A Simple Model of the Long-Term Interest Rate

by

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ABSTRACT

This paper presents a simple model of the long-term interest rate. The model represents John Maynard Keynes’s conjecture that the central bank’s actions influence the long-term interest rate primarily through the short-term interest rate, while allowing for other important factors. It relies on the geometric Brownian motion to formally model Keynes’s conjecture. Geometric Brownian motion has been widely used in modeling interest rate dynamics in quantitative finance. However, it has not been used to represent Keynes’s conjecture. Empirical studies in support of the Keynesian perspective and the stylized facts on the dynamics of the long-term interest rate on government bonds suggest that interest rate models based on Keynes’s conjecture can be advantageous.

KEYWORDS: Long-Term Interest Rate; Bond Yields; Monetary Policy; Short-Term Interest Rate; John Maynard Keynes

JEL CLASSIFICATION: E12; E43; E50; E58; E60; G10; G12; G41
1. INTRODUCTION

John Maynard Keynes conjectured that a country’s central bank has a decisive influence on the long-term interest rate on risk-free government bonds, mainly through its policy rate and monetary policy actions. He argued that the central bank’s decisions on the policy rate set the short-term interest rate, which has a crucial effect on the long-term interest rate of risk-free government bonds. Keynes’s views on the central bank’s influence on government bond yields and the shape of government bond yield curves were articulated in his *Treatise on Money* (1930a, 1930b) and *General Theory* ([1936] 2007).

Keynes’s conjecture on interest rates is based on both theory and stylized empirical facts (Kregel 2011). The theoretical basis rest on his analysis of the central bank’s policy rate, open market operations, and balance sheet policies (Keynes 1930a, 185–220; 1930b, 362–364, 369–373), as well as his theory of interest rates (Keynes 1930b, 352–361; [1936] 2007, 165–185, 222–244) and liquidity preference (Keynes [1936] 2007, 194–209). The empirical basis for the conjecture comes from Riefler’s (1930) statistical study of money markets and bond markets in the United States in the 1920s and Keynes’s (1930b, 355–356) observations of the same markets in the United Kingdom during that period.

In recent years, the Keynesian approach to modeling long-term government bond yields has been revived and revitalized. The Keynesian approach has contested the conventional position that relies on the loanable funds theory of interest rates, which holds that increased fiscal deficit and higher government debt ratios exert upward pressures on government bond yields, and that higher deficit and debt ratios increase the likelihood of sovereign debt defaults. Reinhart and Rogoff (2009) embody and epitomize the conventional position; Simoski (2019, 8–21) provides a critical review of both the conventional and the Keynesian approaches.

The Keynesian approach, however, has not yet deployed the substantial theoretical work on interest rate modeling that exists in the quantitative finance literature. In the same vein, the existing quantitative finance literature has yet to appropriate Keynes’s valuable insights on the dynamics of the long-term interest rate based on his deep and original analysis of financial
markets, investors’ long-term expectations, and institutional features of advanced capitalism, even though there is a growing body of literature that provides empirical support for the Keynesian approach.

This paper addresses this crucial gap in interest rate modeling. It develops a Keynesian model of interest rate dynamics based on geometric Brownian motion using stochastic differential equations. This paper also relates the long-term interest rate, the short-term interest rate, the central bank’s policy rate, other relevant macroeconomic variables, and stochastic shocks in a simple model that provides a useful understanding of interest rate dynamics based on Keynes’s insights.

The paper progresses as follows. Section 2 briefly describes Keynes’s view on interest rates. Section 3 presents a model based on the geometric Brownian motion in the quantitative finance literature to represent Keynes’s conjecture that relates the central bank’s decisive influence on the long-term interest rate through its policy rate, the short-term interest rate, and other relevant variables. Section 4 draws attention to some stylized facts that show the evolution of interest rates in advanced countries are consistent with the simple Keynesian model. Section 5 discusses the policy relevance of Keynes’s conjecture in contemporary debates in macroeconomic theory and policy issues. Section 6 provides a conclusion.

2. KEYNES’S VIEWS ON INTEREST RATES

Keynes was acutely aware of the institutional features of capitalist production processes and financial systems, as pointed out in Kregel (1980). Thus, he understood the central bank’s ability to influence the long-term interest rate and the shape of the yield curve in actual capitalist economies with money, financial assets, and financial institutions. In actual capitalist economies, investors who are engaged in business investment, financing, and financial speculation play a critical role. He stated:
a. “The efficacy of Bank-rate for the management of a managed money was a great discovery and a most novel one … its precise modus operandi and the varying results to be expected from its application in varying conditions were not clearly understood—and have not been clearly understood … down to this day.” (Keynes 1930a, 17)

b. “The main, direct influence of the Banking System is over the short-term rate of interest. But when it is a question of controlling the rate of investment, not in working capital but in fixed capital, it is the long-term rate of interest which chiefly matters.” (Keynes 1930b, 352)

c. “[E]xperience shows that … the influence of the short-term interest rate on the long-term rate is much greater than anyone … would have expected. … [T]here are some sound reasons based on the technical character of the market, why it is not unnatural that this should be so.” (Keynes 1930b, 353)

d. “[S]hort-term rates influence long-term rates more than the reader might expect … it is not difficult to find sufficient explanation for this observed fact.” (Keynes 1930b, 362)

e. “[T]here is no reason to doubt the ability of a Central Bank to make its short-term rate of interest effective in the market.” (Keynes 1930b, 363)

f. “It should not be beyond the power of a Central Bank (international complications apart) to bring down the long-term market rate of interest to any figure at which it is itself prepared to buy long-term securities.” (Keynes 1930b, 371)

g. “[T]he long-term market rate of interest will depend, not only on the current policy of the monetary authority, but also on the market expectations concerning its future policy. The short-term interest rate is easily controlled by the monetary authority. … But the long-term rate may be more recalcitrant.” (Keynes [1936] 2007, 202–203)

h. “If the monetary authority were prepared to deal both ways on specified terms in debts of all maturities, and even more so if it were prepared to deal in debts of varying degrees of risk, the relationship between the complex rate of interest and the quantities of money would be direct.” (Keynes [1936] 2007, 205)

i. “[A] complex offer by the central bank to buy and sell at stated prices gilt-edged bonds of all maturities, in place of the single bank rate for short-term bills, is the most important practical improvement which can be made in the technique of monetary management.” (Keynes [1936] 2007, 206)

j. “If the monetary authority deals only in short-term debts, we have to consider what influence the price, actual and prospective, of short-term debts exercise on debts of longer maturity.” (Keynes [1936] 2007, 206)

Keynes argued that the interest rate is a return for the willingness to give up holding cash or bank money rather than a return to saving or waiting. He repudiated the loanable funds theory. Liquidity preference is the basis for interest rates. It arises because of: (1) “uncertainty as to the future of the rate of interest”; and (2) difference among investors regarding their assessments of the uncertain and unknown future. He classifies several motives for holding liquid assets: (1) income motive; (2) business motive; (3) precautionary motive; (4) speculative motive; and (5) finance motive.

Keynes points out that since the central bank controls the policy rate, it has direct influence on the short-term interest rate. However, he also argues that the short-term interest rate has a decisive and noticeable effect on the long-term interest rate, much more than appears warranted. He wrote: “For whilst it is reasonable that long-term rates should have a definite relation to the prospective short-term rates, quarter-by-quarter, over the years to come, the contribution of
current three-monthly period to this aggregate expectation should be insignificant in amount—so one might suppose” (Keynes 1930b, 352).

However, the influence of the current short-term interest rate on the long-term interest rate is much larger than a simple model of the long-term interest rate. In that model, it is a function of the current short-term interest rate and the expected path of the future short-term interest rate or forward rates. The current short-term interest rate is only one factor in that model, and its role would be fairly limited compared to the future short-term interest rates or forward rates.

Keynes (1930b, 353–362; [1936] 2007, 147–164) explains this apparent anomaly in terms of various technical characteristics of financial markets and institutional attributes of banks, money managers and financial institutions, investors’ long-term expectations, psychological conventions and sociological considerations, and rational herding in financial markets. Above all, fundamental uncertainty prevents investors from having well-defined mathematical expectations about for the future and investment opportunities.

The influence of the short-term interest rate on long-term interest rates has been well-documented in various studies on government bond yields. Table 1 provides references documenting this relationship.

**Table 1: Recent Studies That Document the Connection Between the Short-Term Interest Rate and the Long-Term Interest Rate**

<table>
<thead>
<tr>
<th>Country/Region</th>
<th>Studies</th>
</tr>
</thead>
<tbody>
<tr>
<td>United States</td>
<td>Akram and Li (2016, 2017, 2019b); Akram and Das (2019b); Levrero and Deleidi (2019)</td>
</tr>
<tr>
<td>Eurozone</td>
<td>Akram and Das (2017)</td>
</tr>
<tr>
<td>Japan</td>
<td>Akram and Das (2014), Akram and Li (2018, 2019a)</td>
</tr>
<tr>
<td>Canada</td>
<td>Akram and Das (2020)</td>
</tr>
<tr>
<td>Australia</td>
<td>Akram and Das (Forthcoming)</td>
</tr>
<tr>
<td>Latin America</td>
<td>Simoski (2019), Akram and Uddin (2020)</td>
</tr>
<tr>
<td>India</td>
<td>Vinod, Chakraborty, and Karun (2014); Akram and Das (2015, 2019a); Patra, Pattanaik, and Behera (2016)</td>
</tr>
<tr>
<td>Others</td>
<td>Malliaropoulos and Migiakis (2018)</td>
</tr>
</tbody>
</table>
3. A KEYNESIAN MODEL OF THE LONG-TERM INTEREST RATE DYNAMICS

Several of the well-known interest rate models in the quantitative finance literature are analytically simple and elegant. The best known of these interest models are by Vasicek (1977), Dothan (1978), Cox, Ingersoll, and Ross (1985), Heston (1993), Ho and Lee (1986), Hull and White (1990a, 1990b), Black, Derman, and Toy (1990), Heath, Jarrow, and Morton (1992), and Brace, Gatarek, and Musela (1997). Rebonato (1996, 2004) provides a detailed summary and overview of most of them.

None of these models, however, incorporate Keynes’s fundamental insights and observations about the relationship between the central bank’s monetary policy actions and the long-term interest rate. In recent years, research on the long-term interest rate, inspired by the Keynesian approach, has been renewed. These empirical studies—including those cited in section 2—are still disconnected from the interest rate models in the quantitative finance literature.

Hence, the model presented here attempts to bridge the gap between Keynes’s insightful observations about interest rate dynamics and financial markets, and the interest rate modeling literature in quantitative finance. It does so by emphasizing the connections between the central bank’s policy rate, the short-term interest rate, the long-term interest rate, and the overall macroeconomy. The model below is quite similar to that in Heston (1993) but has incorporated some crucial insights drawn from Keynes (1930a, 1930b, [1936] 2007).

Notation
The long-term interest rate is $r_{LT}$, the short-term interest rate is $r_{ST}$, and the central bank policy rate is $r_{CB}$. Volatility is $V$ and various macroeconomics factors, such as the rate of core inflation, the growth of industrial production, and the ratio of government debt to nominal GDP, are represented as $W_i$. The correlation between the Weiner process, $dz$, and the $i$th macroeconomic factor is $\rho_i$. 
Equations

The interest rate model is represented in the following four equations:

\[ dr_{LT} = \mu r_{ST} dt + \sqrt{V} r_{ST} dz \]  

\[ dr_{ST} = \alpha (r_{CB} - r_{LT}) dt + \varepsilon_t \]  

\[ dV = \kappa (\theta - V) dt + \sigma \sqrt{V} \sum_{i=1}^{N} dW_i \]  

\[ dzdW_i = \rho_i dt \forall i \]

Equation 1 states that the long-term interest rate follows a geometric Brownian motion that satisfies the above stochastic differential equation. Here, \( dz \) is a Weiner process; \( \mu \) is the drift, and \( V \) is the volatility. Both the drift and the volatility terms are constant.

Equation 2 states that the short-term interest rate, \( r_{ST} \), is a mean reverting function of the central bank’s policy rate, \( r_{CB} \), at a pace of \( \alpha \). Here, \( \varepsilon_t \) is an error term.

Equation 3, the equation for volatility, implies that the volatility is a mean reverting to \( \theta \) at a rate set by \( \kappa \). Here, \( \sigma \) is the standard deviation of the volatility and \( W_i \) is a random variable that represents the shock from the \( i \)th macroeconomic variable.

Equation 4 indicates that \( \rho_i \) is the correlation between the Weiner process, \( dz \), and the \( i \)th macroeconomic factor, \( dW_i \).
4. STYLIZED FACTS

The short-term interest rate very closely tracks the central bank’s policy rate. The long-term interest rate generally moves in lockstep with the short-term interest rate. The figures below reveal these stylized facts about the dynamics of interest rates in major advanced countries and regions, including the United States (figure 1), Canada (figure 2), the eurozone (figures 3 and 4), the United Kingdom (figure 5), and Japan (figure 6), along with selected emerging markets, such as China (figure 7), India (figure 8), Brazil (figure 9), and Mexico (figure 10). Even though the short-term interest rate and the central bank’s policy rate are very strongly correlated, and the long-term interest rate and the short-term interest rate are strongly correlated, there are some random variations in the underlying trends due to a wide range of incoming macroeconomic information, technical characteristics of financial markets, policy pronouncements and regulatory changes, policy uncertainty, business cycles, and other factors.

Figure 1: The Evolution of Key Interest Rates in the United States, 2000–19

Source: Macrobond
Figure 2: The Evolution of Key Interest Rates in Canada, 2000–19

![Canada Overnight Rate and Government Benchmark Yields](chart1)

Source: Macrobond

Figure 3: The Evolution of Key Short-Term Interest Rates in the Eurozone, 2000–19

![Euro Zone, ECB Policy Rates and EONIA](chart2)

Source: Macrobond
Figure 4: The Evolution of Long-Term Interest Rates in Major Eurozone Economies, 2000–19

![Graph showing the evolution of long-term interest rates in Major Eurozone Economies, 2000–19.](image)

Source: Macrobond

Figure 5: The Evolution of Key Interest Rates in the UK, 2000–19

![Graph showing the evolution of key interest rates in the UK, 2000–19.](image)

Source: Macrobond
Figure 6: The Evolution of Key Interest Rates in Japan, 2000–19

![Graph showing the evolution of interest rates in Japan from 2000 to 2019. The graph includes the BOJ Policy Rate, Government Benchmark Yields, and other relevant interest rates. Source: Macrobond]

Source: Macrobond

Figure 7: The Evolution of Key Interest Rates in China, 2000–19

![Graph showing the evolution of interest rates in China from 2000 to 2019. The graph includes the China PBOC Policy Rate, Government Benchmark Yields, and other relevant interest rates. Source: Macrobond]

Source: Macrobond
Figure 8: The Evolution of Key Interest Rates in India, 2000–19

Figure 9: The Evolution of Key Interest Rates in Brazil, 2010–19
5. POLICY RELEVANCE

The simple model for the long-term interest rate developed here is relevant not just for macroeconomic and finance theory but also for monetary and fiscal policy. In this model, a higher (lower) short-term interest rate is associated with a higher (lower) long-term interest rate on government bonds. The central bank affects government bond yields through policy rates, such as the overnight financing rate, the interest rate on reserves, and so forth. The central bank’s policy rate decision is affected by its statutory mandates, inflationary pressures, inflation expectations, current economic and financial conditions, and its forecast of the economic outlook and the balance of risks. In the final analysis, the central bank is a key driver of the long-term interest rate and the shape of the yield curve. Under monetary sovereignty, as defined in Wray (2012), the central bank has the operational ability and flexibility to effectively influence the long-term interest rate on government bond yields on government debt in local currency, provided that a regime of floating exchange rate is maintained.
This model emphasizes the fundamental role of the central bank in influencing the long-term interest rate and the shape of the Treasury yield curve, in contradistinction to the loanable funds theory, which holds that the interest rate is simply the price of loanable funds and depends on the demand and supply of funds. This model of the interest rate can be enhanced and extended to illuminate policy issues and discussions related to government debt management, fiscal sustainability, fiscal policy, and the central bank’s ability to control long-term interest rates on government bonds. It also can be used to assess the efficacy of monetary policy and the monetary transmission mechanism.

A policy framework that recognizes the Keynesian approach and models the dynamics of interest rates accordingly can contribute to various debates, such as those related to the following:

(1) The effects of government fiscal variables on government bond yields (Ardagna, Caselli, and Lane 2007; Gruber and Kamin 2012; Horioka, Nomoto, and Terada-Hagiwara 2014; Hoshi and Ito 2013; Jaramillo and Weber 2013; Min et al. 2003; Poghosyan 2014; Tkačevs and Vilerts 2019; Turner 2002);
(2) asset prices (Kurihara 2015);
(3) operational issues on central banking and government debt management (Bindseil 2004; Das et al. 2010; Fullwiler 2016, [2008] 2017; Mattos et al. 2019);
(4) fiscal theory of price (Bölükbaş 2018; Sims 2013);
(5) functional finance (Lerner 1943);
(6) fiscal and monetary policy (Sau 2018); and
6. CONCLUSION

This paper presents a simple model of the long-term interest rate. This model is based on Keynes’s conjecture that the central bank’s actions influence the long-term interest rate primarily through the short-term interest rate. The central bank’s policy rate has a decisive influence on the short-term interest rate, according to Keynes. The short-term interest rate, along with central bank actions and various macroeconomic factors, is a key driver of the long-term interest rate. The model applies geometric Brownian motion, as represented in stochastic differential equations—which are widely used in modeling interest rate dynamics in quantitative finance—to represent Keynes’s conjecture. Various empirical studies and the stylized facts on the evolution of the long-term interest rate on government bonds in advanced countries and selected emerging markets support Keynes’s conjecture. This suggests that a simple model of the long-term interest rate, based on the Keynesian approach, might be useful not merely for understanding interest rate dynamics but also for addressing a wide range of theoretical questions in macroeconomics and finance, and in contemporary debates on monetary, fiscal, and other economies policies.
REFERENCES


