



Working Paper No. 773

Keynes's Employment Function and the Gratuitous Phillips Curve Disaster

by

Egmont Kakarot-Handtke*

Institute of Economics and Law, University of Stuttgart

August 2013

* Correspondence: AXEC-Project, Egmont Kakarot-Handtke, Hohenzollernstraße 11, D-80801 München, Germany; e-mail: handtke@axec.de

The Levy Economics Institute Working Paper Collection presents research in progress by Levy Institute scholars and conference participants. The purpose of the series is to disseminate ideas to and elicit comments from academics and professionals.

Levy Economics Institute of Bard College, founded in 1986, is a nonprofit, nonpartisan, independently funded research organization devoted to public service. Through scholarship and economic research it generates viable, effective public policy responses to important economic problems that profoundly affect the quality of life in the United States and abroad.

Levy Economics Institute
P.O. Box 5000
Annandale-on-Hudson, NY 12504-5000
<http://www.levyinstitute.org>

Copyright © Levy Economics Institute 2013 All rights reserved

ISSN 1547-366X

ABSTRACT

Keynes had many plausible things to say about unemployment and its causes. His “mercurial mind,” though, relied on intuition, which means that he could not strictly prove his hypotheses. This explains why Keynes’s ideas immediately invited bastardizations. One of them, the Phillips curve synthesis, turned out to be fatal. This paper identifies Keynes’s undifferentiated employment function as a sore spot. It is replaced by the structural employment function, which also supersedes the bastard Phillips curve. The paper demonstrates in a formal and rigorous manner why there is no trade-off between price inflation and unemployment.

Keywords: New Framework of Concepts; Structure-centric; Axiom Set; Say’s Regime; Keynes’s Regime; Market Clearing; Full Employment; Product Price Flexibility; Intertemporal Budget Balancing; Multiplier; Trade-Off; Price Inflation; Wage Inflation

JEL Classifications: E12, E24

Keynes had many plausible things to say about unemployment and its causes. His “mercurial mind” (Skidelsky, 2009, p. 57), though, relied on intuition which means in a more critical vein that he held many opinions but could not prove them convincingly. The problem of Keynesianism is not that it lacks micro-foundations but that it lacks its own foundations, that is to say, an agreed upon set of consistent propositions that clearly distinguishes it from all other approaches. This explains why Keynes’s ideas immediately invited bastardizations. One of them, the Phillips curve synthesis, helped to phase out the once dominant Keynesian policy paradigm.

The present paper identifies Keynes’s undifferentiated employment function as a sore spot. The structural employment function as superior alternative establishes not only the relation between employment and effective demand but, in addition, between employment and wage rate, price, and productivity. This comprehensive employment function, which is entirely free of any behavioral assumptions, works in inflationary and deflationary environments and also supersedes the bastard Phillips curve.

It almost goes without saying that the customary formal points of departure have to be abandoned. Behavioral assumptions, rational or otherwise, are not solid enough to be eligible as first principles of theoretical economics (for details see (2013a)). This crucial methodological point is not always duly observed.

By having a vague theory it is possible to get either result. ... It is usually said when this is pointed out, ‘When you are dealing with psychological matters things can’t be defined so precisely’. Yes, but then you cannot claim to know anything about it. (Feynman, 1992, p. 159), see also (Hudik, 2011, p. 147)

Keynesianism does not satisfy scientific criteria (for a formal proof see (2013b)). Hence all endeavors to lay the formal foundation on a new site and at a deeper level actually need no further vindication. The customary behavioral assumptions are indefensible and therefore in the present paper replaced by *structural* axioms.

Structural axiomatization provides a comprehensive employment function. This, in turn, makes it possible to rigorously demonstrate why there never was a trade-off between price inflation and unemployment. In Sections 1 to 3 the formal foundations of Say’s and Keynes’s regimes are explicitly stated and compared. Say’s regime is free of marginalistic preconceptions and fully supplants the general equilibrium model. In Keynes’s regime effective demand determines employment. In Sections 4 and 5 the properties of the original, the bastard version, and the structural Phillips curve are compared. The neoclassical synthesis version is refuted. Section 6

summarizes the consistent formal interconnections of the employment function, the multiplier, and the generalized Phillips curve and spells out the policy implications. Section 7 concludes.

1. BEGIN AT THE AXIOMATIC BEGINNING

There are still economists obsessed with the idea that economic theory must be built on the assumptions of utility-maximizing and profit-maximizing of individual agents. For these economists, the news is that the formal foundations of theoretical economics must be nonbehavioral and epitomize the interdependence of real and nominal variables that constitutes the monetary economy.

The first three structural axioms relate to income, production, and expenditures in a period of arbitrary length. The period length is conveniently assumed to be the calendar year. Transparency demands that we begin with one world economy, one firm, and one product. All quantitative and temporal extensions have to be deferred until the implications of the most elementary economic configuration are fully understood. Axiomatization is about ascertaining the *minimum* number of premises.² Three suffice for the beginning.

Total income of the household sector Y in period t is the sum of wage income, i.e. the product of wage rate W and working hours L , and distributed profit, i.e. the product of dividend D and the number of shares N .

$$Y = WL + DN \quad |t \quad (1)$$

Output of the business sector O is the product of productivity R and working hours.

$$O = RL \quad |t \quad (2)$$

The productivity R depends on the underlying production conditions. The 2nd axiom should therefore not be misinterpreted as a linear production function.

Consumption expenditures C of the household sector is the product of price P and quantity bought X .

² Methodologically this is no different from J. S. Mill's program: "They [Einstein and Dirac] agreed that science was fundamentally about explaining more and more phenomena in terms of fewer and fewer theories, a view they had read in Mill's *A System of Logic*." (Farmelo, 2009, p. 137)

"The basic concepts and laws which are not logically further reducible constitute the indispensable and not rationally deducible part of the theory. It can scarcely be denied that the supreme goal of all theory is to make the irreducible basic elements as simple and as few as possible without having to surrender the adequate representation of a single datum of experience." (Einstein, 1934, p. 165)

$$C = PX \quad |t \quad (3)$$

The axioms represent the pure consumption economy, that is, no investment expenditures, no foreign trade, and no government. All axiomatic variables are measurable in principle. No nonempirical concepts like utility, equilibrium, decreasing returns or perfect competition are put into the premises.

The economic meaning is rather obvious for the set of structural axioms. What deserves mention is that total income in (1) is the sum of wage income and *distributed profit* and not of wage income and profit. Profit and distributed profit are quite different things. Eqs. (1) to (3) are the non-Euclidean economic axioms that Keynes called for (1973, p. 16) but did not supply.

2. PRELIMINARIES: A RIGOROUS FORMAL UNDERPINNING FOR SAY'S REGIME

Keynes made Say's Law the fundamentalistic bone of contention between his new approach and the "classical" mindset (Keen, 2011, p. 210). Say's Law was, of course, never a law in the sense of the natural sciences. The classical economists were so impressed by Newton's achievement that they termed any remotely plausible relationship a law. To obviate confusion and to end the abuse of terms (Hutchison, 1960, p. 63) it is advisable to replace "law" with "regime" which indicates a possible but not a necessary structure of interconnections.

Say struggled to give his basic idea a precise meaning and produced several verbal versions which left ample room for diverse interpretations (Kates, 1998, pp. 180-196). The basic idea, though, is rather straightforward. Since all markets are in a continuous process of clearing there can be no such thing as a general overproduction or deficiency in demand. Left to themselves, markets will remove partial imbalances if prices are allowed to move freely. The most important policy implication of Say's Law was that, if there is unemployment, workers have to accept a lower price for their commodity. For Keynes this point was as infeasible as it was for the classics (1973, p. 17), (Klant, 1994, p. 105). As we shall see, this old recipe for full employment rests on the fallacy of composition.

The idea of overall market clearing is in the following restated in structural axiomatic terms for the simplest economic configuration. Say's Law, the core of general equilibrium theory (Quiggin, 2010, p. 85), has been stated long before it became a habit to couch human behavior in marginalistic terms. A formal restatement that comes as close as possible to the original and, more

important, to reality has to go without marginalism. Homo economicus is not merely unrealistic as has been observed by almost everybody (Stiglitz, 2010, pp. 249-255), but is indefensible for deeper methodological grounds (for details see (2013a)).

Market clearing in period t , then, is defined as equality of quantity produced and sold, i.e.

$O = X$. In a general form we can write for the relation of both quantities:

$$\rho_X \equiv \frac{X}{O} \quad |t. \quad (4)$$

Market clearing is then defined as a sales ratio of $\rho_X = 1$. This is condition #1. From (3) and (2) then follows the market clearing price as:

$$P = \frac{C}{RL} \quad \text{if} \quad \rho_X = 1 \quad |t. \quad (5)$$

The market clearing price for the economy as a whole is determined by consumption expenditure, productivity, and employment. Say's regime implies that the price in the product market is perfectly flexible, such that the market clearing condition is always satisfied. Perfect product price flexibility, as expressed by (5), is condition #2. Price changes exactly mimic changes in nominal demand, therefore business cycles in real output are excluded from the start (cf. (Gordon, 1990, p. 1126)).

Budget balancing is in the simplest case defined as equality of consumption expenditure and total income, i.e. $C = Y$. In a general form we can write for the relation of both nominal magnitudes:

$$\rho_E \equiv \frac{C}{Y} \quad |t. \quad (6)$$

Now, the market clearing price of (5) can be rewritten as:

$$P = \rho_E \frac{W}{R} \left(1 + \frac{DN}{WL} \right) \quad \text{if} \quad \rho_X = 1 \quad |t. \quad (7)$$

Budget balancing is realized with an expenditure ratio of $\rho_E = 1$. This is condition #3. It keeps saving and dissaving out of the picture for the moment. From (7) then follows:

$$P = \frac{W}{R} \left(1 + \frac{DN}{WL} \right) \quad \text{if} \quad \rho_X = 1, \rho_E = 1 \quad |t. \quad (8)$$

To further compactify the equation the distributed profit ratio is defined as:

$$\rho_D \equiv \frac{DN}{WL} \quad |t. \quad (9)$$

In sum: the market clearing price in (10) is higher than unit wage costs $\frac{W}{R}$ provided $\rho_D > 0$. In this case, profit per unit is greater than zero. If distributed profit is zero, i.e. $\rho_D = 0$, then profit per unit is zero and by consequence total profit is zero. It goes without saying that distributed profit cannot be < 0 . By consequence, the business sector as a whole never makes a loss if $\rho_E = 1$. Up to now, Say's regime, stated in structural axiomatic terms, involves:

$$P = \frac{W}{R}(1 + \rho_D) \tag{10}$$

if $\rho_X = 1, \rho_E = 1 \quad |t.$

Note that the market clearing price for the economy as a whole is completely determined by *objective* conditions. There is no room left for speculation about utility maximization, market power, notional demand–supply functions and the like.

The form of (10) invites the interpretation of mark-up pricing. It could be said that the mark-up on unit wage costs $\frac{W}{R}$ seems to be determined by the income distribution which is captured by the distributed profit ratio ρ_D . Since the market clearing price follows from three objective conditions that are constituents of Say's regime this *behavioral* interpretation is beside the point.

Conditions #1 to #3 refer to the product market. What about employment? Taken together, eqs. (9) and (10) assert that if employment L increases the distributed profit ratio ρ_D decreases and the market clearing price falls. In order to neutralize the effect of employment on the market clearing price we at first define the ratio of number of shares to employment:

$$\rho_N \equiv \frac{N}{L} \quad |t. \tag{11}$$

Neutrality is formally expressed as $\rho_N = \text{const.}$ or, in other words, the number of shares is supposed to move with employment. This is condition #4. Under this condition employment may change in any direction without affecting the market clearing price if the other variables in (10), i.e. W, R, D , remain unchanged.

The next question is: how do employment variations affect profit? Monetary profit is defined as difference between sales revenues and costs:

$$Q_m \equiv PX - WL \quad |t. \tag{12}$$

Taking the price from (10) gives:

$$Q_m = WL(1 + \rho_D) - WL \quad \text{if} \quad \rho_X = 1, \rho_E = 1 \quad |t. \quad (13)$$

The profit *ratio*, which is more general than the profit *rate*, is defined as:

$$\rho_Q \equiv \frac{Q_m}{WL} \quad |t. \quad (14)$$

In combination with (13) follows that the profit ratio is, under the given conditions, equal to the distributed profit ratio:

$$\rho_Q = \rho_D \quad (15)$$

$$\text{if} \quad \rho_X = 1, \rho_E = 1 \quad |t.$$

The profit ratio ρ_Q does not change if employment changes. This is due to condition #4. We want, in addition, keep the profit ratio neutral under *all* conditions for a start. To achieve this, the relation of dividend to wage rate is at first defined as:

$$\rho_V \equiv \frac{D}{W} \quad |t. \quad (16)$$

Under the condition #5, i.e. $\rho_V = \text{const.}$, the dividend moves always with the wage rate. With the conditions #4 and #5, the profit ratio in (15) is constant, no matter what happens to employment and the wage rate. Say's regime implies the conservation of the initially given income distribution which is expressed by ρ_D . It should be obvious that this is a limiting case; conditions #4 and #5 may be dropped at any time.

No matter what the level of employment or the wage rate, the profit ratio is always the same according to (15). With regard to the profit ratio the single firm that represents the business sector can therefore remain completely *indifferent* between various employment levels. To break the indifference and to tip the balance in the right direction a behavioral assumption is required (the only one!). It is assumed that, given the profit ratio ρ_Q , the firm prefers a greater absolute profit Q_m . This is condition #6. It says, in other words, that the firm prefers to be larger than smaller if the profit ratio is equal. With regard to employment this implies that the firm grows whenever possible:

$$L \uparrow \quad \text{as long as} \quad u > 0 \quad |t. \quad (17)$$

The firm hires workers at the going wage rate until the labor market is cleared, that is, until there is no more labor supply $L^\theta - L$ at the going wage rate or, in still other words, until the unemployment rate u is zero. The unemployment rate is defined as usual (Blanchard, 2000, p. 118):

$$u \equiv \frac{L^\theta - L}{L^\theta} \quad |t. \quad (18)$$

Condition #6 guarantees full employment in Say's regime. This condition is more general than profit maximization. The latter presupposes decreasing returns. Whether this is the case in the real world is an open question that can never be decided *a priori*. To assume decreasing returns in order to make the profit maximization assumption applicable is a quite ordinary *petitio principii* (Mill, 2006, p. 820). Note that the profit ratio (15) does not depend on the productivity R , that is, on constant, increasing or decreasing returns. The move from unemployment to full employment makes the firm and absolute profit larger but leaves the profit ratio unchanged, no matter how the productivity develops on the way.

What about the other side of the labor market? The real wage follows from (10) as:

$$\frac{W}{P} = \frac{R}{1 + \rho_D} \quad |t. \quad (19)$$

If productivity and the distributed profit ratio stay unchanged the real wage is not altered by a change of employment. In the case of constant returns the real wage is the same on all levels of employment. The real wage, among many other factors, influences the quantity of voluntarily offered working hours L^θ which, however, remains necessarily below some physical maximum:

$$\frac{W}{P} \rightarrow L^\theta \leq L^{\max} \quad |t. \quad (20)$$

The leisure rate is defined as a correspondence to the unemployment rate:

$$l \equiv \frac{L^{\max} - L^\theta}{L^{\max}} \quad |t. \quad (21)$$

It is impossible to say *a priori* how L^θ varies with the real wage. A general theory covers all possible cases. In the case of constant returns the real wage remains unchanged on the way from unemployment to full employment which is defined by (18) as $u = 0$. Hence we can refrain from further behavioral speculation. It is not terribly important how L^θ is determined by the workers in (20). It is only important that $L = L^\theta$ is realized in (17) and that the labor supply L^θ has an

upper limit.

The wage rate W is immaterial for the determination of the real wage. Eq. (16) and condition #5 see to it that all wage rate variations are neutralized. The distributed profit ratio ρ_D remains constant and since the profit ratio ρ_Q does not change according to (15) the wage rate cannot affect employment. Neither wage rate stickiness nor perfect downward flexibility has any employment effect in Say's regime. Price flexibility in the product market is entirely sufficient.

Eq. (10) makes it clear that inflation can only occur if the wage rate increases permanently or if productivity decreases. The latter case can safely be ignored for practical purposes. However fast the wage rate increases, neither the profit ratio (15) nor the real wage (19) is affected by the corresponding price increase. In Say's regime inflation has no effect on employment (for details about the monetary side see (2011)).

To sum up, in Say's regime the product and labor market always clear. The real wage and the profit ratio are equal on all employment levels. Because of the flexibility of the product market price all output variations are easily absorbed. No glut or demand deficiency ever appears. There is no trade-off between inflation and unemployment. Neither wage rate flexibility nor stickiness has any effect on employment. Say's regime, as defined by the structural axiom set and the six conditions enumerated above, is an elementary economic configuration that is reproducible in principle for an indefinite time span on any employment level. There is, obviously, no claim that exactly the full employment configuration will be realized with the inexorable necessity of a law. Say's regime should be seen, first of all, as an elementary, objective, explicit and consistent benchmark economy that exhibits all desirable properties. The structural axioms and the six conditions formally define the actions of the invisible hand. Say's regime is free of marginalistic preconceptions and fully supplants the obsolete general equilibrium model.

3. CONDITIONS AND IMPLICATIONS OF KEYNES'S REGIME

In Say's regime full employment is always realized. There is no such thing as an employment function. Or, in other words, the employment function is a vertical through the full employment point on the x-axis. Keynes introduced the employment function as an inverse to the aggregate supply function (1973, p. 280). In the most basic form the employment function defines the dependency of employment from effective demand.

Here we do not start with a supply function, which implies indefensible behavioral assumptions, but with the objective structural axioms and the condition of product market clearing. The crucial alteration vis-à-vis Say's regime consists in making the price P the independent variable. This implies that employment L becomes the dependent variable. Instead of (5) we now have:

$$L = \frac{C}{PR} \quad \text{if} \quad \rho_x = 1 \quad |t. \quad (22)$$

This change of the direction of dependency makes *all the difference* between Say's and Keynes's regime. There is not more about it. Employment is dependent on consumption expenditure, price and productivity. Consumption expenditure C represents effective demand. Other components of total effective demand are absent in the pure consumption economy.

From the elementary form of the structural axiomatic employment function (22) one gets by substituting (6):

$$L = \frac{DN}{\frac{PR}{\rho_E} - W} \quad \text{if} \quad \rho_x = 1 \quad |t. \quad (23)$$

Employment L depends, all other variables fix, on the expenditure ratio ρ_E which is a superior indicator of effective demand.³ This structural employment function is certainly more informative than Keynes's inverse supply function.

The first case to consider is $\rho_E = 1$ given P, R, W, D, N . If employment happens to be equal to labor supply, i.e. $L = L^o$, there is no difference to Say's regime. This is the benchmark case. With the same values of the variables we arrive at the same outcome with different dependencies. Initially, Say's and Keynes's regime are indistinguishable. This helps to identify the crucial differences.

If $\rho_E < 1$ then employment L falls in period t according to (23) below full employment. This is, of course, always possible. There is no reason to assume that $\rho_E = 1$ in each single period. What is plausible as a first approximation is that the value of the expenditure ratio is randomly distributed around unity in successive periods, such that the deviations cancel

³ The explicit inclusion of the consumption function determines the expenditure ratio as follows: $\rho_E = \frac{a}{Y} + b$. The consumption function is not needed for our present purposes.

out over a longer time span. The condition for intertemporal budget balancing in the pure consumption economy reads:

$$\sum_{t=1}^{\infty} C_t = \sum_{t=1}^{\infty} Y_t. \quad (24)$$

This condition holds in the proverbial “long run” and leaves plenty of room for various time sequences of ρ_E . In fact, there are infinitely many sequences that satisfy the immutable balanced budget condition (24). The best case, clearly, is small deviations and fast alterations between $\rho_E < 1$ and $\rho_E > 1$ that average out over a time span the business and the household sector can cope with. Problems arise if there is a longer random sequence of $\rho_E < 1$. Under the condition of pure randomness, i.e. with positive or negative feed-backs excluded, this can always happen. In other words, it is to be expected that a Keynesian unemployment configuration occurs as a quite normal event because of a temporary fall of effective demand. That this configuration will be reversed, albeit at some unknown date, is made sure by condition (24).

The characteristics of a Keynesian configuration are, first: employment in (23) is below initial full employment if $\rho_E < 1$, with the severity of unemployment depending on the span $1 - \rho_E$.

The profit ratio in Keynes’s regime is derived from (14) and (12):

$$\rho_Q = \rho_E (1 + \rho_D) - 1 \quad \text{if} \quad \rho_X = 1 \quad |t. \quad (25)$$

The profit ratio falls below the benchmark of $\rho_Q = \rho_D$ at $\rho_E = 1$. The distributed profit ratio ρ_D is kept constant just as in Say’s regime. That is to say, the effects of income redistribution are at first kept out of the picture. Distribution is a separate issue.

The household sector’s saving and dissaving hovers around zero if the random changes of the expenditure ratio ρ_E are distributed around unity. Monetary saving is defined as:

$$S_m \equiv Y - C \equiv (1 - \rho_E)Y \quad |t. \quad (26)$$

In sum: unemployment, lower profit, and higher saving characterize all periods with below average effective demand, i.e. $\rho_E < 1$. Overemployment, higher profit, and dissaving characterize all periods with above average effective demand, i.e. $\rho_E > 1$. Note that the product market is cleared by assumption as in Say’s regime. Because of $\rho_X = 1$ there is no need to bother with inventory changes.

It is rather obvious that we live in Keynes's regime. The household sector's budget is not balanced in each single period. Since the intertemporal budget balancing condition (24) is assumed to hold with certainty all deviations $\rho_E \neq 1$ are temporary if the expenditure ratio varies around unity, such that the weighted expenditure ratio over all periods is unity. This, though, is an inchoate conclusion until the actual length of "temporary" is specified. As long as it cannot be convincingly demonstrated that there exists some "force" that impedes longer or extreme downward deviations of the expenditure ratio from unity there is a rationale for Keynesian measures under the given conditions.

Let us introduce government as additional agent, such that the overall expenditure ratio is the sum of the household and government sector's respective ratios:

$$\rho_E \equiv \rho_{EH} + \rho_{EG} \quad |t. \quad (27)$$

The formula for a perfectly counter cyclical government policy then reads:

$$\rho_{EG} = 1 - \rho_{EH} \quad |t. \quad (28)$$

This formula works if ρ_{EH} is distributed around unity and government manages to exactly compensate the household sector's expenditure swings. In the ideal case employment stays at the initial full employment level and the government's dissaving and saving averages out "in the long run" due to (24). That is, budget deficits are not permanent. In the real world, of course, we have lags and (28) has to be replaced by a more sophisticated time-form (Phillips, 1954). Seen over *all* periods, however, Keynes's regime is not different from Say's regime – it contains the latter as a special case. On a deeper level both regimes are in harmony (after all, they depend on the same set of axioms).

However, up to this point Keynes's regime has been discussed under the condition that P, R, W, D, N in (23) are fixed at their initial values and that only effective demand varies. While Keynes rightly stressed the natural volatility of ρ_E he ignored the factors that are a constituent part of (23) but not of the employment function of the *General Theory*. There is room for further generalization.

4. FROM THE EMPLOYMENT FUNCTION TO THE PHILLIPS CURVE

The structural employment function (23) can alternatively be expressed by applying the definition of unemployment (18):

$$u = 1 - \frac{DN}{\left(\frac{PR}{\rho_E} - W\right)L^\theta} \quad \text{if } \rho_X = 1 \quad |t. \quad (29)$$

The factor cost ratio is defined as:

$$\rho_F \equiv \frac{W}{PR} \quad |t. \quad (30)$$

Combined with (16) and (30) unemployment in Keynes's regime is finally given by:

$$u = 1 - \frac{\Theta}{\frac{1}{\rho_E \rho_F} - 1} \quad (31)$$

$$\text{if } \rho_X = 1, \text{ with } \Theta \equiv \frac{\rho_V N}{L^\theta} \quad |t.$$

The unemployment rate u depends on effective demand ρ_E and the factor cost ratio ρ_F . The latter is composed of wage rate W , price P and productivity R . Both ratios taken together determine employment/unemployment and, most important, their changes may counteract or reinforce each other. Multipliers like $\frac{1}{1-s}$ make these effects invisible; they are the result of formal misspecifications.

To get the rates of change, the respective period values can be formally decomposed as follows :

$$\rho_{Ft} \equiv \left(\frac{W_{t-1}}{P_{t-1} R_{t-1}} \right) \frac{(1 + \ddot{W}_t)}{(1 + \ddot{P}_t)(1 + \ddot{R}_t)}. \quad (32)$$

Eqs. (31) and (32) together deliver the relation of unemployment rate u and changes of the wage rate \ddot{W} in period t if the rates of change of price and productivity are set to zero for the beginning, i.e. $\ddot{P} = 0$ and $\ddot{R} = 0$ in (32). Then (31) reproduces the correlation of the *original* Phillips curve. An increase of the wage rate, i.e. $\ddot{W} > 0$, leads to *higher* employment, i.e. to a lower unemployment rate u . The *original* Phillips curve is a straightforward logical implication of the structural employment function.

Keynes focused – in structural-axiomatic terms – on the relation between effective demand ρ_E and unemployment u in (31). He ignored, and could not do other than to ignore the factor cost ratio ρ_F because nothing of the sort appeared in his inverse supply function. The lack

of a comprehensive employment function should eventually provoke disaster.

5. THE RISE AND FALL OF THE BASTARD PHILLIPS CURVE

The *original* Phillips curve is about the relation of the rate of unemployment and the rate of change of the wage rate (Phillips A. W., 1958). Phillips studied more than a century's worth of data and established the stable inverse relation for the United Kingdom. Phillips's original curve was a remarkable empirical finding. It has to be emphasized that Phillips "had not made an explicit link between inflation and unemployment" (Ormerod, 1994, p. 120).

The original curve was transformed by Samuelson with the simple formula: rate of inflation = rate of wage growth – rate of productivity growth (Samuelson & Nordhaus, 1998, p. 590); in our notation $\ddot{P} = \ddot{W} - \ddot{R}$. This formula, according to Samuelson an "important piece of inflation arithmetic," says that price inflation runs in tandem with wage inflation and that both have basically the same effect on employment respectively the rate of unemployment. The difference between the original and the bastard Phillips curve consists of a 1 percent productivity growth. This naïve arithmetical exercise led to the far-reaching policy conclusion that there exists an exploitable trade-off between inflation and unemployment. Needless to say that this basic version experienced refinement, reinterpretation and qualification in the sequel. It was completely overlooked in the ensuing filibuster, though, that, to begin with, the underlying arithmetic was fallacious. The lack of a sound theoretical foundation did not prevent the application of the Samuelson-Solow Phillips curve. Quite the contrary. Phillips is said to have remarked, "if I had known what they would do with the graph I would never have drawn it." (Quiggin, 2010, p. 91).

The rest of the story has been told and retold from different perspectives (Blanchard, 2000, pp. 154-156), (Skidelsky, 2009, pp. 102-110), (Akerlof & Shiller, 2009, pp. 43-46), (Quiggin, 2010, pp. 90-107), (Keen, 2011, pp. 195-202). In the 1960s the menu choice between inflation and unemployment was part and parcel of the then dominant Keynesian policy mindset: "it was the Golden Age of the Phillips Curve" (Bruno & Easterly, 1996, p. 1). Troubles arose with the acceleration of inflation in the 1970s. Friedman suggested that the long-term Phillips curve is a vertical at some natural rate of unemployment. This thesis was sharpened with the the help of the rational expectation hypothesis. The final outcome was that the bastard Phillips curve was successively discredited in academic economics during the 1970s and with it Keynesian macroeconomics.

Keynes anticipated the vertical Phillips curve as a special case (1973, p. 289) which, by the way, is perfectly compatible with Say's regime while a natural rate of unemployment, in the strict sense, amounts to a recantation of the classical full employment promise. All this, though, did not help much. The first weakness of Keynes's approach was that his employment function did not explicitly account for the employment effects of changes of wage rate W , price P , and productivity R . The second mishap was that Post-Keynesians did not spot the elementary mistake in Samuelson's inflation arithmetic but jumped on the bandwagon. The lack of a consistent employment function like (23), not the superiority of theoretical alternatives, eventually sealed the fate of Keynesianism as dominant policy paradigm.

6. ALL IN ONE: THE EMPLOYMENT FUNCTION, THE MULTIPLIER, AND THE PHILLIPS CURVE

In order to include investment as the second important component of effective demand the axioms and definitions have first to be differentiated for two industries. This formal exercise is referred to the Appendix. The structural Phillips curve is derived as (41) and here reproduced:

$$u = 1 - \frac{1}{\underset{\text{multiplier}}{1 - \rho_E \rho_{FC}}} \left(\overset{\text{investment exp.}}{\frac{I}{P_I R_I} + \frac{\rho_E Y_D}{P_C R_C}} \right) \frac{1}{L^\theta} \quad (33)$$

$$\text{if } W_C = W_I = W, \rho_{XC} = 1, \rho_{XI} = 1 \quad \text{with} \quad \rho_{FC} \equiv \frac{W}{P_C R_C} \quad |t.$$

The structural Phillips curve asserts in detail (all other variables are fixed in each case):

- An increase of the wage rate W leads to *higher* employment, i.e. to a lower unemployment rate u . This is in accordance with the correlation of Phillips's *original* study but clearly beyond the comprehension of the marginalistic mindset. For the sequence of employment levels see (2011).
- A price increase is conducive to *lower* employment. This is in *discordance* with the Samuelson-Solow version of the Phillips curve, that is, with the trade-off between unemployment and inflation. The stagflation of the 1970s is generally seen as an empirical refutation of the neoclassical synthesis version of the Phillips curve (Davidson, 2009, pp. 175-179), (Forder, 2010, p. 330). This refutation of "perverted" Keynesian economics (Davidson, 2009, pp. 18-20) indirectly *corroborates* (33).

- Provided that wage rate, price and distributed profit all change with the same rate, $\ddot{W} = \ddot{P} = \ddot{Y}_D$ and $\ddot{R} = 0$ in the simplified case (40), there is no effect on employment. This is in accordance with the NAIRU and rational expectations *interpretation* of the Phillips curve (Blanchard & Katz, 1997), (Gordon, 1990, p. 1125), i.e. in this cases the Phillips curve is a vertical. This does not imply that the underlying behavioral speculations are adopted here. The configurations that make for a vertical Phillips curve are extremely improbable: “It is not news that NAIRU theory is a failure.” (Hall, 2011, p. 446). Eq. (40) explains verticality without recourse to expectations (rational or otherwise) or any other nonempirical behavioral assumption.
- If the configuration of price and wage rate changes is such that the denominator in the simplified case (40) remains unchanged then employment stays where it is, no matter how large wage rate and price changes are. In this case, perfect wage-price flexibility has no impact on employment (Hahn & Solow, 1997, p. 134). Again, the structural Phillips curve is a vertical at the *actual* unemployment rate.
- An increase of the expenditure ratio ρ_E leads to higher employment. An expenditure ratio $\rho_E > 1$ presupposes the existence of a banking system (for details see (2011)).
- A productivity increase leads to lower employment. This covers the case of technological unemployment.
- As the difference in the denominator approaches zero employment goes (formally) off to infinity. This singularity is an implicit property of the economy as given by the structural axiom set.
- Investment expenditures I exert a positive influence on employment.
- Distributed profits Y_D exert a positive influence on employment.
- Price inflation and wage inflation have *opposite* effects on employment. This vitiates Samuelson’s naïve inflation arithmetic. The net effect depends on the *relative* rates of change in (32). The absolute rates are of no consequence if they are equal (with $\ddot{R} = 0$, of course). Hyperinflation at full employment is possible as a mathematical limiting case, i.e. $\ddot{W} = \ddot{P}$. The net employment effect of inflation is positive if $\ddot{W} > \ddot{P}$; in contradistinction $\ddot{W} < \ddot{P}$ produces stagnation or rising unemployment. The structural Phillips curve covers *both* cases and explains the instability of the bastard Phillips curve.

These conclusions follow without regress to indefensible behavioral assumptions from the axiom set (1) to (3), the dependency assumption (22), and the laws of algebra. If the axioms capture reality the logical implications that are given with (33) are observable. The structural Phillips curve has the formal status of a theorem and contains twelve observables, that is, vastly more than the methodologically inferior bastard version. All that has to be done is to take the data and to carry out the crucial experiment. In other words:

Reason gives the structure to the system; the data of experience and their mutual relations are to correspond exactly to consequences in the theory. On the possibility alone of such a correspondence rests the value and the justification of the whole system. (Einstein, 1934, p. 165)

7. CONCLUSION

The Keynesian as well as the anti-Keynesian arguments about the relationship of inflation and unemployment rest on logically deficient formal foundations. The structural-axiomatic approach recovers the original Phillips curve and explains the breakdown of the bastard Phillips curve.

REFERENCES

- Akerlof, G. A., & Shiller, R. J. 2009. *Animal Spirits*. Princeton, NJ, Oxford: Princeton University Press.
- Blanchard, O. 2000. *Macroeconomics* (2nd ed.). Saddle River, NJ: Prentice Hall.
- Blanchard, O., & Katz, L. F. 1997. "What We Know and Do Not Know About the Natural Rate of Unemployment." *Journal of Economic Perspectives*, 11(1), 51-72.
- Bruno, M., & Easterly, W. 1996. "Inflation and Growth: In Search of a Stable Relationship." *Review Federal Reserve Bank of St. Louis, May/June*, 1-8.
- Davidson, P. 2009. *The Keynes Solution*. New York: Palgrave Macmillan.
- Einstein, A. 1934. "On the Method of Theoretical Physics." *Philosophy of Science*, 1(2), pp. 163-169.
- Farmelo, G. 2009. *The Strangest Man*. London: Faber and Faber.
- Feynman, R. 1992. *The Character of Physical Law*. London: Penguin.
- Forder, J. 2010. "Friedman's Nobel Lecture and the Phillips Curve Myth." *Journal of the History of Economic Thought*, 32(3), 329-348.
- Gordon, R. J. 1990. "What is New-Keynesian Economics?" *Journal of Economic Literature*, 28(3), pp. 1115-1171.
- Hahn, F. H., & Solow, R. M. 1997. *A Critical Essay on Modern Macroeconomic Theory*. Cambridge, MA, London: MIT Press.
- Hall, R. E. 2011. "The Long Slump." *American Economic Review*, 101(2), 431-469.
- Hudik, M. 2011. "Why Economics is Not a Science of Behaviour." *Journal of Economic Methodology*, 18(2), pp. 147-162.
- Hutchison, T. W. 1960. *The Significance and Basic Postulates of Economic Theory*. New York: Kelley.
- Kakarot-Handtke, E. 2011. "Keynes's Missing Axioms." *SSRN Working Paper Series*, 1841408, 1-32.
- _____. 2013a. "Crisis and Methodology: Some Heterodox Misunderstandings." *Real-World Economics Review*, 63, pp. 98-117.
- _____. 2013b. "Why Post Keynesianism is Not Yet a Science." *Economic Analysis and Policy*, 43(1), pp. 97-106.
- Kates, S. 1998. *Say's Law and the Keynesian Revolution. How Macroeconomic Theory Lost its*

- Way. Cheltenham, Northampton: Edward Elgar.
- Keen, S. 2011. *Debunking Economics* (rev. ed.). New York, NY: Zed Books.
- Keynes, J. M. 1973. *The General Theory of Employment Interest and Money*. London, Basingstoke: Macmillan.
- Klant, J. J. 1994. *The Nature of Economic Thought*. Aldershot, Brookfield: Edward Elgar.
- Mill, J. S. 2006. *A System of Logic*. Indianapolis: Liberty Fund.
- Ormerod, P. 1994. *The Death of Economics*. London: Faber and Faber.
- Phillips, A. W. 1954. "Stabilisation Policy in a Closed Economy." *Economic Journal*, 64(254), pp. 290–323.
- _____. 1958. "The Relation Between Unemployment and the Rate of Change of Money Wage Rates in the United Kingdom." 1861-1957. *Economica*, 25, 283-299.
- Quiggin, J. 2010. *Zombie Economics. How Dead Ideas Still Walk Among Us*. Princeton, NJ, Oxford: Princeton University Press.
- Samuelson, P. A., & Nordhaus, W. D. 1998. *Economics* (16th ed.). Boston, Burr Ridge, etc.: Irwin, McGraw-Hill.
- Skidelsky, R. 2009. *Keynes. The Return of the Master*. London: Allen Lane.
- Stiglitz, J. E. 2010. *Freefall. America, Free Markets, and the Sinking of the World Economy*. New York, NY: Norton.

APPENDIX

The differentiated structural axiom set follows naturally from (1) to (3) and is given by:

$$Y = W_C L_C + W_I L_I + \underbrace{D_C N_C + D_I N_I}_{\equiv \dot{Y}_D} \quad |t. \quad (34)$$

$$\begin{aligned} O_C &= R_C L_C \\ O_I &= R_I L_I \end{aligned} \quad |t. \quad (35)$$

$$\begin{aligned} C &= P_C X_C \\ I &= P_I X_I \end{aligned} \quad |t. \quad (36)$$

Combined with (4), (6) and (30) this yields the structural income multiplier under the condition of market clearing:

$$Y = \frac{1}{1 - \rho_E \rho_{FC}} (\rho_{FI} I + Y_D) \quad (37)$$

if $\rho_{XC} = 1, \rho_{XI} = 1 \quad |t.$

The condition of zero profit in both industries gives the simplest form of the income multiplier:

$$Y = \frac{1}{1 - \rho_E} I \quad (38)$$

if $\rho_{XC} = 1, \rho_{XI} = 1, \rho_{FC} = 1, \rho_{FI} = 1, Y_D = 0 \quad |t.$

This is as close as possible to the textbook multiplier. It is a remarkable property of the textbook multiplier that it holds only in a zero profit economy. In the general version (37) the multiplicative effect is smaller than in (38).

Total employment with two industries is given by $L = L_C + L_I$. From the differentiated axiom set follows the structural employment function as:

$$L = \frac{1}{1 - \rho_E \rho_{FC}} \left(\frac{I}{P_I R_I} + \frac{\rho_E Y_D}{P_C R_C} \right) \quad (39)$$

$$\text{if } W_C = W_I = W, \rho_{XC} = 1, \rho_{XI} = 1 \quad |t.$$

Employment depends on effective demand, i.e. ρ_E and investment expenditure I , as well as the configuration of wage rate, price, and productivity, i.e. the factor cost ratio ρ_{FC} . The structural employment function yields (23) as limiting case:

$$L = \frac{Y_D}{\frac{P_C R_C}{\rho_E} - W} \quad (40)$$

$$\text{if } I = 0, \rho_{XC} = 1 \quad |t.$$

The crucial determinants of employment are effective demand and the configuration of wage rate, price, and productivity. All changes that keep the fraction constant fixate employment at the current level whatever it is. This effect is sometimes interpreted as hysteresis.

Together with the definition of unemployment, i.e. (18), eq. (39) yields the structural Phillips curve:

$$u = 1 - \frac{1}{1 - \rho_E \rho_{FC}} \left(\frac{I}{P_I R_I} + \frac{\rho_E Y_D}{P_C R_C} \right) \frac{1}{L^\theta} \quad (41)$$

$$\text{if } W_C = W_I = W, \rho_{XC} = 1, \rho_{XI} = 1 \quad |t.$$

The structural Phillips curve depends on the structural employment function which in turn depends on the expenditure and the factor cost ratio as constituents of the multiplier.