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The Financial Instability Hypothesis: a Stochastic Microfoundation Framework

Carl Chiarella and Corrado Di Guilmi

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The Hyman P. Minsky Conference June 29, 2010 - Levy Economics Institute

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| The context           |                   |                     |                      |                         |              |
| Aim                   |                   |                     |                      |                         |              |

• To consistently microfound the models by Minsky (1975) and Taylor and O'Connell (1985) in which investments drive instability.

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- Minsky (1963): Financial Instability Hypothesis:
  - Different types of economic units with respect to their financial soundness;

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 "Shifts of firms among classes as the economy evolves in historical time underlie much of its cyclical behavior. This detail is rich and illuminating but beyond the reach of mere algebra" [Taylor and O'Connell, 1985].

- Two different methods for model solution:
  - the agent based model with numerical simulation;
     the stochastic dynamic aggregation framework

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    - [Aoki and Yoshikawa, 2006].

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- Two different methods for model solution:
  - the agent based model with numerical simulation, the stochastic dynamic appropriation framework.
    - [Aoki and Yoshikawa, 2006].

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- The context
- Agent based modelHypotheses
- 3 Stochastic dynamics
  - Set up
  - Master equation
- ④ Simulations
  - Results
- 5 Concluding remarks
  - Future research



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"The system is pretty sound. It only collapses every three or four years"

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- Minsky (1975): firms decide on investment based on the difference between the shadow-price of capital P<sub>k</sub> and its selling price P<sub>i</sub>;
- Taylor and O'Connell (1985):
  - P<sub>k</sub> = f(ρ): ρ is the expected difference of anticipated return to capital with respect to the current level;
  - $\rho$  influences the demand for equities.
- Our contribution:
  - $\rho^j$  is the expected difference of return to capital for the firm j with respect to a common minimum level;

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| Hypotheses            |                             |                     |                      |                         |              |
| Firms                 |                             |                     |                      |                         |              |

$$I^{j}(t) = aP_{k}^{j}(t)$$

)

where

• the shadow-price of capital is

$$P_{k}^{j}(t) = \frac{(r(t) + \rho^{j}(t))P}{i(t)}$$
(2)

- ρ<sup>j</sup> is the expected difference of return to capital for the firm j with respect to the minimum level r;
- *i* is the interest rate, *P* is the final good price and *a* is a parameter.

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| Hypotheses   |                   |                     |             |                    |              |

- Firms prefer to finance their investments:
  - first with retained earnings  $A^{j}$  and, then
  - with new equities  $E^j$  or debt  $D^j$  (in a proportion dependent on the level of interest rate)
- Firms are classified into two groups according to their level of debt  $D^j$ :
  - state z = 1: speculative firms:  $D^{j}(t) > 0$
  - state z = 2: hedge firms:  $D^{j}(t) = 0$
- Correspondingly, there are two types of shares in the market, with prices  $P_{e,1}$  and  $P_{e,2}$ .

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| Hypotheses   |                   |                     |             |                    |              |

- A firm fails if  $D^{j}(t) > cK^{j}(t)$ , with c > 1.
- The probability of new firm entering the system is directly proportional to the variation in the aggregate output observed in the previous period.

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| Hypotheses            |                             |                     |                      |                         |              |
| Investor              | rs                          |                     |                      |                         |              |

- Two possible types of investors: chartists (proportion n<sup>c</sup>) and fundamentalists (proportion 1 - n<sup>c</sup>);
- chartists on average favour the speculative firms, so a higher proportion of chartists implies a higher ρ for speculative firms;
- the proportion of chartists in the market *n<sup>c</sup>* is randomly drawn in each period from a uniform distribution.

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| Investor              | ſS                          |                           |                      |                         |              |

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| Hypotheses            |                             |                           |                      |                         |              |
| Wealth                | allocation                  |                           |                      |                         |              |

$$\begin{pmatrix} \epsilon_1(i,\rho_1,\rho_2,\psi)W = \mathbf{P}_{e,1}E_1\\ \epsilon_2(i,\rho_1,\rho_2,\psi)W = \mathbf{P}_{e,2}E_2\\ \beta(\mathbf{i},\rho_1,\rho_2,\psi)W = D\\ \Psi(i,\rho_1,\rho_2,\psi)W = \mathbf{M}\\ \mathbf{W} = P_{e1}E_1 + P_{e2}E_2 + D + M \end{cases}$$
(3)

- the parameter  $\psi$  reflects the preference for liquidity and the capacity of the system to generate endogenous money;
- *i* is the interest rate, *M* the demand for money, *D* the debt and *E*<sub>1</sub>, *E*<sub>2</sub> are the quantity of shares.

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| Hypotheses            |                             |                           |                      |                         |              |
| Wealth                | allocation                  |                           |                      |                         |              |

$$\begin{cases} \epsilon_{1}(i, \rho_{1}, \rho_{2}, \psi)W = \mathbf{P}_{e,1}E_{1} \\ \epsilon_{2}(i, \rho_{1}, \rho_{2}, \psi)W = \mathbf{P}_{e,2}E_{2} \\ \beta(\mathbf{i}, \rho_{1}, \rho_{2}, \psi)W = D \\ \Psi(i, \rho_{1}, \rho_{2}, \psi)W = \mathbf{M} \\ \mathbf{W} = P_{e1}E_{1} + P_{e2}E_{2} + D + M \end{cases}$$
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| Hypotheses            |                             |                           |                      |                         |              |
| Wealth                | allocation                  |                           |                      |                         |              |

$$\begin{cases} \epsilon_{1}(i,\rho_{1},\rho_{2},\psi)W = \mathbf{P}_{\mathbf{e},1}E_{1}\\ \epsilon_{2}(i,\rho_{1},\rho_{2},\psi)W = \mathbf{P}_{\mathbf{e},2}E_{2}\\ \beta(\mathbf{i},\rho_{1},\rho_{2},\psi)W = D\\ \Psi(i,\rho_{1},\rho_{2},\psi)W = \mathbf{M}\\ \mathbf{W} = P_{\mathbf{e}1}E_{1} + P_{\mathbf{e}2}E_{2} + D + M \end{cases}$$
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| Hypotheses            |                             |                     |                      |                         |              |
| Wealth                | allocation                  |                     |                      |                         |              |

$$\begin{cases} \epsilon_{1}(i,\rho_{1},\rho_{2},\psi)W = \mathbf{P}_{\mathbf{e},1}E_{1} \\ \epsilon_{2}(i,\rho_{1},\rho_{2},\psi)W = \mathbf{P}_{\mathbf{e},2}E_{2} \\ \beta(\mathbf{i},\rho_{1},\rho_{2},\psi)W = D \\ \Psi(i,\rho_{1},\rho_{2},\psi)W = \mathbf{M} \\ \mathbf{W} = P_{\mathbf{e}1}E_{1} + P_{\mathbf{e}2}E_{2} + D + M \end{cases}$$
(3)

- the parameter  $\psi$  reflects the preference for liquidity and the capacity of the system to generate endogenous money;
- *i* is the interest rate, *M* the demand for money, *D* the debt and *E*<sub>1</sub>, *E*<sub>2</sub> are the quantity of shares.

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| Hypotheses            |                             |                     |                      |                         |              |
| The var               | riable $ ho$                |                     |                      |                         |              |

- The key variable for the allocation of wealth is  $\rho^{j}$ . It influences:
  - the level of firms' **investment** through the shadow price  $P_k^j(t) = \frac{(r(t)+\rho^j(t))P}{i(t)}$ ;
  - the prices of **shares**  $P_{e,1}$  and  $P_{e,2}$  in system (3), reflecting the investors' expectations on the different firms.

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| Set up                |                   |                           |                      |                         |              |
| The two               | o dynamics        |                           |                      |                         |              |

- Using the mean field approximations ρ<sub>1</sub> and ρ<sub>2</sub> it is possible to replicate the model for a *representative hedge firm* and for a *representative speculative firm*;
- thus the model is able to generate dynamics in two different ways:
  - an **agent based** approach with *N* different agents;
  - a **stochastic approximation**, with 2 different firms: one "good" and one "stressed".

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| Master equation       |                   |                           |                      |                         |              |

• Replicating the model for the two representative firms, the stochastic dynamics of the the proportion of the two types of firms can be described by a **master equation**:

 $\frac{dp(N_z, t)}{dt} = \text{influx of probability into state } z - \text{outfluxes of probability from } z$ 

- Using the asymptotic solution [Di Guilmi, 2008], the dynamics of the economy can be represented by a dynamical system with two ODEs:
  - one describes the evolution of the proportion of speculative firms;

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Figure: Capital (upper panel) and share of speculative firms (lower panel). Agent based model (black continuous line) and stochastic dynamics (red dashed line).





Figure: Debt/capital ratio (left axes) and aggregate capital (right axis). Simulation of the agent based model.



Figure: Aggregate capital, variance of fluctuations, interest rate and wealth for different values of  $\psi$  (Monte Carlo agent based simulation).

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| Results               |                   |                           |                      |                         |              |



Figure: Aggregate capital, variance of fluctuations, interest rate and wealth for different values of c.

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| Future research       |                   |                     |                      |                    |              |

- the identification of the conditions under which the system generates speculative bubbles and how they burst;
- the integration with an asset pricing model to study the feedback effects from the financial market;
- the study of the effects of the shifting of debt: introduction of banking and public sector.

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