_F)ml + \left[(1-rr)\alpha_{dep}(\ell + d_W + ml) - d_W - ml\right]\}\right.$$

$$\left. + \theta\left({m_{NF}(c+g+\delta)}\big/{(1+m_{NF})} - \delta\right)\right)$$

$$\dot{\ell}_U \equiv -g_u^{\ell}\ell_U = -f_u^{\ell}(g, \alpha_{\ell_U} p)\ell_U$$

$$\dot{c} = -\alpha_c\left(c - \chi_1\left(\frac{(1-\tau)y_K - \alpha_{pr_S}\left(pr_S - \left({m_{NF}(c+g+\delta)}\big/{(1+m_{NF})} - \delta\right)\cdot f_{PE}\left(\overset{+}{\widetilde{ml}}, \overset{+}{\tilde{g}}\right)\right)N_S}{pr_S} + pr_S g\right.\right.$$

$$\left. - \chi_2\left(\frac{(\ell + d_W + ml) + pr_S}{w^T} - 1\right) - (y_W + pr_U\dot{d}_W + d_W(\widehat{pr}_U + g)))]\right)$$

$$\dot{pr}_u = \dot{m}_{NF}\omega_U\ell_U + (1+m_{NF})(\dot{\omega}_U\ell_U + \omega_U\dot{\ell}_U)$$

$$\dot{g} = -\alpha_g\left(g - \varphi ArcTan\left(\gamma\left(c + g + \delta - u_{EQ}\right) + \kappa\left({m_{NF}(c + g + \delta)}\Big/{(1 + m_{NF})} - \delta - \pi^T\right) + \varsigma(1 - \theta)\left({m_{NF}(c + g + \delta)}\Big/{(1 + m_{NF})} - \delta\right)\right)\right)$$

$$\dot{k} \equiv gk$$

$$\dot{N}_S = gN_S$$

$$\dot{d}_W = \alpha_D(f_{BW}(P, (1 + m_F) \cdot \bar{\imath} \cdot consumerprem)(y_W - \bar{\imath}consumerprem(1 + m_F)d_W) - d_W)$$

$$\dot{\alpha}_{dep} = -\alpha_{dep,dep}(\alpha_{dep} - \alpha^*_{dep})$$

$$\dot{G} = \theta(\sigma_E - \mu(mitig))(NI + C + \Delta) - \zeta G, mitig > 0$$

$$\dot{N} = f_n\left(\frac{N - L_P - (c + g + \delta)k\ell_U}{N}\right)$$

$$\dot{l} \equiv p + (\widehat{pr}_U + \bar{\imath}\widehat{pr}_U - 1)\left[(1 - rr)\alpha_{dep}(\ell + d_W + ml) - d_W - ml\right] - \tau\left(\frac{y_K + y_W}{1 - \tau}\right) - \left(\frac{\widehat{pr}_U + g}{pr_U k}\right)$$

$$p\dot{r}_S = -\alpha_{pr_S}\left(pr_S - \left({m_{NF}(c + g + \delta)}\Big/{(1 + m_{NF})} - \delta\right) \cdot f_{PE}\left(\overbrace{ml}^{+}, \overbrace{g}^{+}\right)\right)$$

$$\dot{ml} = -\alpha_{ml}\left((\ell + d_W + ml + pr_S)f_{ML}\big((1+m_F)\cdot\bar{\imath}, P\big) - ml\right)$$

$$\dot{m}_{NF} = -\alpha_{NF}\left[m_{NF} - f_{mNF}\left(\overbrace{g + c + \delta - u_{EQ}}^{-}, (1-\theta)\left(\frac{m_{NF}(c+g+\delta)}{(1+m_{NF})} - \delta\right) - g\right)\right]$$

Either option 1 or option 2 below:

option 1: P-sector led

$\widehat{\omega}_P = \bar{g}_\omega \geq 0$

$\widehat{\omega}_U = \widehat{\omega}_P$

option 2: "wage Phillips curve"

$$\widehat{\omega}_U = -\alpha_\omega\left[n^{-1}\int_{t-n}^{t}\left(\frac{N - L_p - L_u}{N}\right)\cdot e^{s-t}\,ds\right]^5$$

$\widehat{\omega}_P = \widehat{\omega}_U$

(B) Difference equations (The model contains these equations, which define finite differences in 3 of the variables mentioned above as functions of the two jump variables *rareshock* and *s*, which are explained below. These variables take on the value zero at almost all times *t*.)

$\Delta ln(pr_S) = s$

$\Delta \text{ml} = \Delta\alpha_{dep}(\ell + d_W + ml)$

$\Delta\alpha_{dep} = [1 - \alpha_{dep} - rareshock]^-$

where $[x]^{-}$ *is defined as the nonpositive part of* x; *that is, if* $x > 0, [x]^{-} \equiv 0$ *and if* $x \leq 0, [x]^{-} \equiv x$

(C) Time-dependent behavioral equation

This equation governs the evolution of P over time

$$P = f_P\left(\overbrace{\tilde{T}}^{-}, \overbrace{\alpha_F p}^{+}, \overbrace{c + g + \delta}^{-}\right), \qquad T \equiv \int_{\tau_{-1}}^{t} 1\, dt$$

(D) Stochastic model

$$\lambda = f_\lambda\left(\overbrace{c + g + \delta}^{-}, f_{FF}\left(\overbrace{(y_W - \bar{\imath}consumerprem(1 + m_{NF})d_W)/d_W}^{-}, \overbrace{(y_K - \bar{\imath}(1 + m_{NF})ml)/_{ml}}^{-}, \overbrace{\tilde{\ell}}^{-}\right)\right)$$

where λ is the intensity variable in the Poisson model. Then, we have that

For any t_1 >0 and Δt>0

$$P(0\ financial\ shocks\ in\ (t_1, t_1 + \Delta t) | shock\ at\ t = t_1) = exp\left\{-\int_{t_1}^{t_1 + \Delta t} \lambda(y) dy\right\}$$

where the function λ(t) is introduced to emphasize that λ is a function of time, and *y* is a variable of integration. The following equations determine the sizes of the random variables, *s* and *rareshock:*

At any time t, given that a Poisson jump occurs, that is, that s ≠ 0,

$$rareshock \equiv \frac{1}{2}(ArcTan(s) + 1)$$

$s \sim F(s) = c_1 s^{-\alpha}\ for\ s \geq 0$, where α >0 and $c_1 > 0$