

**A cautionary tale**  
**Consistency on the estimation of Indices with**  
**Endogenous weights**

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In **economics** and other **social sciences**, we are often interested in constructing indices that help us measure concepts that are otherwise difficult to identify.

This may happen because:

- the inner complexity of the topic, ie attitudes or preferences,
- because the data to measure those topics could be hard to capture from the interviewees (wealth)

When this occurs, we often rely on proxy variables or instruments, that help identify these measures indirectly.

In general, these composite indices can be constructed using a large set of methodologies:

- Principal component analysis
- Factor analysis
- Item Response Theory
- Structural equation analysis
- and Latent variable analysis, to name a few.

Some of these methods fall within (unsupervised) Machine Learning methods, some within data-reduction methods, while others focus on the identification of unobserved latent parameters.

Most of these strategies construct the desired index using a kind of weighted average:

$$I(v_1, v_2, \dots, v_k) = \sum_{k=1}^K w_k f(v_k)$$

$f()$  is some monotonic transformation of  $v$ , and  $w_k$  are weights given to different variables/instruments.

These weights can be defined Exogenously or endogenously defined (***Optimal weights***).

# How are weights defined?

Exogenous weights, defined by expert opinions or expert consensus.

- Using individual variables/instruments, assumes  $w_k = 1$  for the variable of interest, and 0 otherwise.

Endogenous variables, instead, assign higher weights to variables with higher inter-correlation. Uncorrelated variables may receive 0 weight.

The advantage: They will be able to exploit intercorrelations otherwise ignored by the researcher.

# How are endogenous weights defined?

Assume that ALL instruments have the following structure:

$$v_k = g(l^*) + \varepsilon_k$$

where  $l^*$  is the latent variable of interest, and  $\varepsilon_k$  an uncorrelated component.

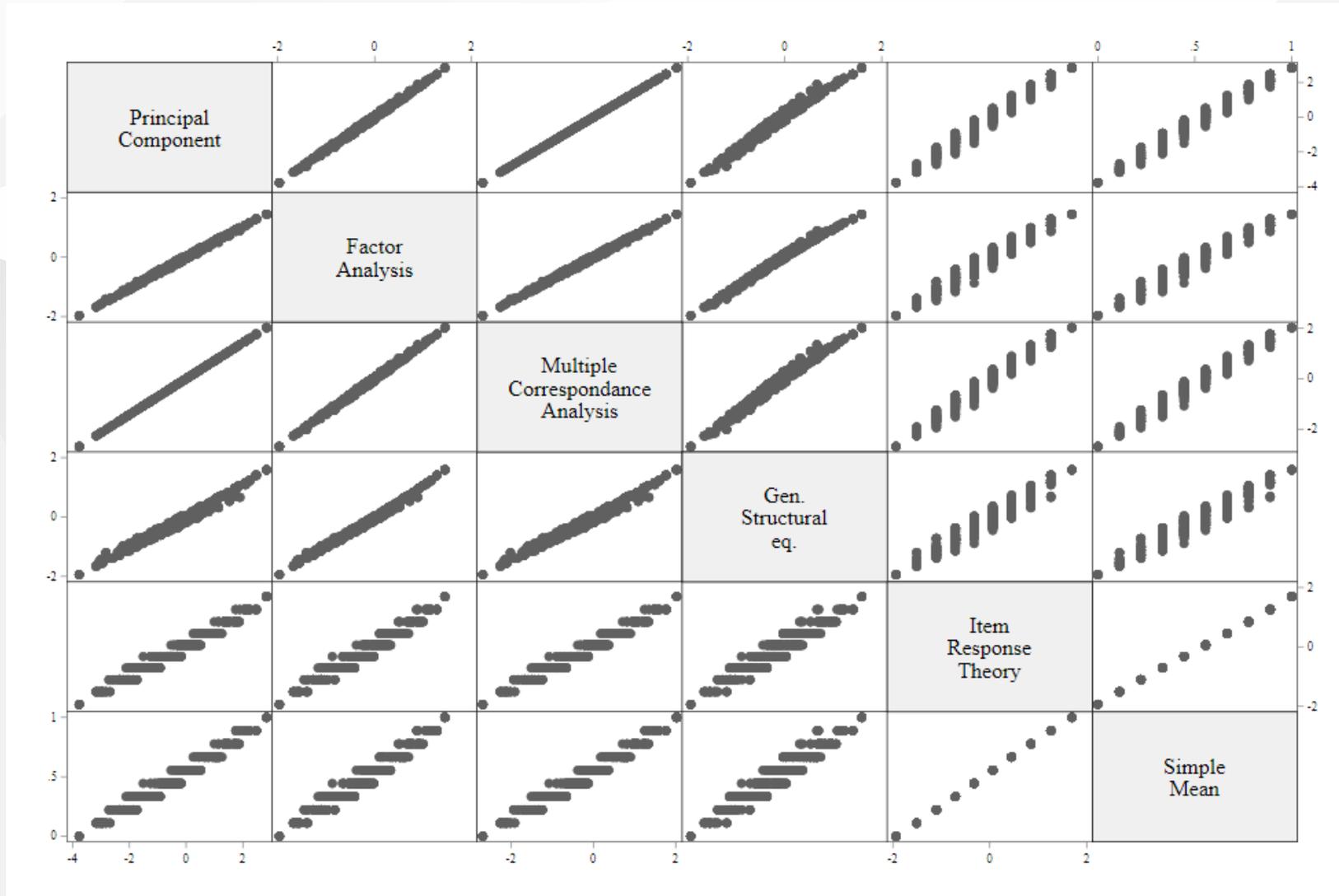
- If the only common factor is given by  $l^*$ , covariances can be used to define these weights.
- In other words, weights are such that:

$$(\omega = w_1, \dots, w_k) = \text{Min}_\omega (I(\omega) - l^*)^2$$

If a clear common *latent* component exists, there will be strong consistency across constructed indices regardless of the method used for its identification.

Namely, all methods will identify similar indices, which would be highly correlated.

# Indices constructed based on simulated data:



## Why is this a problem?

In most analyses, the assumption of a single common factor may not be necessarily true.

Consider the case where the instruments,  $v_k$ , are caused by multiple latent variables :

$$v_k = \sum \theta_q^k l_q^* + \varepsilon_k$$

where  $\theta_q^k$  is the weight latent component  $l_q^*$  is given on instrument  $v_k$

In this case, weights based on cross-variable correlations cannot be used to identify a single latent variable.

# Why is this a problem?

Theoretically, endogenous weights will identify a composite latent factor.

- Some of these latent factors may be related to the topic of interest.
- Some may be completely unrelated
- Yet, some may be driven by systematic errors (how data was collected)

The latter is particularly common for the analysis of topics regarding subjective topics (ie preferences or attitudes) due to systematic measurement errors during data collection.

# Consequences?

Different methodologies define weights  $w_k$  differently.

Thus, different methodologies may identify different indices, which represent a combination of latent indices.

Composite indices will appear to be inconsistent across methodologies, showing low correlations across constructed indices.

$$I = \sum \omega_k v_k \rightarrow \sum \omega_k \left( \sum \theta_q^k l_q^* + \varepsilon_k \right)$$

# The case of Ethiopia

Data: Afrobarometer - Ethiopia 2013

Index: Patriarchal views

Questions:

- Men only as leaders vs. women leaders OK
- Women have equal rights vs. subject to traditional laws
- Education recipient priority: boy vs ability



# Solutions

No clear solution exists.

1. Better selection/collection of instruments, as well as better question framing and data quality.
2. Instruments may need cross-validation to ensure one is measuring the dimensions of interest.
3. Consider exogenous weights, consistent with work previously done in the literature, if such exists.
4. When no consensus exists, consider using multiple measures as a robustness check.

Endogenous weights are useful to start with but are important to verify results align with expectations and common sense.