An Empirical Stock-Flow Consistent Macroeconomic Model for Denmark

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ABSTRACT

This paper emphasizes the need for understanding the interdependencies between the real and financial sides of the economy in macroeconomic models. While the real side of the economy is generally well explained in macroeconomic models, the financial side and its interaction with the real economy remains poorly understood. This paper makes an attempt to model the interdependencies between the real and financial sides of the economy in Denmark while adopting a stock-flow consistent approach. The model is estimated using Danish data for the period 1995–2016. The model is simulated to create a baseline scenario for the period 2017–30, against which the effects of two standard shocks (fiscal shocks and interest rate shocks) are analyzed. Overall, our model is able to replicate the stylized facts, as will be discussed. While the model structure is fairly simple due to different constraints, the use of the stock-flow approach makes it possible to explain several transmission mechanisms through which real economic behavior can affect the balance sheets, and at the same time capture the feedback effects from the balance sheets to the real economy. Finally, we discuss certain limitations of our model.

KEYWORDS: Empirical Stock-Flow Consistent Models; Denmark; Open Economy

JEL CLASSIFICATIONS: E17; E12; F41
I. INTRODUCTION

The global financial crisis (GFC) revealed the fact that economic growth in many countries was driven to a certain extent by a sharp expansion in balance sheets, occurring due to new credit creation along with asset price booms. This resulted in an extremely heavy reliance on debt-led growth. The expansions in balance sheets prior to the GFC did not receive considerable attention, or at least were not considered harmful by policymakers or macroeconomic modelers. Most macroeconomic models prior to the GFC tended to focus on the real side of the economy while overlooking the important role played by balance sheet structures.\(^1\) The GFC, however, revived interest in examining the link between finance and the real economy, as a key lesson from the crises was that finance matters and balance sheets do play an important role in the economy (Borio 2014).

Appropriate understanding of the link between the financial and real sectors is essential for adopting correct macroprudential measures. These measures can minimize risks in the economy and ensure stability of the financial system. Given the history of recurrent financial crises, there are reasons to believe that none of the measures will guarantee a full prevention of a crisis in open economies. That is, in practice, there might be situations where the effects of a crisis are inevitable and adverse global shocks will eventually propagate in the economy through different channels. However, a good understanding of the interaction between the real and financial sectors can enable policymakers to react to early signs and take preventive measures to reduce the adverse effects of shocks.

The ultimate goal of this paper is to propose a framework that links the financial and real sectors of the economy, which can be useful for macroeconomic discussions of policy relevance. In this regard, we attempt to address a broader question: What are the structural linkages through which the financial sector interacts with the real sector in a small, open economy with a fixed exchange rate? The transmission channel explaining the positive relationship between financing and economic growth is obvious, but what exactly are the driving forces behind this interaction that eventually makes it unsustainable? What measures

\(^1\) For example, some of the famous mainstream macro models, such as Christiano, Eichenbaum, and Evans (2005) and Smets and Wouters (2007), assumed frictionless financial markets, in which balance sheets do not affect real economic behavior.
can be taken in the future to achieve stable growth? To answer these questions, this paper attempts to develop a benchmark macroeconomic model for the Danish economy following a stock-flow consistent (SFC) approach. The focal point of this study is the macroeconomic system as a whole from a sectoral perspective rather than the direct actions of individual agents. Due to the fact that Denmark is a small, open economy with a fixed exchange rate regime, the rest of the world is treated as exogenous. We model the structural linkages between the real and financial sectors of the economy. The model is first simulated to obtain a baseline scenario and is then analyzed for a standard fiscal shock and interest rate shocks. While the model proposed in this paper has a more elaborative household sector, the framework can easily be extended in various directions as data becomes available.

Our model, largely linked to the Post-Keynesians, is inspired by the recent work in SFC modeling. The SFC framework is better at detecting instabilities in balance sheet structures and their subsequent adverse effects on the economy than mainstream models. In this framework, the real and financial sectors are linked through standard accounting principles, and the dynamics of the data are explained through behavioral equations. This allows us to understand the whole economy as one system. Like any large-scale macro model, this has the advantage of setting up several scenarios within one framework. Our model is greatly influenced by studies in the Post-Keynesian SFC tradition, including, amongst others, Godley and Zezza (1992), Godley (1999), Godley et al. (2007), Papadimitriou, Nikiforos, and Zezza (2013), and Burgess et al. (2016). Despite the recent popularity of SFC models, the number of empirical SFC models is very limited in the existing literature. Thus, our paper also contributes to the scarce literature on empirical SFC models.

The rest of the paper is organized as follows. Section 2 provides a brief review of the current macro models used at various policy institutions in Denmark. Section 3 explains the process of data construction to be used in our model. Section 4 explains the structure of the model. Section 5 explains the results of the model. Section 6 concludes this paper.
II. TRADITION OF MACRO MODELING IN DENMARK

In terms of macroeconomic modeling, the recent objective of policymakers in Denmark is to develop a new hybrid macroeconomic model for the Danish economy (MAKRO). The motivation is to switch from the traditional structural econometric model (SEM) to models based on a foundation of a forward-looking overlapping generations (OLG) setup. The underlying objective, as mentioned in Stephensen et al. (2017), is to have a model that can be used to analyze the short-run effects of economic policy, and also create medium- and long-term fiscal projections. According to the authors, the proposed model is in that sense a hybrid between the short-run model and the long-run OLG model. Within the short run, it is described to be a hybrid between a dynamic stochastic general equilibrium (DSGE) model and a SEM.

While the performance of this model is yet to be seen, the move toward DSGE modeling is perceived as a positive development by those involved in building the MAKRO model. At the first seminar on the model’s development in Copenhagen on December 6, 2017, Olivier Blanchard praised the model for being ambitious, but also cast doubts that a single model can be capable of carrying out as many objectives as described above.²

In order to understand the motivation behind building MAKRO, a review of current macro models used at various policy institutions in Denmark is needed. Currently, there are different types of models used at various institutions in Denmark that can broadly be classified into general equilibrium models (including DSGE and OLG) and SEMs. A key difference between SEMs and DSGE (including OLG) models, among other things, is the choice of expectations. While the expectations in SEMs are usually backward-looking, the expectations in a standard OLG or DSGE model are forward-looking in order to analyze the long-term structural effects of changes in economic policy, e.g., how changes in life expectancy affect the choice of consumption, saving, and labor supply made by the households.

On the other hand, the Annual Danish Aggregated Model (ADAM) used by Statistics Denmark argues that forward-looking expectations (despite their immunity to the Lucas

² One of us attended the conference and made notes of these comments.
critique) should not be implemented in ADAM (Danmarks Statistik 2012). Apart from the complexity associated with integrating forward-looking expectations in ADAM, another reason cited for not including this feature in the model is the lack of empirical support for such a choice. In particular, ADAM points out the period before the crisis, where forward-looking expectations failed to foresee the crises. ADAM follows the traditions of SEMs models (including adaptive expectations), since all the behavioral equations are estimated individually (Danmarks Statistik 2012). ADAM is demand led in the short run due to sticky prices and wages, while in the long run it is a neoclassical equilibrium model determined by the supply side.

Following the tradition of SEMs, ADAM is not stochastic like many DSGE models. DSGE models are typically log-linearized around a steady state path, which has the implication that the model path must therefore be interpreted as being close to a steady state (Stephensen et al. 2017). In the last decade, ADAM has deviated from the traditional SEMs in one central aspect: to some extent, the model has become more micro-founded. This can be observed by the large disaggregation of goods and services in the production sectors. However, the micro-foundation in ADAM is still not as stringent as in DSGE models, where rational agents use intertemporal optimization under different kinds of uncertainty. The behavioral equations in ADAM are typically estimated individually, whereas the estimation strategy differs when it comes to DSGE models. Estimation in these models is often carried out by different approaches to system estimation, such as a structural vector autoregression (SVAR) approach.

Overall, the modeling tradition in Denmark is slowly shifting toward general equilibrium models. At some point this might lead to a complete loss of interest in SEMs, thereby following the same trajectory as many countries did prior to the crisis. While using DSGE models has advantages, it is important to point out that these models have also received harsh criticism for various reasons from some notable academics, for example, Hendry and Mizon (2014), Romer (2016), Hendry and Muellbauer (2018), and Stiglitz (2018), among others. Overall, the critiques have mostly pointed out the lack of attention paid to the financial sector in these models. Some have stepped forward to write in defense of the models while

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3 Of course, this might include some exceptions, but the criticism is usually aimed at some of the benchmark models that became very famous and inspired a whole generation of academics. Moreover, models that did
accepting the most common criticism (e.g., Lindé [2018] and Christiano, Eichenbaum, and Trabandt [2018]). Some academics, such as Blanchard (2018) and Wren-Lewis (2018), seem to support a more pluralistic approach to modeling. The former argues that different macroeconomic models should serve different purposes. Wren-Lewis (2018) in particular argues that if SEMs had coexisted alongside micro-founded DSGE models, this would have improved the understanding of the links between the financial and real sides of the economy before the financial crisis. SEMs were largely replaced by DSGE models; however, those that existed or still exist also lacked some important features. For example, Hendry and Muellbauer (2018) argue that the Bank of England’s 1999 medium-term macro model (MTMM) lacked some important features and, if in use, would have failed to identify the credit boom prior to the crisis. The same argument applies to the quarterly macro model (QMM), still actively used by the Central Bank of Iceland, which also failed to identify financial instabilities in the Icelandic economy prior to the crisis.

The above discrepancies in SEMs can, to a certain extent, be overcome by the use of the empirical SFC approach to modeling. The structure of SFC models is built around the notion of stock-flow interactions. The behavioral equations in a dynamic empirical SFC model are usually estimated using time series data on transaction flows and balance sheets. In that sense, some empirical SFC models are also SEMs, however, the theoretical foundation is largely based on Post-Keynesian theory in which the linkages between balance sheets and transaction flows play a central role. This feature is central to the case of Denmark, where the household debt-to-income ratio has reached a very high level. In this paper, we propose a benchmark model that can coexist alongside other macro models in Denmark. This model can be reestimated for quarterly data and can easily be extended in several different directions to study other issues. While studying the interaction of financial and real sectors remains a core component of SFC models, their application is not only limited to these issues. Most recently, the models have been extended to study climate change and economic growth.5

include a financial sector modeled the banking sector in a way that did not reflect the crucial aspects of a banking sector in practice. These issues have been raised in BIS (2011) and Jakab and Kumhof (2019), and are beyond the scope of this paper.

4 See Caverzasi and Godin (2014), Byrialsen (2018), and Nikiforos and Zetta (2017) for comprehensive surveys on the SFC approach to modeling.

III. DATA

Before explaining the structure of our model, we first explain the key steps involved in developing an empirical SFC model using Danish data. In developing a large-scale empirical model, access to data plays a central role. In this section, we describe the steps in constructing the dataset that we use in our model. The primary data source is the sectoral national account from Eurostat. Some of our exogenous price deflators are taken from Statistics Denmark.

3.1 Balance Sheets of the Economy

Following the sectoral national account, financial assets are divided into several groups: monetary gold and special drawing rights \((F_1)\); currency and deposits \((F_2)\); debt securities \((F_3)\); loans \((F_4)\); equity and investment fund shares \((F_5)\); insurance, pensions, and standardized guarantee schemes \((F_6)\); financial derivatives and employees stock options \((F_7)\); and other accounts \((F_8)\). Due to the use of nonconsolidated data, a particular stock can often appear as an asset as well as a liability for a given sector, e.g., equities appear on both the asset and liability sides of the nonfinancial corporations (NFCs).\(^6\) However, the problem is that the counterparty of a particular asset or liability is not always clear, e.g., the stock of equities held by households can be found in the data, but it is not clear which sector issues these equities. The same is the case for the capital income associated with these assets, i.e., one cannot see what proportion of the outflow from sector \(x\) is an inflow into sector \(y\). This issue is not limited to the domestic economy, but is also a problem when dealing with the foreign sector.

To overcome these challenges, we make a few simplifying assumptions. First, we reduce the number of financial assets by aggregating them into fewer subcategories. As shown in table 1, we consider three financial assets in our model, namely interest-bearing assets \((IB)\), equity \((EQ)\), and pensions \((PEN)\).

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\(^6\) The combination of five sectors and eight financial stocks (which can be held as both an asset and a liability by each sector) leads to potentially forty financial gross positions, which can be quite difficult to explain within a single model.
Table 1: Data Aggregation

<table>
<thead>
<tr>
<th>Assets</th>
<th>Description</th>
</tr>
</thead>
<tbody>
<tr>
<td>Interest bearing (IB)</td>
<td>$F_1, F_2, F_3, F_4, F_7$, or $F_8$</td>
</tr>
<tr>
<td>Net interest bearing (NIB)</td>
<td>$F_1, F_2, F_3, F_4, F_7$, or $F_8$</td>
</tr>
<tr>
<td>Net equities (NEQ)</td>
<td>$F_5$</td>
</tr>
<tr>
<td>Pension (PEN)</td>
<td>$F_6$</td>
</tr>
</tbody>
</table>

Second, with the exception of the household sector, we determine the net value for every financial asset as well as the net capital income associated with that financial asset for each sector. In the case of the household sector, we consider the gross position of all financial assets and liabilities. This choice is mainly explained by our initial interest in the effect of household gross debt. While considering gross positions for the households, we make some assumptions regarding the counterparties. In particular, it is assumed that the stock of interest-bearing assets in the household sector is placed as a liability on the balance sheet of financial sectors, just like the stock of loans for the households is placed as an asset in the financial sector. All the financial assets in our dataset evolve according to the following identity:

$$\text{Financial asset}_t = \text{Financial asset}_{t-1} + \text{Transactions}_t + \text{Capital gains}_t$$

The identity simply implies that changes in the stock of an asset can be traced back to its transactions as well as changes in the price of that asset, i.e., capital gains.

Regarding the accumulation of fixed assets, the identity presented above for financial stocks is augmented by including capital depreciations as follows:

$$\text{Fixed asset}_t = \text{Fixed asset}_{t-1} + \frac{\text{Transactions}_t - \text{Depreciation}_t}{\text{Net investment}} + \text{Capital gains}_t$$

The identity implies that changes in fixed assets are due to net investments and capital gains. Regarding the household sector, the total stock of fixed assets is assumed to be in dwellings. Thus, capital gains in the above identity for the household sector also represent changes in
house prices. Our constructed data for changes in house prices closely resembles the data published by Statistics Denmark, as shown in figure 1.

**Figure 1: Change in House Prices**

![Graph showing change in house prices](image)

*Source:* Statistics Denmark and authors’ calculations.

The balance sheet matrix of the economy is presented in table 2. It can be seen that there are some simplifying assumptions dealing with the distribution of financial assets, primarily due to the lack of information in the data. Since the government sector only holds one net asset, namely net interest-bearing assets, the financial net wealth is equal to net interest-bearing assets in this case. This is a strong assumption, as the government sector also holds a significant part of its wealth as equities. In our dataset, this stock of equities is integrated into the stock of net interest-bearing assets of the government. As a result, we also adjust the balance sheets of the financial corporations (FCs) and NFCs accordingly.\(^7\)

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\(^7\) In order to keep consistency, the adjustment of \(\text{NIB}^G\) must also be carried out with regards to financial transactions of \(\text{NIB}^G\) as well as capital gains on \(\text{NIB}^G\), just like these adjustments affect \(\text{NEQ}^N\), \(\text{NEQTR}^N\), \(\text{NEO}_{CG}^N\), \(\text{NIB}^N\), \(\text{NIBTR}^N\), and \(\text{NIB}_{CG}^N\).
Our aggregated balance sheets, consisting of three financial assets, are presented in gross terms for each sector in figures 2–6. The stock of the different financial assets is represented as a percentage of annual GDP over the period 1995–2016. In general, one can clearly observe expansions in the balance sheet of all the sectors. The balance sheet expansion is more pronounced in the years before the GFC, which coincides with high economic growth in that period, as was the case in many other open economies.

For the household sector, figure 2 shows that both assets and liabilities have expanded significantly since the 1990s. Regarding the composition of assets, the stock of interest-bearing assets as a percentage of GDP seems to be quite stable over the period 1995–2016, while the stock of equities and pensions has increased. The increase in the stock of pensions as a percentage of GDP can be explained by the introduction of the Danish labor market pension system in 1991, as a result of which the economy started building up pension stocks by accumulating a constant share of gross income. Thus, the buildup of the pension stock is relatively new as compared to the traditional financial assets held by households.
On the liability side, interest-bearing liabilities (mostly mortgage loans) have increased in general but more so during the period 2000–9, which has garnered some attention (see, e.g., Smidova [2016] and IMF [2017]). In the postcrisis period, the stock of debt as a share of
GDP has fallen because the debt level has stabilized while GDP has increased. Overall, the net financial wealth of the household sector has mostly been positive. The net financial wealth experienced a fall during the crisis, mainly due to a sharp fall in asset prices. An important point to highlight here is that the asset side of the households’ balance sheet seems to be more sensitive to conditions in the financial market than its liability side. Thus, positive net financial wealth as an indication of financial stability can be misleading, as we can see that the GFC had a strong contractionary effect on the asset side of the balance sheet as compared to the liability side.

Turning to the development in the NFCs, figure 3 shows that the stocks of both financial assets and liabilities have experienced an expansion since the 1990s. In particular, the expansion in assets and liabilities relative to the size of the economy has been massive since 2004. The balance sheet expansion in the years before the crisis was primarily driven by equities, while interest-bearing stocks remained relatively stable. The 2008 crisis had a strong effect on equities as asset prices collapsed, leading to a balance sheet contraction overall. However, in the postcrisis period, the size of the balance sheet relative to the economy has significantly increased, primarily due to an increase in the stock of equities. It is important to highlight that there were significant share buy-backs in the Danish economy in 2012, as reported by Friedrichsen (2019). These share buy-backs contributed to the increase in asset prices, which in turn has induced balance sheet expansions mostly via the capital gains channel. Overall, the accumulation of liabilities exceeds the accumulation of assets most of the time, thus financial net wealth is mostly negative.
Figure 4: Financial Balance Sheets for FCs

![Financial Balance Sheets for FCs]

Source: Eurostat and authors’ calculations.

Figure 5: Financial Balance Sheets for the Government Sector

![Financial Balance Sheets for the Government Sector]

Source: Eurostat and authors’ calculations.

Figure 4 shows the balance sheet structure of the FCs, where a general increase in the size of the balance sheet relative to the economy can be observed. This persistent balance sheet expansion is consistent with the process of financialization in most countries. On the asset side, both interest-bearing stocks as well as equities have increased. Regarding the composition of assets, it can be seen that there was a strong increase in interest-bearing assets.
during 2000–9, coinciding with the increase in household debt. In the postcrisis period, there was a shift in asset composition, where the asset-side expansion of the balance sheet was driven by the stock of equities, while interest-bearing stocks remained relatively stable.

On the liability side, the expansion of the balance sheet roughly follows the same pattern as discussed above. That is, the stock of all liabilities relative to GDP expanded more aggressively until the crisis, and then slowed down in the postcrisis period. Regarding the composition of liabilities, the stock of liabilities—along with interest-bearing assets and equities—also consists of pension stock, which is an asset for the household sector. In the postcrisis period, one can observe a shift in the balance sheet composition, following a similar pattern to what we observed in the case of the composition of assets. That is, the liability-side expansion of the balance sheet is driven by equities while interest-bearing stocks have remained relatively stable.

Figure 5 shows the balance sheet development of the public sector. During the expansion of the public sector in the 1950s and 1960s, the public sector managed to balance their income and expenditures. In the 1970s, however, a high level of unemployment put pressure on both expenditures and tax revenues, leading to deficits and thereby accumulation of debt. The combination of public debt, high interest rates, and low economic activity deteriorated the public sector’s balance in the 1980s and the first half of 1990s. Since automatic stabilizers are high in Denmark, business cycle fluctuations explain a major proportion of the movement in the public balance. Against this background, the fall in unemployment in the middle of the 1990s improved the public sector’s balance, which enabled a fast repayment of the debt, as can be seen by the fall in the stock of interest-bearing liabilities in figure 5. The stock of debt fell until 2007 as a result of a positive balance. In the period 2007–12, the stock of debt increased again due to deficits; these deficits were the result of expansionary policies during the first period of the crisis. Despite a small deficit since the crisis, the stock of interest-bearing liabilities has decreased, which can be explained by the fall in the stock of interest-bearing assets (balance sheet contraction). Regarding the stock of equities, this seems to be relatively constant since 1995, which indicates that this stock is not being used as a financial tool for placing wealth or financing deficits.
The balance sheet for the rest of the world is presented in figure 6. Note that the balance sheet is represented from the perspective of the rest of the world. Thus, assets (liabilities) in this case are liabilities (assets) for Denmark.

Figure 6: Financial Balance Sheets for the Rest of the World

Source: Eurostat and authors’ calculations.

Being a small, open economy, the interaction with the rest of the world plays a central role in the Danish economy. Denmark ran persistent current account deficits during the period between the 1950s and 1989, mainly due to wage increases, inflations, and high private and public borrowing. This resulted in the accumulation of foreign debt. Since 1989, the economy started experiencing current account surpluses due to increased competitiveness as well as the introduction of the pension system, which induced household saving. The effect of current account surpluses can be seen in the development of net financial wealth as Denmark became a net creditor to the rest of the world.

Overall, there was a general expansion in both the accumulation of assets and liabilities from 1995 to 2016. On the asset side, the period from 1995 to 2010 is characterized by a small and steady increase in both the stock of interest-bearing assets and equities. From 2010 onwards, the stock of interest-bearing asset fell, while the stock of equities increased. On the liability side, rest of the world accumulated interest-bearing liabilities and also issued equities to finance the negative net lending vis-à-vis Denmark.
3.2 Real and Financial Transactions in the Economy

We now turn to explaining our data regarding flows on the real side of the economy. Our constructed transaction flow matrix is presented in table 3. In our model, all production takes place in the NFC, which means that all wages are paid by the NFC to the domestic and foreign labor force. The gross operating surplus is shared among the domestic sectors. Most economic transactions on the real side—such as consumption \((C)\), government expenditure \((G)\), investment \((I)\), net exports \((X - M)\), wages \((W)\), and gross operating surplus—are reported in a standard way.

In order to simplify our dataset, some transactions are aggregated up to a certain extent. Regarding the flow of taxes, three flows (namely taxes on wealth and income, taxes on production, and other taxes on production\(^8\)) have been merged into an aggregated tax variable. The transactions related to subsidies, other subsidies,\(^9\) other current transfers, social contributions, and social benefits have been merged into one transaction called transfers. It should be highlighted that the aim of the model is not to explain each and every transaction, but to focus on the most relevant flows.

Table 3: Transaction Flow Matrix

<table>
<thead>
<tr>
<th>NFC</th>
<th>Current</th>
<th>Capital</th>
<th>FC</th>
<th>Current</th>
<th>Capital</th>
<th>G</th>
<th>Current</th>
<th>Capital</th>
<th>H</th>
<th>Current</th>
<th>Capital</th>
<th>ROW</th>
<th>Current</th>
<th>Capital</th>
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</thead>
<tbody>
<tr>
<td>Private Consumption</td>
<td>+C</td>
<td>-1^n</td>
<td>+I</td>
<td>-I^p</td>
<td>+G</td>
<td>-G</td>
<td>-I^R</td>
<td>-X</td>
<td>+M</td>
<td>-1^n</td>
<td>-1^n</td>
<td>+1^n</td>
<td>0</td>
<td>0</td>
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<tr>
<td>Government Consumption</td>
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<tr>
<td>Investment</td>
<td></td>
<td></td>
<td>+X</td>
<td>-M</td>
<td>-</td>
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<td>Exports</td>
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<td>(GDP)</td>
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<tr>
<td>Taxes</td>
<td>-I^n</td>
<td>-</td>
<td>-G</td>
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<tr>
<td>Gross Operating Surplus</td>
<td>-</td>
<td>-</td>
<td>+I^p</td>
<td>+</td>
<td>+</td>
<td>+</td>
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<tr>
<td>Capital Income</td>
<td>+</td>
<td>+</td>
<td>+K^p</td>
<td>+</td>
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<td>+</td>
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<td>Savings</td>
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\(^8\) “Other taxes on production (D.29) consist of all taxes that enterprises incur as a result of engaging in production, independent of the quantity or value of the goods and services produced or sold” (Eurostat European Commission 2013, 93).

\(^9\) “Other subsidies on production (D.39) consist of subsidies except subsidies on products which resident producer units may receive as a consequence of engaging in production” (Eurostat European Commission 2013, 96).
Turning to capital income in our model, the income associated with assets originates from three financial assets, namely, net interest-bearing assets, net equities, and pensions, as discussed earlier. These income flows are determined in the following way:

\[ \text{net capital income}_t = r_{t-1} \times (\text{net stock}_{t-1}) \]

The above equation simply describes that capital income flow is equal to the previous value of stock times the rate of return on that stock. However, rates of return are not available in the data and need to be computed as well. For each financial asset we calculate our own rate of returns, and take into account any discrepancy between the flows reported in the income account and the flows calculated using our computed rates of return. For example, the interest rate on interest-bearing assets for the household sector is computed as follows:

\[ r^{H}_{I_{t-1}} = \frac{\text{interest received}_{t}}{IBA^{H}_{t-1}} \]

Following the above procedure, we calculate three interest rates, i.e., interest rates on household assets, interest rates on household liabilities, and interest rates on net interest-bearing stocks. The same procedure is followed to calculate the rate of return on the stock of pensions and equities. We consider one rate of return on equities and one rate of return on pension stocks. Our computed rates of return are plotted in figure 7. The discrepancies (or error terms) between the flows reported in the income account and the flows calculated using our rate of return are plotted as a percentage of GDP in the appendix. Overall, these error terms are very small and not worthy of further discussion.

Figure 7: Computed Rates of Return

Source: Eurostat and authors’ calculations.
The real economic transactions for the Danish economy in 2015 are visualized in figure 8. The diagram clearly shows the origin and destination of different flows. The width of the flow represents the magnitude of a flow relative to other flows in the economy.

For the household sector, it clearly gives an idea about the importance of each component of income; wages are by far the largest source of income, followed by social transfers. Inflows associated with financial assets and gross operating surplus from production also contribute to income. On the expenditure side, along with taxes, consumption accounts for a considerable part of the expenditures, whereas the expenditures on investment and interest on debt are relatively small. For the NFCs, wages, imports, taxes, interest on liabilities, and distribution of gross operating surplus are the main expenditures, while the primary source of income comes from selling goods domestically and abroad. For the FCs, inflows associated with financial assets (i.e., capital income) are the major source of income, while interest paid to the other sectors together with changes for adjustment of pension entitlements form the main flows on the expenditure side. For the government sector, public consumption, social transfers (mostly toward the households), and investment are the main expenditures, whereas the interest expenditures are relatively small due to lower levels of public debt. On the income side, the vast majority of income comes from taxes paid by other sectors. Finally, the rest of the world pays a higher capital income to Denmark than it receives, since Denmark is a net creditor and has a current account surplus.
We also visualize the financial transactions for the Danish economy in 2015 as shown in figure 9. These transactions need to be explained with caution. With the exception of the household sector, for all other sectors the transactions are represented on net basis. Overall, most of these transactions are largely consistent with the way the balance sheet structures have evolved. For example, household outflows for the purpose of purchasing an asset includes pensions, interest-bearing stocks, and equities. Household inflows for the purpose of borrowing includes interest-bearing loans. The only transaction that seems to be at odds with the balance sheet structure is the net equities in the NFC sector, i.e., in general the NFC sector has net equities as a liability, but in 2015, this sector purchased more equities as assets than they issued as liabilities. Therefore, the net equity transaction appears on the asset side of the NFC sector in the figure. This could be explained by the improved current account balance, where the surplus was invested in financial assets abroad. This is further evident by the net capital inflows received by the rest of the world originating from Denmark. In particular, the rest of the world receives a relatively large net capital flow, mostly in the form of net equities.
IV. MODEL STRUCTURE

We now proceed to presenting the structure of the model.

4.1 Nonfinancial Corporations (NFC)

We assume all production takes place in the NFC sector. The total production in nominal terms is determined in the standard way as follows:

\[ Y_t = C_t + I_t + G_t + X_t - M_t \]

10 Since all production is assumed to take place in the same sector, any distribution of the gross operating surplus cannot be determined within the model. Since the flows of this surplus provide an important income for all sectors, this flow is kept exogenous for FCs, households, and the government sector, while the surplus for the NFC sector is a residual. For the government sector however, the gross operating surplus is equal to the consumption of fixed capital, so it should be possible to make this endogenous.
This equation can be rewritten to express the total sales in domestic economy:

\[ S_t = C_t + I_t + G_t + X_t \]

Value of real output:

\[ y_t = c_t + i_t + g_t + x_t - m_t \]

GDP deflator:

\[ P_t^y = \frac{Y_t}{y_t} \]

Firms pay taxes (including production taxes) to the government sector, and wages \((WB)\) to households in Denmark and abroad. The wage bill is defined as a product of the wage rate \((W_t)\) and the level of employment \((N_t^N)\), where the wage rate is assumed to be the same for Denmark and the rest of the world. The level of employment is the sum of domestic employment and net foreign employment (foreign citizens employed in Denmark minus Danish citizens employed abroad).

The wage bill paid by the NFC sector to its employees is \(N^N:\)

\[ WB_t^N = W_t(N_t^N) \]

Since the majority of taxes paid by the firms are taxes on production, it is further assumed that the level of taxes in our model changes accordingly with variations in total production:

\[ T_t^N = \beta_t(Y_t) \]

From an accounting perspective, the gross operating surplus is the residual between GDP, net taxes on production, and employee compensation. Since net taxes as described earlier are merged into the flows “taxes” and “transfers” (subsidies), the gross operating surplus for the total economy is assumed to be described as a share of GDP as follows:
\[ B_{2t} = \beta Y_t \]

The nominal stock of capital of the NFC sector is determined by the following accounting identity:

\[ K_t^N = K_{t-1}^N + I_t^N - D_t^N + K_{CGt}^N \]

where the level of depreciation (D) depends on the rate of depreciation and the stock of fixed capital in last period.

\[ D_t^N = \delta(K_{t-1}^N) \]

The real stock of capital is determined by deflating the nominal stock with the capital price deflator.

Real stock of capital:

\[ k_t^N = \frac{K_t^N}{P_t^i} \]

We now focus on explaining the level of real investment in the NFC sector. According to Godley and Lavoie (2012), empirical work seems to suggest that capacity utilization is an essential component of the decision to invest. The theoretical argument is that a high rate of capacity utilization motivates firms to raise their capital stock by increasing investment and vice versa. Thus, capacity utilization in that sense also carries the accelerator effect. The level of real investment in our model is determined by the rate of capacity utilization, which in turn is proxied by dividing the level of economic activity (measured by real output) with the real stock of capital in the NFC sector. Our investment function and measure of capacity utilization is similar to the one used in the SFC model for the United Kingdom by Burgess et al. (2016).
Real investment:

$$\ln(i_t^N) = \beta_i + \ln P_t, \left(\frac{Y_t-i}{k_t^N}\right)$$

Theoretically, $\beta_i$ in the above equation has also been interpreted by several authors as reflecting “animal spirits” (e.g., Fujita [2018] and Jesus, Araujo, and Drumond [2018]).

Nominal investment in fixed assets:

$$I_t^N = i_t^N (P_t^i)$$

The savings of the firms can be computed from the primary and secondary incomes:

$$S_t^N = Y_t - WB_t^N + (B_{2,t}^N - B_{2,t}) + r_{Nt-1}(NIB_{t-1}^N) + \chi_t(NEQ_{t-1}^N) - T_t^N + STR_t^N + \epsilon^N$$

The net lending of the firms is the difference between savings and investment adjusted for the exogenously determined capital transfers ($KTR_t^N$) and net acquisitions of nonproduced nonfinancial assets ($NP_t^N$).

Net lending/borrowing:

$$NL_t^N = S_t^N - I_t^N - NP_t^N + KTR_t^N$$

On the financial side of the economy the firms finance their expenditures with two different financial assets: net interest-bearing assets and net equities. In the current version of the model, the transaction of net equities in the NFC sector plays a passive role and accommodates the demand for new equities originating from other sectors. The transaction of net interest-bearing assets is described as the difference between total net lending and transaction for equities.

Net equities:

$$NEQ_t^N = NEQ_{t-1}^N + NEQTR_t^N + NEQ_{CG_t}^N$$
Net interest-bearing stocks (assets minus liabilities) held by the firms:

\[ NIB_t^N = NIB_{t-1}^N + NIBTR_t^N + NIBCG_t^N \]

Net interest-bearing financial transactions:

\[ NIBTR_t^N = NL_t^N - NEQTR_t^N \]

The financial net wealth of the firms can be written as the sum of the two assets, as explained above:

\[ FNW_t^N = NIB_t^N + NEQ_t^N \]

The total net wealth of the firms can then be expressed as the sum of financial net wealth and the stock of fixed capital:

\[ NW_t^N = FNW_t^N + K_t^N \]

4.2 Household Sector

We now turn to explaining the household sector, which is the main endogenous sector of the economy in our model. The household sector receives income mainly from four sources: wages from firms \((WB_t^H)\), gross operating surplus from production \((B_{2t}^H)\), social transfers \((STR_t^H)\), and capital income. The capital income of the households originates from interest-bearing assets \((IBA_t^H)\), pensions \((PENAH_t^H)\), and equities \((EQA^H)\).

The total income for households can be written as:

\[
Y_t^H = WB_t^H + B_{2t}^H + r_{At-1}^H(IBA_{t-1}^H) - r_{Lt-1}^H(IBL_{t-1}^H) + \chi_t(EQA_{t-1}^H) + \psi_t(PENAH_{t-1}^H) + STR_t^H + \epsilon_t^H
\]

where \(r_{At}^H\) and \(r_{Lt}^H\) represent interest rates on assets and liabilities, respectively. \((\chi_t)\) and \((\psi_t)\) represent returns on equities and pensions, respectively.
Social transfers received by the household sector in the above equations are the sum of social contributions ($SCON^H$) paid by the households, social benefits ($SBEN_t^H$), and other transfers ($OTR_t^H$) received by the households:

Social transfers:

\[
STR_t^H = SBEN_t^H + OTR_t^H - SCON_t^H
\]

Households are assumed to pay a constant proportion of their income in taxes ($T_t^H$). Subtracting this tax payment from the gross income gives us disposable income ($YD_t^H$) as follows:

\[
YD_t^H = Y_t^H - T_t^H
\]

The aggregate level of taxes paid by households is determined as a fraction of their disposable income:

\[
T_t^H = \beta_1(YD_t^H)
\]

Social contributions paid by the household sector are assumed to be a time-varying fraction of previous household disposable income.\(^{11}\)

Social contributions:

\[
SCON_t^H = \beta_7(YD_{t-1}^H)
\]

The level of benefits received by the household sector is determined by two main indicators, namely the level of unemployment ($UN_t$) and the wage rate ($W_t^H$).

\[\text{In that sense, it can simply be thought of as an exogenous variable in the model.}\]
Social benefits received by the households:

\[ \ln(SBEN_t^H) = \beta_i + \beta_i \ln(U_N) + \beta_i \ln(W_{t-1}^H) \]

This equation implies that a higher level of unemployment increases the level of social benefits through an increase in unemployment benefits, which are a major component of social benefits in a welfare state like Denmark. The level of social benefits is also directly affected by a change in the wage rate, since the compensation rate (ratio of unemployment benefits to the wage rate) is legally determined as a share of the wage rate. Thus, theoretically, the effect of an increase in the wage rate on social benefits is expected to be positive. This feature is consistent with our theoretical SFC model for Denmark proposed in Byrialsen and Raza (2018), and also in line with the empirical SFC model for Denmark by Godley and Zezza (1992).

Real disposable income:

\[ yd_t^H = \frac{YD_t^H}{P_t^c} \]

where \( P^c \) represents the price index for consumption.

Real consumption for the households follows a standard consumption function, where real consumption depends on real disposable income \((yd_t^H)\) and real net wealth \((nw_t^H)\) in the last period.

Real consumption by the households:

\[ \ln(c_t) = \beta_i + \beta_i \ln(yd_{t-1}^H) + \beta_i \ln(nw_{t-1}^H) \]

Nominal consumption:

\[ C_t = c_t(P_t^c) \]
The consumption price index \( (P^c) \) in the model is assumed to be determined by the wage rate and the prices on imports \( (P^m) \), which reflects the fact that Denmark as a small, open economy has a high degree of trade openness with the rest of the world:

\[
lnP^c_t = \beta_i ln(W_t) + \beta_i ln(P^m_t)
\]

The level of housing investment is determined by the incentive to invest in new housing and real disposable income. The incentive to invest in new housing—known as Tobin’s q for housing—is usually defined as the ratio of house prices to construction costs. The argument is that an increase in house prices relative to construction costs would induce investments in housing (Kohlscheen, Mehrotra, and Mihaljek 2018).

Real investment in fixed assets (housing):

\[
ln(i^H_t) = \beta_i + \beta_i ln(i^H_{t-1}) + \beta_i ln(P^H_t) + \beta_i ln(yd^H_{t-1})
\]

The intuition of the above equation is straightforward, i.e., an increase in house prices motivates households to invest more in the construction of new houses, while an increase in the construction costs would lower housing investment. Finally, an increase in real disposable income—which, like house prices, is a procyclical indicator—will increase the level of investment in housing.12 Our model of housing investment in this regard is in line with the theoretical arguments and empirical evidence presented in several studies such as Gattini and Ganoulis (2012), Caldera and Johansson (2013), and Kohlscheen, Mehrotra, and Mihaljek (2018).

Nominal investment in fixed asset can be written as:

\[
I^H_t = i^H_t (P^I_t)
\]

where \( P^I_t \) represents the price deflator for investment.

\[12\] This behavior is similar to the model proposed in Zezza (2008), where an increase in expected disposable income positively affects the demand for houses.
The change in the nominal stock of housing \( (K^H) \) follows the basic accounting identity:

\[
K_t^H = K_{t-1}^H + I_t^H - D_t^H + K_{CG_t}^H
\]

This equation simply implies that a change in the stock of housing can occur due to new investment in housing \( (I^H) \), depreciation \( (D^H) \) of capital, and/or capital gains on housing \( (K_{CG}^H) \). Capital gains in the above equation reflect the change in housing stock occurring due to the change in house prices, i.e., we can express realized capital gains as follows:

\[
K_{CG}^H = \Delta P_t^H (K_{t-1}^H)
\]

From the capital gains equation above, we calculate our housing price index, which we also used in the housing investment function. The change in house prices can be written as follows:

\[
\Delta P_t^H = \frac{K_{CG}^H}{K_{t-1}^H}
\]

As demonstrated earlier, our measure of the change in house prices is similar to the one provided by Statistics Denmark.

The nominal stock of capital can be rewritten as follows:

\[
K_t^H = K_{t-1}^H (1 + \Delta P_t^H) + I_t^H - D_t^H
\]

We adjust the nominal stock of capital for the investment price deflator to obtain the real stock of capital as:

\[
k_t^H = \frac{K_t^H}{P_t^i}
\]
The households savings \( (S^H) \) can be found as the difference between disposable income and consumption plus the adjustment for the change in pension entitlements \( (CPEN^H) \):

\[
S_t^H = YD_t^H - C_t^H + CPEN_t^H
\]

Net lending/borrowing is written as the difference between savings and investment, adjusted for net acquisitions of nonproduced nonfinancial assets \( (NP_t^H) \) and capital transfers \( (KTR_t^H) \):

\[
NL_t^H = S_t^H - I_t^H - NP_t^H + KTR_t^H
\]

We now turn to explaining the households’ investment decisions in the financial markets. The overall development of the financial markets in our model is primarily driven by household demand for credit (loans) and assets (interest-bearing equities and pensions). In our behavioral equations, we attempt to explain the financial transactions made in acquiring particular stocks, and then let those transactions (along with capital gains) determine the stocks in the model. It should be highlighted that capital gains on financial assets in our model are exogenous.

We begin by describing the financial balance of the households, which can be written as the difference between the accumulation of financial assets and financial liabilities:

\[
FNL_t^H = FATR_t^H - FLTR_t^H
\]

The total transaction of financial assets \( (FATR^H) \), is the sum of three financial transactions: interest-bearing asset transactions \( (IBATR^H) \), equities transactions \( (EQATR^H) \), and pension transactions \( (PENATR^H) \):

\[
FATR_t^H = IBATR_t^H + EQATR_t^H + PENATR_t^H
\]

The demand for new equities is inspired by Tobin’s portfolio theory in the sense that a household is faced with the choice of investing in different financial assets.\(^{13}\) The investment

\(^{13}\) See Godley and Lavoie (2012: ch 4) for the integration of Tobin’s portfolio theory is SFC models.
decision is mainly determined by the relative return on each financial asset and real disposable income. In our model, the households invest in three financial assets, namely interest-bearing assets, equities, and pensions. After the introduction of the Danish pension system in the 1990s, a portion of wealth is held in pensions regardless of the return on other financial assets. Thus, the households in our model are typically faced with a choice of allocating their savings between interest-bearing stocks and equities. Equities transactions are determined by the return on equities \( \delta \) and the return on interest-bearing stocks \( r^H_i \) as well as the credit available to the households and can be written as:

\[
EQAHTR_t = \beta_i + \beta_i(\delta_t) + \beta_i(r^H_{i,t-1}) + \beta_i(IBLTR^H_t)
\]

An increase in the return on equities would induce investment in equities whereas an increase in the interest rate on interest-bearing assets would reduce the demand for new equities as households would allocate their savings to interest-bearing assets. Finally, the link between the demand for equities and accumulation of new loans needs to be explained with caution: an important element of the Danish tax system is that households that are subject to interest payments on loans are entitled to a tax reduction. This reduces the cost of loans, which, according to the Nationalbanken (2016), may have created an incentive to increase the stock of loans and the stock of financial assets at the same time. Since a part of the accumulation of equities is financed through new loans, the demand for new equities is therefore expected to have a positive relationship with the accumulation of loans.

Pension wealth transactions are determined by the wage bill \( WB^H_t \) in the economy along with the return on pensions \( \psi_t \). In other words, an increase in the wage bill (either due to an increase in employment or the wage rate) would increase pension transactions. Similarly, an increase in the rate of return on pensions would induce households to allocate more savings to pensions.

Pension transactions can be written as:

\[
PENATR^H_t = \beta_i + \beta_i(\psi_t) + \beta_i WB^H_t
\]
The demand for new loans by households \((IBLTR^H)\) is assumed to be a function of investment in housing, the stock of debt from last period, the transaction of financial assets, and the interest rate on interest-bearing liabilities.

Interest-bearing liability transactions can be written as:

\[
IBLTR^H_t = \beta_1(I^H_{t-1}) + \beta_2(IBL^H_{t-1}) + \beta_3(FATR^H_t) + \beta_4(r^H_{t-1})
\]

The above equation shows the relationship between the decision to invest in housing and the demand for new loans by the households. This relationship also captures the effect of house prices on household debt, as widely discussed in the literature, i.e., an increase in house prices creates an incentive to invest in housing, which in turn, would induce an increased demand for loans. The stock of loans from last period is expected to contribute negatively to the demand for new loans due to two main reasons. First, as presented in Godley and Lavoie (2012), agents are driven by stock-flow norms, such as wealth-to-income (or debt-to-income) ratios. For a given desired norm for wealth-income ratios, a higher level of debt would lead to higher savings and thereby lower net accumulation of financial liabilities. Second, from the supply side, a high level of debt may result in low collateral or creditworthiness and thereby lower access to credit for the households.

In our model, the demand for new loans is also linked to the accumulation of financial assets. This relationship can be explained from different theoretical perspectives. First, as explained earlier, the lower cost of loans via reduced taxation creates an incentive to borrow new loans while acquiring new financial assets. Second, this also captures the transmission channel of households savings to investment, i.e., an increase in gross savings (implying an increase in financial asset transactions) will induce an increased supply of credit—a portion of which is then assumed to finance household investment, leading to a positive relationship between savings and investment.\(^{14}\) Third, an increase in the accumulation of financial assets is an indication of better creditworthiness. This implies that households have more collateral to borrow against to finance their expenditures. Following these arguments, the relationship

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\(^{14}\) At this point, it is important to highlight that investment is not constrained by savings, however, an increase in savings can induce investment. Similarly, an increase in investment can also lead to an increase in savings, implying a bidirectional causality. This is in line with the empirical evidence in Raza, Zoega, and Kinsella (2018), who used national savings (gross) and investment for 17 OECD countries in their study.
between the demand for loans and the transactions of financial assets should be positive, resulting in a positive relationship between household debt and financial assets at a macroeconomic level. Our assumption of a positive relationship in this regard is also in line with the empirical evidence found for individual households in the literature (see e.g., Brown and Taylor [2008] and Brown, Garino, and Taylor [2013]). Finally, high interest rates on loans are expected to contribute negatively to the demand for new loans.

The demand for deposits by the households (interest-bearing assets) is modeled as a residual in this model:

\[
IBATR_t^H = NL^H + IBLTR_t^H - EQATR_t^H - PENATR_t^H
\]

These transactions of financial assets and liabilities lead to changes in the stock of each financial asset.

The stock of interest-bearing assets at time t can be written as the sum of the stock in period t-1, the transaction of interest-bearing assets in period t, and capital gains in period t:

\[
IBA_t^H = IBA_{t-1}^H + IBATR_t^H + IBA_{CG}^H
\]

The stock of equities, pensions, and interest-bearing liabilities can be written in the same way:

\[
EQA_t^H = EQA_{t-1}^H + EQATR_t^H + EQA_{CG}^H
\]

Pension assets can be written as:

\[
PENA_t^H = PENA_{t-1}^H + PENATR_t^H + PENA_{CG}^H
\]

Interest-bearing liabilities can be written as:

\[
IBL_t^H = IBL_{t-1}^H + IBLTR_t^H + IBL_{CG}^H
\]
Total financial assets in this model is the sum of the three financial assets and can be expressed as:

\[ FA_t^H = IBA_t^H + EQA_t^H + PENA_t^H \]

Note that the total stock of financial liabilities in the household sector is equal to the stock of interest-bearing liabilities in the household sector:

\[ FL_t^H = IBL_t^H \]

The difference between total financial assets and total financial liabilities determines the financial net wealth as follows:

\[ FNW_t^H = FA_t^H - FL_t^H \]

We now obtain total net wealth by simply adding housing to financial net wealth:

\[ NW_t^H = FNW_t^H + K_t^H \]

Real financial wealth is shown as:

\[ fnw_t^H = \frac{FNW_t^H}{P_t^c} \]

Real wealth can be written as:

\[ nw_t^H = \frac{NW_t^H}{P_t^c} \]
4.3 Financial Sector

The financial sector in this model is the main provider of credit in the economy, which means that capital income plays a major role for the savings of the financial sector. The FC’s savings are determined by the standard accounting identity, i.e., savings ($S^F_t$) can be expressed as the sum of the net capital income, gross operating surplus ($B^F_{2,t}$) (received), social transfers ($STR^F_t$) minus taxes paid to the government ($T^F_t$), and the changes in pension entitlements ($CPEN^F_t$) paid to the households:

\[
S^F_t = B^F_{2,t} + r^F_{A,t-1}(IBA^F_{A,t-1}^H) - r^F_{t,t-1}(IBL^F_{t-1}^H) + r^F_{N,t-1}(NIB^F_{t-1}) + \chi^F_t(NEQ^F_{t-1}) \\
- \psi_t(PEN^F_{t-1}) - T^F_t + STR^F_t - CPEN^F_t + \epsilon^F
\]

The FCs’ stock of fixed assets is determined in the standard way, as explained for other sectors above:

\[
K^F_t = K^F_{t-1} + I^F_t - D^F_t + K^F_{C,t-1}
\]

After taking net acquisitions of nonproduced nonfinancial assets ($NP^F_t$) and capital transfers into account ($KTR^F_t$), the net lending/borrowing of FCs can be expressed as the difference between savings and investment as follows:

\[
NL^F_t = S^F_t - I^F_t - NP^F_t + KTR^F_t
\]

The financial balance ($FNL^F_t$) (which is equal to the net lending) is calculated as the difference between the accumulation of financial assets and financial liabilities:

\[
FNL^F_t = IBATR^F_{t}^H + NIBTR^F_t + NEQTR^F_t - IBLTR^F_{t}^H - PENLTR^F_t
\]

Interest-bearing inflows and outflows in the above equation are only linked to the balance sheets of the household sector. That is, an interest-bearing transaction ($IBATR^F_{t}^H$) for the sake of acquiring an asset by the banks is equal to the new loans received by the households. Similarly, transactions for the sake of accumulating interest-bearing liabilities ($IBLTR^F_{t}^H$) by the bank are equal to the transaction for interest-bearing assets by the households. Thus, these transactions, by construction, are determined as follows:
Note that from the FCs’ perspective, the development in both $IBATR_{t}^{F-H}$ and $IBLT_{t}^{F-H}$ is entirely determined by households’ demand for new loans and their allocation of savings, respectively.

The FCs’ interactions with all other sectors that involve transactions for the purpose of acquiring interest-bearing stocks are captured through net interest-bearing transactions ($NIBTR^F$). Hence, the transactions involving net interest-bearing stocks are determined as follows:

$$NIBTR^F_t = -(NIBTR^N_t + NIBTR^G_t + NIBTR^W_t)$$

where $NIBTR^N$, $NIBTR^G$, and $NIBTR^W$ represent the net interest-bearing stock of the NFCs, the government sector, and the rest of the world, respectively.

We now turn to explaining the accounting identities involved in determining the stock of interest-bearing assets. The change in the FCs’ stock of interest-bearing assets is determined by the corresponding transaction along with capital gains:

$$IBA_{t}^{F-H} = IBA_{t-1}^{F-H} + IBATR_{t}^{F-H} + IBA_{CG_t}^{F-H}$$

Similarly, the change in interest-bearing liabilities is expressed as follows:

$$IBL_{t}^{F-H} = IBL_{t-1}^{F-H} + IBLT_{t}^{F-H} + IBL_{CG_t}^{F-H}$$

The change in the stock of net interest-bearing stocks is equal to the transaction of net interest-bearing assets and net capital gains:

$$NIB_{t}^{F} = NIB_{t-1}^{F} + NIBTR_{t}^{F} + NIB_{CG_t}^{F}$$
The overall development in \( PENLTR^F \) is mainly explained by household contributions to pensions, as discussed earlier:\(^{15}\)

\[
PENLTR^F_t = PENATR^H_t + NPENTR^W
\]

We now turn to the FCs’ last financial asset. The transaction of net equities (\( NEQTR^F_t \)) is modeled as a residual between net lending and the sum of the transaction of other financial assets:

\[
NEQTR^F_t = NL_t^F + IBLTR_t^{F-H} + PENLTR_t^F - IBATR_t^{F-H} - NIBTR_t^F
\]

This transaction leads to a change in the stock of net equities:

\[
NEQ_t^F = NEQ_{t-1}^F + NEQTR_t^F + NEQ_{CGt}^F
\]

The difference between the stock of total financial assets and total financial liabilities is equal to the financial net wealth:

\[
FNW^F_t = NIB_t^F + NEQ_t^F + IBA_t^{F-H} - IBL_t^{F-H} - PENL_t^F
\]

We can calculate the total net wealth by adding fixed assets to financial wealth as follows:

\[
NW_t^F = FNW_t^F + K_t^F
\]

### 4.4 Government Sector

The government sector in Denmark is characterized as a welfare state with a high level of public expenditures, which are financed through taxes.

The total tax revenue received by the government is equal to the taxes paid by all other sectors:

\[
T_t^G = T_t^{NF} + T_t^H + T_t^F + T_t^W
\]

\(^{15}\) \( NPENTR^W \) is exogenous in our model. It is a very small proportion of the total transaction.
A major expenditure for the government sector apart from consumption is social transfers. The social transfers paid by the government sector are equal to the sum of social transfers received by the other sectors:

\[ STR_t^G = -(STR_t^H + STR_t^{NF} + STR_t^E + STR_t^{W}) \]

The government sector’s savings are an accounting identity, which also depends on government consumption (\( G_t \)) and the interest payment on public debt. The savings identity (\( S_t^G \)) can be written as follows:16

\[ S_t^G = B_{2t}^G + \tau_{N_t-1}(NIB_{t-1}^G) + T_t^G + STR_t^G - G_t + e^G \]

The government sector can also affect the aggregate demand through public investment (\( I_t^G \)) in fixed assets, which is also treated as exogenous in this model. The stock of fixed capital is determined in the same way as discussed for the other sectors:

\[ K_t^G = K_{t-1}^G + I_t^G - D_t^G + K_{G_{t-1}}^G \]

After taking capital transfers and NP into account, the difference between savings and investment determines net lending:

\[ NL_t^G = S_t^G - I_t^G - NP_t^G + KTRt^G \]

On the financial side of the economy, the government is assumed to finance its deficit through net interest-bearing assets. The total transaction of this stock determines the financial net lending as follows:

\[ FNL_t^G = NIBR_t^G \]

16 Like for other sectors, the government sector also receives a share of the gross operating surplus from the production sector.
Since net interest-bearing assets are the only financial asset, the net transaction of net interest-bearing assets is determined by the size of net lending:

$$NIBTR^G_t = NL^G_t$$

The change in the net stock of interest-bearing assets is determined by its corresponding transactions along with capital gains:

$$NIB_t^G = NIB_{t-1}^G + NIBTR^G_t + NIB_{CG_t}$$

4.5 Balance of Payments and Trade

Denmark is a small, open economy, and its interaction with the rest of the world plays a big role. From the 1950s until 1989, the economy experienced persistent current account deficits and therefore accumulated a large stock of foreign debt. Since 1989, the economy has been running persistent current account surpluses, resulting in the accumulation of external wealth.

We now proceed to explaining our model for trade flows in Denmark. The import equation in our model is pretty standard; that is, imports are affected by relative prices and private demand.

Real imports:

$$\ln(m_t) = \beta_i + \beta_i \ln\left(\frac{p_{t-1}^y}{p_{t-1}^m}\right) + \beta_i \ln(c_{t-1} + i_{t-1} + x_{t-1})$$

The export function is based on the Armington (1969) model, where the market share of Danish exports is explained by relative prices. This relationship is formulated in the equation below, where $\beta$ indicates the price elasticity, $x_t$ is real exports, and $\frac{p_{t-1}^x}{p_{t-1}^m}$ indicates the relative prices of tradeables; $m_t^W$ is an import index of the Danish trading partners. Thus, $\frac{x_t}{m_t^W}$ represents the share of Danish exports in the market:
The link between relative prices \( \frac{p_{t}^{x}}{p_{t}^{m}} \) and the share of Danish exports \( \frac{x_{t}}{m_{t}} \) between 1995–2016 is presented below, where there is a clear pattern pointing to an inverse relationship, i.e., an increase in relative prices is associated with a fall in export market shares due to a loss of competitiveness.

Figure 10: Exports and Relative Prices

![Graph showing exports and relative prices over time from 1995 to 2016.]

Source: Eurostat and authors’ calculations.

The equation above can be transformed to express the real level of exports as a function of relative prices and the export market index:

\[
\ln(x_{t}) = \beta_{i} + \beta_{i} \ln\left(\frac{p_{t}^{x}}{P_{t-1}^{m}}\right) + \beta_{i} \ln(m_{t}^{w})
\]

Nominal imports:

\[
M_{t} = m_{t}(P_{t}^{m})
\]
Nominal exports:

\[ X_t = x_t(P_t^x) \]

Import prices \((P_t^m)\) are expressed in domestic currency assuming a fixed exchange rate of one. Export prices \((P_t^x)\) are determined within the model as a function of unit labor cost and import prices. The inclusion of import prices in this equation is based on the fact that Denmark imports semimanufactured goods and has a high degree of trade openness.

Price of exports:

\[ \ln(P_t^x) = \beta_t + \beta_t \ln(P_t^m) + \beta_t \ln(ULC_{t-1}) \]

The sectoral balance for the foreign sector can be written as an identity containing exports, imports, capital income, net wages received from Denmark, net taxes paid to Denmark, and net social transfers.

Savings of the rest of the world:

\[ S_t^W = M_t - X_t + \chi_t(NEQ_{t-1}^W) + \Psi_t(NPEN_{t-1}^W) + r_{Nt-1}(NIE_{t-1}^W) + WB_t^W - T_t^W + STR_t^W + \epsilon_t^W \]

Finally, we express all the accounting identities for the rest of the world following the same principle as we did for the other sectors.

The rest of the world’s net lending:

\[ NL_t^W = S_t^W - NP_t^W + KTR_t^W \]

Current account balance:

\[ CAB_t = -NL_t^W \]
Financial account balance:

\[
FN_{t}^{W} = NIB_{t}^{W} + NEQTR_{t}^{W} + NPENTR_{t}^{W}
\]

Net interest-bearing stocks:

\[
NIB_{t}^{W} = NIB_{t-1}^{W} + NIB_{t}^{W} + NIB_{CG_{t}}^{W}
\]

Net equity stocks:

\[
NEQ_{t}^{W} = NEQ_{t-1}^{W} + NEQTR_{t}^{W} + NEQ_{CG_{t}}^{W}
\]

Net pension stocks:

\[
NPEN_{t}^{W} = NPEN_{t-1}^{W} + NPENTR_{t}^{W} + NPEN_{CG_{t}}^{W}
\]

Net interest-bearing transactions:

\[
NIBTR_{t}^{W} = NL_{t}^{W} - NEQTR_{t}^{W} - NPENTR_{t}^{W}
\]

The rest of the world’s net financial wealth:

\[
FNW_{t}^{W} = NIB_{t}^{W} + NEQ_{t}^{W} + NPEN_{t}^{W}
\]

4.5 Labor Market

We now turn to explaining the labor market, which determines wages and employment in the model. First, we determine GDP at factor cost to determine adjusted wage shares. Then we derive a measure for the unit labor cost as a ratio of the wage share and GDP at factor cost.

GDP at factor cost:

\[
Y_{t}^{F} = WB_{t}^{N} + B_{2_{t}}
\]
Wage share:

\[ WS_t = \frac{WB_t^N}{Y_t^F} \]

Unit labor cost:

\[ ULC_t = \frac{WS_t(Y_t)}{Y_t^F} \]

The number of unemployed individuals is defined as the difference between the total labor force and the number of employed individuals. The number of individuals employed domestically is explained by the level of economic activity as well as labor force participation, which is exogenous in the model.

Number of unemployed individuals:

\[ UN_t = LF_t - N_t \]

The ratio between the number of unemployed and the labor force measures the rate of unemployment:

\[ UR_t = \frac{UN_t}{LF_t} \]

The number of employed individuals in the domestic economy is assumed to be determined by real economic activity and the actual size of the labor force:

\[ \ln(N_t) = \beta_t + \beta_t\ln(y_{t-1}) + \beta_t\ln(LF_t) \]

The total number of individuals employed by the firms is the sum of domestic labor and foreign labor employed in Denmark:

\[ N_t^N = N_t + N_t^W \]
The domestic wage bill received by the household sector is simply the product of the wage rate and the number of employed individuals domestically. The wage rate is determined by the unemployment rate, i.e., a rise in the unemployment rate would reduce the wage rate and vice versa.

Wage bill of the household sector:

\[ WB_t^H = W_t(N_t) \]

The wage rate is modeled through a Phillips curve relation, i.e., it is a function of the change in the rate of unemployment. The changes in the unemployment rate can roughly be interpreted as a measure of the change in the bargaining power of labor, i.e., a higher unemployment rate will imply weaker bargaining power and vice versa:

\[ W_t = \beta_t + \beta_t UR_{t-i} \]

Finally, the number of individuals hired from abroad can be deducted from the ratio between the total wage bill paid abroad and the wage share, which is assumed to be the same for all employed persons:

\[ N_t^W = \frac{WB_t^W}{W_t} \]

V. CONFRONTING THE MODEL WITH THE DATA

Our model has a number of structural parameters that are estimated using annual Danish data from 1995–2016, following an autoregressive distributive lag (ARDL) model. Before estimating the regression equations, we test our variables for a unit root using the augmented Dickey-Fuller (ADF) and Phillip-Perron tests. If we suspect a strong structural break in the data, we supplement our analysis with a Zivot and Andrew’s unit root structural break test. If a variable is found to be nonstationary, we estimate the equation in first differences and test
for cointegration, given that the prerequisites for establishing cointegrating relationships are fulfilled.

To estimate the equations, in most cases we start our estimation by including two lags, due to small sample size. We then follow general-to-specific methodology and fit a parsimonious model. We also account for any significant structural breaks in our estimations. While our estimation technique is entirely econometric in nature and aimed at obtaining statistically valid estimators, our choice of variables in every equation is purely theoretical, as discussed earlier. The number of variables in our econometric equations are to some extent limited by data availability. Overall, we did not encounter any contradictions between our theoretical and empirical relationships that are worthy of consideration. In some cases, the variables of interest revealed an insignificant relationship; however, the estimators in all cases had the right signs. All econometric results are reported in the appendix.

After estimating the structural parameters, we simulate the model and compare our model’s overall performance with the actual data. The use of simulation is essential to characterize the dynamic properties of any given model. Here we only focus on our key variables. Overall, the estimated behavioral equations are able to explain the dynamics of the data to some extent.

**Figure 11: Real Consumption**

![Real Consumption Graph](image)

*Source: Eurostat and simulated data.*
Figure 12: Real Investment (NFCs)

Source: Eurostat and simulated data.

Figure 13: Real Investment (households)

Source: Eurostat and simulated data.
Figure 14: Real Exports

Source: Eurostat and simulated data.

Figure 15: Real Imports

Source: Eurostat and simulated data.
Figure 16: Financial Accounts

Source: Eurostat and simulated data.

Figure 17: Household Demand for Equities

Source: Eurostat and simulated data.
Figure 18: Household Demand for Pensions

Source: Eurostat and simulated data.

Figure 19: Household Demand for Interest-Bearing Assets

Source: Eurostat and simulated data.
Figure 20: Household Stock of Equities

Source: Eurostat and simulated data.

Figure 21: Household Stock of Pensions

Source: Eurostat and simulated data.
Figure 22: Household Stock of Interest-Bearing Assets

Source: Eurostat and simulated data.

Figure 23: Household Demand for Loans

Source: Eurostat and simulated data.
Figure 24: Household Stock of Loans

Source: Eurostat and simulated data.

Figure 25: Household Balance

Source: Eurostat and simulated data.
Figure 26: NFC Balance

Source: Eurostat and simulated data.

Figure 27: FC Balance

Source: Eurostat and simulated data.
Figure 28: Government Balance

Source: Eurostat and simulated data.

Figure 29: Real Output

Source: Eurostat and simulated data.
5.1 Baseline

We now proceed to performing simulations on the model for several periods ahead. It is important to make it clear that the primary objective of our simulations is not to provide a forecast of growth rates, but to explore how an economy that is based on the structure presented earlier might unfold. The result from this simulation is used as our baseline result against which different scenarios are compared in order to analyze the effects of different shocks to the economy.

In order to perform simulations and create sensible outcomes, we follow a simple approach and forecast some key exogenous variables while keeping the estimated parameters constant. It should be highlighted that a combination of exogenous variables can be crucial for determining the growth rate of the economy in the simulations. In particular, we find that choosing a combination of prices is critical, as it determines both inflation and competitiveness in the economy. It is nonsensical to project all the exogenous variables with the same growth rate, and choosing a particular forecasted combination of exogenous variables becomes extremely difficult. The task becomes even more complicated when it comes to projecting capital gains, due to their fragile nature. Thus, we refrain from forecasting capital gains and prefer to analyze their effects in the form of shocks in the future.

For projecting some of the key real economic variables, in some cases we first determine their ratios with GDP and then use the mean of the last 12 years of their ratios. However, the
depreciation of capital stock is determined on the basis of its ratio to the stock of capital (instead of GDP) in the projections. We allow prices to grow at a rate equal to the average growth rate of the last 12 years. However, we do not strictly bind ourselves by the aforementioned criteria. In some cases when a variable shows a mean reverting tendency, we either keep its value constant or zero, depending on how far it has been fluctuating from zero. Regarding the financial side of the economy, we let the rate of returns on stocks (namely, interest rate, return on equities, and return on pension) remain constant, using their latest values. Finally, we do not allow for capital gains on stocks in the baseline simulations for the reasons discussed earlier. We randomly choose to solve the model for 50 periods, which gives us a baseline solution without any nonsensical explosions in any variable. Here we will only focus on our results until 2030.

Figure 31: Real GDP

Source: Simulated data.
Figure 31 shows the baseline scenario for real GDP, which has a growth rate of 0.7 percent to 0.95 percent, whereas the nominal growth rate is around 2 percent to 2.2 percent. The growth rate in our model is lower compared to the baseline growth rate in other models, e.g., the nominal growth rate in Burgess et al. (2016) is around 5 percent.\textsuperscript{17} We now present other components of real GDP as a ratio of GDP.

Focusing on consumption, figure 32 shows the ratio of real consumption to GDP, which follows a slight downward trend, implying that the growth rate of real consumption is slightly lower than that of real GDP.

\textsuperscript{17} The lower growth rate in this model is primarily because of lower household and government consumption. Focusing on household consumption, the growth rate is slightly lower because of slow demographic changes projected for Denmark (i.e., low growth rate in the labor force), higher pension contributions by employed individuals, and zero capital gains on household assets. Focusing on the government sector, we let real government consumption grow by 1.18 percent, which is consistent with the lower growth rates of the previous governments, and also close to the growth rate expected by the Danish Central Bank in the next few years. Note that the sample average growth rate for real government expenditure is 1.6 percent and for nominal government expenditure it is 3.63 percent. In contrast, Burgess et al. (2016) project this variable using its long-term average.
Figure 33: Real Investment

![Graph showing the ratio of real investment to GDP over the simulation period. The post-crisis upward trend continues to drive economic growth.](image)

Source: Simulated data.

Figure 34: Real Exports

![Graph showing real exports and imports as ratios of real GDP over the simulation period. The growth in imports seems to be slightly higher than that of exports.](image)

Source: Simulated data.

Figure 33 shows the ratio of real investment to GDP, where the post-crisis upward trend continues to drive economic growth. The same is the case with trade openness, i.e., both real exports and imports as ratios of real GDP follow an upward trend, as shown in figure 34. The growth in imports seems to be slightly higher than that of exports. However, the overall trade balance continues to be positive and Denmark experiences a current account surplus in the simulation period.
Figure 35: Household Debt

![Household Debt Graph]

Source: Simulated data.

Figure 36: Unemployment Rate

![Unemployment Rate Graph]

Source: Simulated data.

Figure 35 shows the household debt-to-GDP ratio, where we can see a slightly upward trend, indicating that debt is growing faster than GDP. Debt accumulation in our simulation is not as sharp as it was in the precrisis period. Focusing on the labor market, figure 36 shows that the unemployment rate continues to fall in Denmark, despite low economic growth. This result can be explained by the fact that the labor force growth rate in our simulation is very small.
Finally, we present the financial balances of all the sectors in figure 37. It can be seen that all domestic sectors with the exception of FCs have a surplus. The rest of the world shows an increasing deficit vis-à-vis Denmark, which is in line with the persistent current account surpluses experienced by the Danish economy since the 1990s. In our model, despite experiencing an increase in their debt-to-GDP ratio, the household sector manages to achieve a surplus, indicating that there is also strong growth in the accumulation of financial assets. This is in line with the Danish households’ balance sheet presented in section 3.

**Figure 37: Financial Balances of Sectors**

![Graph showing financial balances to GDP over time for different sectors](image)

*Source:* Simulated data.

We now proceed to analyzing the effect of two standard shocks (fiscal shocks and interest rate shocks) in our model in order to explore whether it is able to explain some of the stylized facts.

**5.2 Fiscal Shocks**

We analyze the effects of a fiscal shock in our model by increasing real public consumption by 1 percentage point in 2017 (i.e., in 2017, the growth rate of real government consumption increases to 2.18 percent as compared to the baseline of 1.18 percent). This leads to a permanent increase in the level of public consumption in the simulation sample.
Figure 38 shows real output’s response to a fiscal shock (i.e., a permanent increase in government expenditures that positively affects real economic activity). The multiplier effect is around 0.3 percent. We now turn to explaining the effects of the shock on individual components of GDP. Figure 39 shows the response of household consumption to a fiscal shock, indicating a positive response; however, the magnitude of this response is very small. The increase in consumption in this case can be explained by both an increase in the wage rate and employment. The effect is weaker because a fall in the unemployment rate also affects real consumption due to an increase in consumption prices.

**Figure 38: Shock 1, Real GDP**

Source: Simulated data.

**Figure 39: Shock 1, Real Consumption**

Source: Simulated data.
Figure 40 shows the shock’s effect on real investment. The level of real investment increases in 2018 mainly due to the accelerator mechanism in the investment function, as presented earlier. The shock’s effect is stronger in the short run but then slowly fades away. However, the overall level of investment stabilizes at one slightly above the baseline in the long run.

**Figure 40: Shock 1, Real Investment**

![Graph showing shock effect on real investment](source)

*Source: Simulated data.*

Turning to the trade balance, following a fiscal shock the economy experiences a trade deficit, which in turn leads to a current account deficit. In particular, the level of imports increases strongly in response to a fiscal shock, which can be explained by two main channels...
in our model: i) an increase in the level of economic activity affects the demand for imported goods, as seen in the import function; and ii) since import prices are assumed to be fixed, an increase in domestic prices in response to a fiscal shock adversely affects the real exchange rate, which in turn increases the demand for imports. This effect is very strong in the short run, but then it slowly reduces and stabilizes slightly above the baseline scenario.

**Figure 42: Shock 1, Government Net Debt**

[Graph showing the trend of government net debt from 1995 to 2025 with a peak around 2015.]

*Source: Simulated data.*

**Figure 43: Shock 1, Unemployment Rate**

[Graph showing the trend of unemployment rate from 1995 to 2025 with a sharp decrease around 2015.]

*Source: Simulated data.*
On the financial side, an increase in public consumption leads to a fall in the public sector’s financial balance, as seen in figure 44. An increase in public consumption directly increases the sales of goods and services for NFCs, leading to an increase their financial balance. The effect of a fiscal shock on the financial account balance of the rest of the world was explained earlier. Finally, the effects on the household and financial sectors are negligible.

Focusing on the debt, a fall in the public sector’s financial balance results in an increase in the stock of debt, as shown in figure 42. The effect is very sharp in the short run, but then slowly reduces as more tax revenues flow into the public sector and the number of social benefit recipients falls. These tax revenues originate mostly from NFCs as they experience an increase in sales. Overall, we can conclude that the effects of a fiscal shock in our model are consistent with the theory.

5.3 Interest Rate Shocks
We now analyze the effect of interest rate shocks in our model. Since we have more than one interest rate in the model, we introduce an interest rate shock by increasing the interest rate by 1 percentage point on all interest-bearing stocks. Here we retain the assumption of a fixed exchange rate, and assume that the increase in the interest rate is a response to the rise in foreign interest rates. Figure 45 shows the overall effect of an increase in the interest rate on real GDP and its components. The effect of an interest rate shock on the real economy is contractionary in nature; however, the overall effect on output is not so strong, as will be
explained. The negative effect on GDP is clearly driven by a strong fall in both consumption and investment. Specifically, the fall in consumption in the short run is due to a fall in disposable income, as shown in figure 46. The decline in investment is due to the accelerator mechanism built into the investment function, as well as due to the rising cost of investment (i.e., higher interest rate).

**Figure 45: Shock 2, Demand Components**

Source: Simulated data.

**Figure 46: Shock 2, Real Disposable Income**

Source: Simulated data.
Focusing on the trade balance, interest rate shocks do not have a strong effect on real exports, since we still assume a fixed exchange rate. On the other hand, real imports decline in response to the shock, mainly due to a fall in domestic activity as well as due to a fall in domestic prices. The overall contraction in real economic activity also affects the unemployment rate, which increases by almost 1 percent from 2017 to 2021.

Focusing on the financial balances, figure 48 shows that among the domestic sectors, FCs experience a considerable improvement in their financial balance following an interest rate shock. The financial balance of NFCs falls due to aggregate demand contraction and increased interest payments. The government balance also falls due to falling tax revenues and increased interest payments. Overall, positive interest rate shocks will adversely affect the financial balances of sectors with a negative stock of interest-bearing assets (i.e., NFCs, government, and the rest of the world) and positively affect the financial balances of sectors with a positive net stock of interest-bearing assets (FCs).

We now focus on the household sector, which holds both interest-bearing assets and liabilities. Households’ real disposable income drops as a result of the fact that the stock of interest-bearing liabilities exceeds the stock of interest-bearing assets. Moreover, the adverse effect on disposable income is further induced by a fall in aggregate demand and an increase in the rate of unemployment, as shown in figure 47.

Figure 47: Shock 2, Unemployment Rate

Source: Simulated data.
Figure 48: Shock 2, Financial Balances

Source: Simulated data.

In the short term, the financial balance is negative because the fall in disposable income exceeds the fall in households consumption and investment. However, the effect of the shock on net lending turns positive after a year, mainly due to a fall in household investment, i.e., the improvement in net lending is due to deleveraging.

VI. CONCLUSION

This paper emphasizes the need for understanding the interdependencies between the real and financial sides of the economy in macroeconomic models. While the real side of the economy is generally well explained in macroeconomic models, the financial side of the economy and its interaction with the real economy remains poorly understood. This is partly due to the complexity of financial systems, but more importantly due to methodological neglect of an active financial system in the macroeconomic tradition. This paper makes an attempt to model the interdependencies between the real and financial sides of the Danish economy while adopting a stock-flow-consistent approach. The model is estimated using Danish data for the period 1995–2016.
The model is simulated to create a baseline scenario for the period 2017–30, against which the effects of two standard shocks (fiscal and interest rate) are analyzed. An increase in public consumption affects the economy through different channels. Specifically, a fiscal shock increases domestic demand and adversely affects the current account balance. It also deteriorates the public balance, resulting in a higher level of public sector debt, which in turn improves financial net wealth in the production and foreign sectors. An increase in the interest rate has the effect of contracting aggregate demand; however, the overall effect on output is not so strong. Overall, positive interest rate shocks adversely affect the financial balances of the sectors with a negative stock of interest-bearing assets (i.e., NFCs, government, and the rest of the world) and positively affect the financial balances of the sectors with a positive net stock of interest-bearing assets (FCs). The results of these two shocks are in line with the theoretical arguments.

While the model structure is fairly simple due to different constraints, the use of the stock-flow approach makes it possible to explain the accumulation of different individual financial assets and liabilities in a meaningful way. The financial and real sides of the economy are tied together via net lending (or financial balances), where a surplus on the real side of the economy requires a net accumulation of financial assets, just like a deficit on the real side of the economy requires a net accumulation of financial liabilities. Hence, the real economy has a direct effect on balance sheets, and these balance sheets have a feedback effect on the real side of the economy.

Finally, in common with all studies, our analyses are subject to several limitations. In particular, the supply of credit needs to be explained, since credit creation in our model is demand driven. Moreover, there are no policy reactions in the economy. Hence, the results drawn from this model need to be interpreted with great caution.
REFERENCES


APPENDIX

Consumption equation:

\[
\ln(c_t) = -0.007** + \ln(c_{t-1}) + 0.23\Delta \ln(c_{t-1}) + 0.51*** \Delta \ln(yd_t^H) \\
+ 0.38** \Delta \ln(yd_{t-2}^H) + 0.09** \Delta \ln(nw_{t-1}^H)
\]

Investment by NFC sector:

\[
\ln(i_t^N) = 0.03*** + \ln(i_{t-1}^N) - 0.41** \Delta \ln(i_{t-1}^N) + 3.23*** \Delta \ln(k_{t-1}^N) - 0.25***D_{2009}
\]

The level of investment by the household sector is estimated as follows:

\[
\ln(i_t^H) = -0.11** + \ln(i_{t-1}^H) - 0.3* \Delta \ln(i_{t-1}^H) + 2.54**\left(\frac{p_t^H}{P_t}\right) \\
+ 1.91***\left(\frac{p_{t-1}^H}{P_{t-1}}\right) + 2.6***\ln(yd_t^H) + 3.19***\ln(yd_{t-2}^H)
\]

The level of exports and imports:

\[
\ln(x_t) = 13.73*** - 0.47\ln\left(\frac{p_{x_t}^L}{P_{t-1}}\right) + 0.87***\ln(y^w)
\]

\[
\ln(m_t) = -12.16*** + 0.09**\ln\left(\frac{p_{m_t}^L}{P_{t-1}}\right) + 1.76***\ln(c_t + i_t + x_t) + 0.05***D_{2009}
\]

The demand for new equities is determined by the return on equities and the return on interest-bearing assets:

\[
EQATR_t^H = 427062^*\chi_t - 581223**r_{t-1}^H + 0.23***IBLTR_t^H - 59072***D_{2007} - 64431***D_{2010}
\]

The transaction for pensions is determined by:

\[
PENATR_t^H = 0.24^*PENATR_{t-1}^H - 212033.2 + 2714501.5^*(\psi_t) + 0.16^*WB_t^H
\]
The demand for new interest-bearing liabilities is determined by:

\[ IBLTR_t^H = 1.99*** t_t^H - 0.05*** (IBL_t^H) + 0.67*** (FATR_t^H) - 270042^* t_t^H \]

Total gross operating surplus:

\[ B_{zt} = 175119.1^{**} + 0.189^{**} Y_t + 6362.97^{**} t - 53422.90^{**} D_{2009} \]

Social benefits:

\[ \ln(SBEH_t) = 0.59^{**} \ln(SBEH_{t-1}) + 0.06^{**} \ln(U) + 0.88^{**} \ln(W_{t-1}) - 0.01^{**} t \]

Price of consumption goods:

\[ \ln(P_c^t) = -0.0006^{**} + \ln(P_{c-1}) + 0.462^{**} \ln(\Delta P_{c-1}) + 0.462^{**} \ln(\Delta W_{t-2}) + 0.10^{**} \Delta \ln(P_m^t) \]

+ 0.008^{**} D_{2008}

Price of export goods:

\[ \ln(P_x^t) = 0.0028^{**} + \ln(P_{x-1}) + 0.040^{**} \ln(\Delta P_{x-2}) + 1.05^{**} \ln(\Delta P_m^t) + 0.269^{**} \Delta \ln(ULC_{t-1}) \]

Number of employed individuals:

\[ \ln(N) = -4.54^{**} + 0.232^{**} \ln(Y_{t-1}) + 1.148^{**} \ln(LF) - 0.0016^{**} t \]

Wage rate:

\[ W_t = 0.822^{**} + W_{t-1} + 0.426^{**} \Delta(W_{t-1}) + 0.453^{**} \Delta(W_{t-2}) - 101.47^{**} \Delta(UR) + 88.98^{**} \Delta(UR_{t-1}) - 126.26^{**} \Delta(UR_{t-2}) - 4.691^D_{2011} \]
Figure A1: Error Terms

- Error in equity income (% of GDP)
- Error in interest bearing income (% of GDP)
- Error in pension income (% of GDP)

Source: Simulated data.