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The Empirics of Canadian Government Securities Yields

by

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Dataset availability: The dataset used in the empirical part of this paper is available upon request to bona fide researchers for the replication and verification of the results.

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ABSTRACT

Keynes argued that the short-term interest rate is the main driver of the long-term interest rate. This paper empirically models the relationship between short-term interest rates and long-term government securities yields in Canada, after controlling for other important financial variables. The statistical analysis uses high-frequency daily data from 1990 to 2018. It applies both the cointegration technique and Granger causality within the vector error correction (VEC) framework. The empirical results suggest that the action of the monetary authority is an important determinant of Canadian government securities yields, which supports the Keynesian perspective. These findings have important implications for investors, financial analysts, and policymakers.

KEYWORDS: Canadian Government Bond Yields; Long-Term Interest Rate; Short-Term Interest Rate; Monetary Policy; Cointegration; Granger Causality

JEL CLASSIFICATIONS: E43; E50; E60; G10; G12

I. INTRODUCTION

Motivation

The long-term interest rate on Canadian government securities is an important theoretical and empirical topic. It is germane for macroeconomic theory and public policy, particularly as it concerns these issues: the monetary transmission mechanism, monetary policy, market volatility, inflationary pressures, financial conditions, government debt management and operations, and the effects of higher government debt and deficits ratios on government securities yields. Understanding the empirics of Canadian government securities yields can be useful for investors and portfolio managers in making strategic and tactical asset allocation and investment decisions concerning duration, convexity, speculation, and delta hedging.

John Maynard Keynes (1930, 352–364) argued that the central bank's actions have a decisive influence on the long-term interest rate. He claimed that the central bank's policy rate sets the short-term interest rate, which has a crucial influence on the long-term interest rate for government securities. Keynes wrote (1930, 353): "[T]he influence of the short-term rate of interest on the long-term rate is much greater than anyone … would have expected." He attributed this correlation to fundamental macroeconomic factors, technical characteristics of financial markets, and investors' behavior, including herding and the formation of expectations. Keynes (1930, 363) asserted that "there is no reason to doubt the ability of a Central Bank to make its short-term rate of interest effective in the market."

This paper empirically examines whether Keynes's claim holds true. It analyzes the effects of the short-term interest rate on the long-term interest rate, as measured by Canadian government securities yields. The statistical exercise undertaken in this paper controls for several important factors, including the domestic equity market, oil prices, and the exchange rate of the Canadian dollar. This paper uses daily data to analyze Canadian government securities yields. There are two main benefits of using daily financial data. First, daily data over a long period provides many observations, which ensures a robust degree of freedom. Second, analyzing high-frequency data provides a near real-time fundamental assessment of long-term government securities yields, and thus provides important information to investors, financial analysts, and policymakers.

Relation to Debates in the Literature

This paper contributes to the ongoing debate on the dynamics of government bond yields. The literature on government bond yields contains many substantial but unresolved debates. The two main schools of thought regarding the dynamics of government bond yields represent the neoclassical and Keynesian views.

The neoclassical view holds that government bond yields are the outcome of the demand for and supply of loanable funds. Other exogenous factors, such as government debt and deficit ratios, also influence government bond yields. In the past two decades, scholars have presented their arguments in various studies on the dynamics of government bond yields and various macroeconomic and financial variables, including government debt and deficit ratios. There are numerous studies from the neoclassical perspective. For examples, see Ardagna, Caselli, and Lane (2007); Baldacci and Kumar (2010); Doi, Hoshi, and Okimoto (2011); Elmendort and Mankiw (1998); Gurber and Kamin (2012); Hansen and İmrohoroğlu (2013); Horioka, Nomoto, and Tera-Hagiwara (2014); Hoshi and Ito (2013, 2014); Lam and Tokuoka (2011); Tokuoka (2012); Paccagnini (2016); Poghosyan (2014); Reinhart and Rogoff (2019); and Tkačevs and Vilerts (2019). Their analyses and interpretation of the data support the neoclassical view.

The Keynesian school follows Keynes's ([1936] 2007) argument that interest rates have a psychological and sociological foundation in a world characterized by ontological uncertainty (Davidson 2015). The Keynesian school maintains the liquidity preference view of interest rates as articulated in Keynes's *General Theory* ([1936] 2007). This view holds that the long-term interest rate is primarily determined by the central bank's actions, such as the setting of benchmark policy rates, repurchase and reverse repurchase agreements, forward guidance about policy rates, and decisions concerning the central bank's monetary base and balance sheets. Riefler's (1930) analysis of the dynamics of the short-term interest rate and the long-term interest rate in the United States in the 1920s and 1930s provided the empirical basis for Keynes to formulate this hypothesis. The Keynesian perspective on interest rates and monetary operations, and their relation to fiscal policy, was later developed in Lerner (1947). In recent years, several Keynesian and Post-Keynesian economists have advanced the Keynesian view of interest rates. They argued that an increase (decrease) in government debt and deficit ratios does *not* necessarily lead to higher (lower) government bond yields, particularly in countries with monetary sovereignty (Akram 2014; Kregel 2011; Lavoie 2014; Wray 2012). This is contrary to the claims of the neoclassical view. The strong relationship between the short-term interest rate and the long-term interest rate on government bonds has been empirically shown to hold in several studies. Akram and Das (2014) and Akram and Li (2018, 2019a) argued this for Japan; Akram and Das (2015, 2019a) for India; Akram and Das (2017) for the eurozone countries; Akram and Li (2016, 2017, 2019b), Akram and Das (2019b), and Levrero and Deleidi (2019) for the United States; Akram and Das (forthcoming) for Australia; and Simoski (2019) for several Latin American countries, including Brazil and Mexico.

The aim of this paper is to empirically examine the Canadian case from a Keynesian perspective. This paper contributes to the debate in the empirical literature on government securities yields by analyzing the relationship between short-term interest rates, government securities yields, and other macroeconomic and financial variables in Canada. High-frequency daily data are used here for analyzing the relationship between the short-term interest rate and the long-term interest rate on Canadian government bonds. Very few papers use such high-frequency daily data to study government securities yields (Bollerslev, Cai, and Song 2000; Gürkaynak, Sack, and Wright 2007). Using the higher frequency data to examine the empirics of Canadian government bond yields from a Keynesian perspective is a useful extension of the literature because it furthers the ongoing debate. This is the first paper to use daily data in analyzing Canadian government securities yields from a Keynesian perspective. The findings of this paper are well-aligned with the Keynesian perspective because the results suggest that the short-term interest rate is the key determinant of the long-term interest rate on Canadian government securities.

Outline of the Paper

This paper proceeds as follows. Section 2 provides an overview of the evolution of Canadian government securities yields and key financial variables. Section 3 presents the data used in the paper. It also undertakes unit root tests, describes the econometric methodology, and reports the empirical findings. Section 4 concludes with a summary of the findings and their relevance to debates in economic theory and policy.

2. THE EVOLUTION OF CANADIAN GOVERNMENT SECURITIES YIELDS AND KEY FINANCIAL VARIABLES

It is useful to look at the evolution of Canadian government securities yields and key macroeconomic variables in order to understand the underlying dynamics between these variables. It can also give a useful perspective about the drivers of the long-term interest rate and the underlying relationships between these macroeconomic and financial variables.

Figure 1 shows the evolution of long-term Canadian government securities yields. It shows that government bond yields have progressively declined over time. There appears to be an underlying trend. The decline in government securities yields is partly due to a decline in observed inflation and inflation expectations. Government securities yields were elevated in the early 1990s. Since peaking around 12 percent in the early 1990s, yields have steadily declined. Yields rose sharply in the mid-1990s. Since then, bond yields have steadily declined, though there have been modest increases from time to time. Government securities yields fell notably prior to the global financial crisis. An attentive look at government securities yields' evolution and relating this to concurrent macroeconomic events and trends can be useful in understanding the underlying dynamics of the government securities market.

Figure 1: The Evolution of Yields of Selected Long-Term Canadian Government Securities, 1990–2018

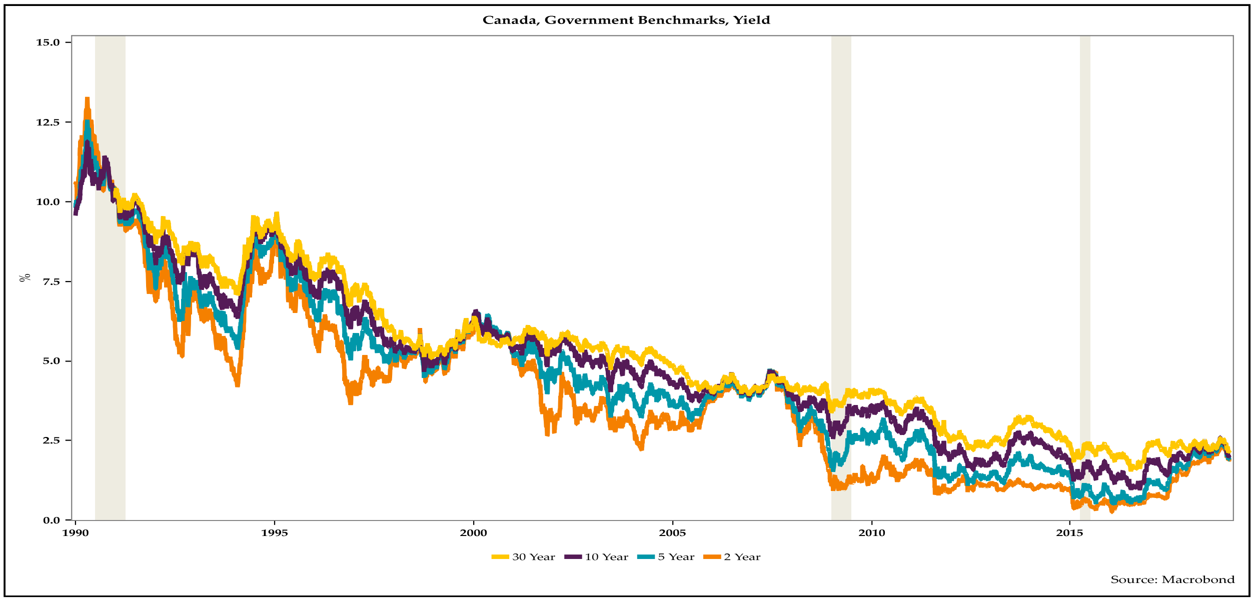


Figure 2 displays the evolution of the central bank's policy rate and the short-term interest rate, as measured by the yield of 3-month Treasury bills. It shows that the yield of 3-month Treasury bills generally moves in tandem with the Bank of Canada's policy rate.

Figure 2: The Evolution of the Policy Rate and the Short-Term Interest Rate in Canada, 1990–2018

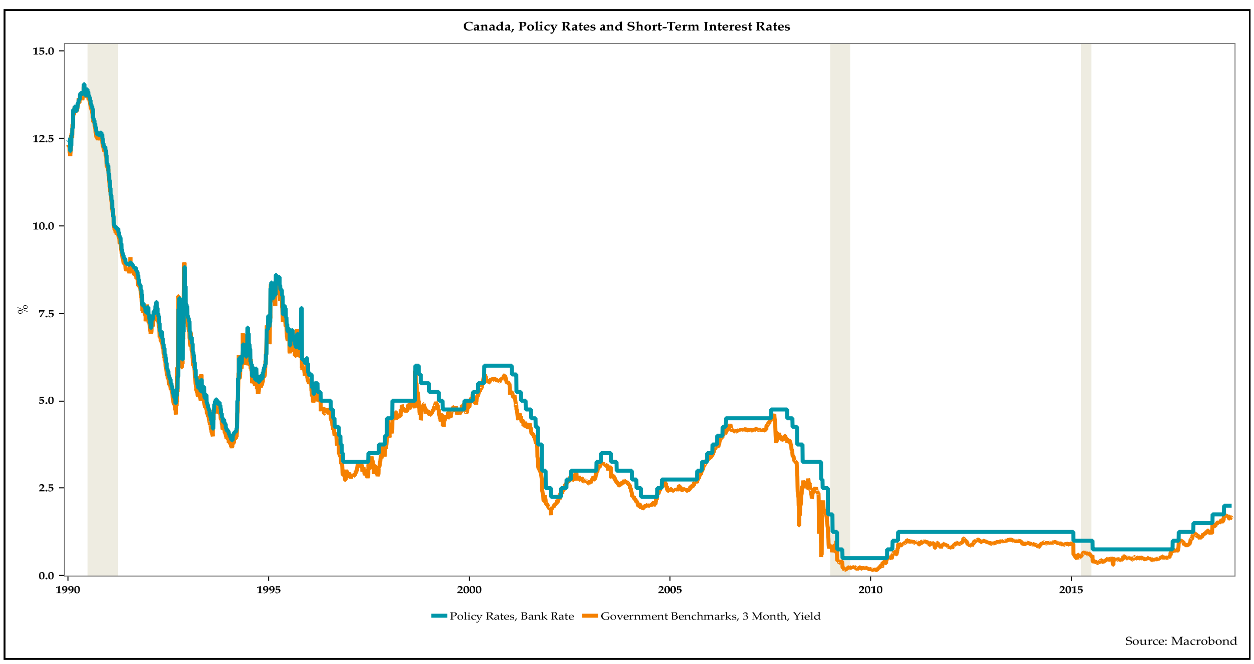


Figure 3 shows the evolution of Canadian equity prices, as measured by Standard and Poor's (S&P) Index for the Toronto Stock Exchange (TSX).

Figure 3: The Evolution of the Canadian Equity Price Index, 1990–2018

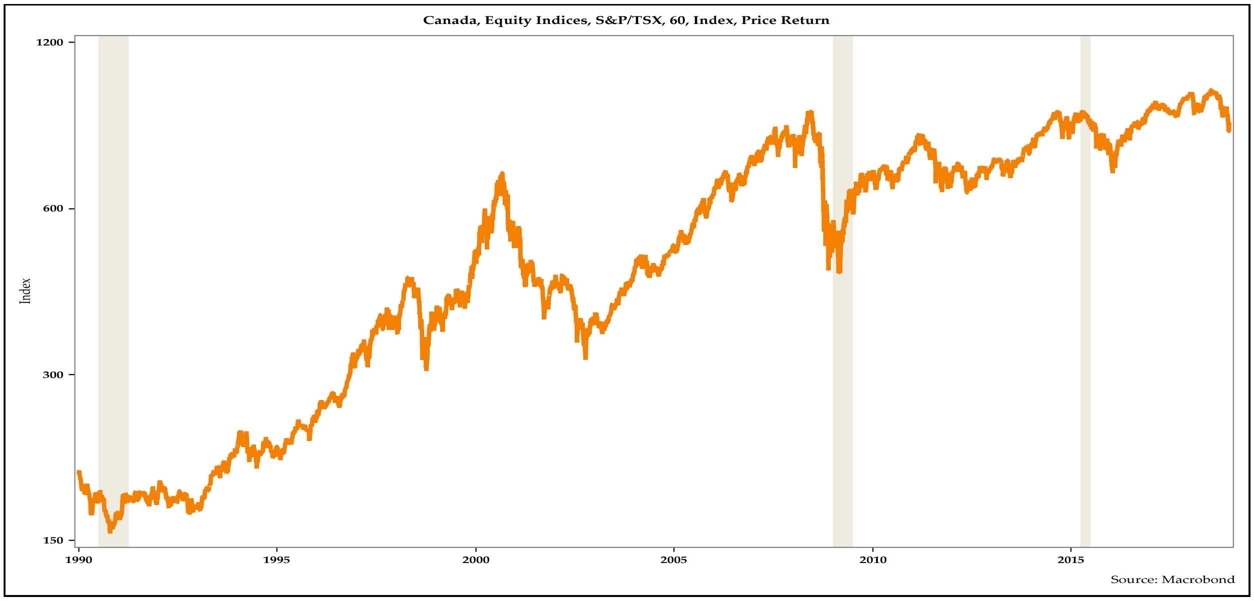


Figure 4 exhibits the evolution of crude oil prices. Monitoring this evolution is useful because oil prices provide information about inflationary pressures emanating from energy inputs. Crude oil prices also provide insights about growth in the global economy and the outlook for global effective demand. Finally, oil prices are important indicators of economic and political risks, particularly related to conditions in the major crude oil producing areas. Higher crude oil prices should increase government securities yields if investors regard higher crude oil prices as a harbinger of domestic and global inflationary pressures and/or strong effective demand in the global economy.

Figure 4: The Evolution of Crude Oil Prices, 1990–2018

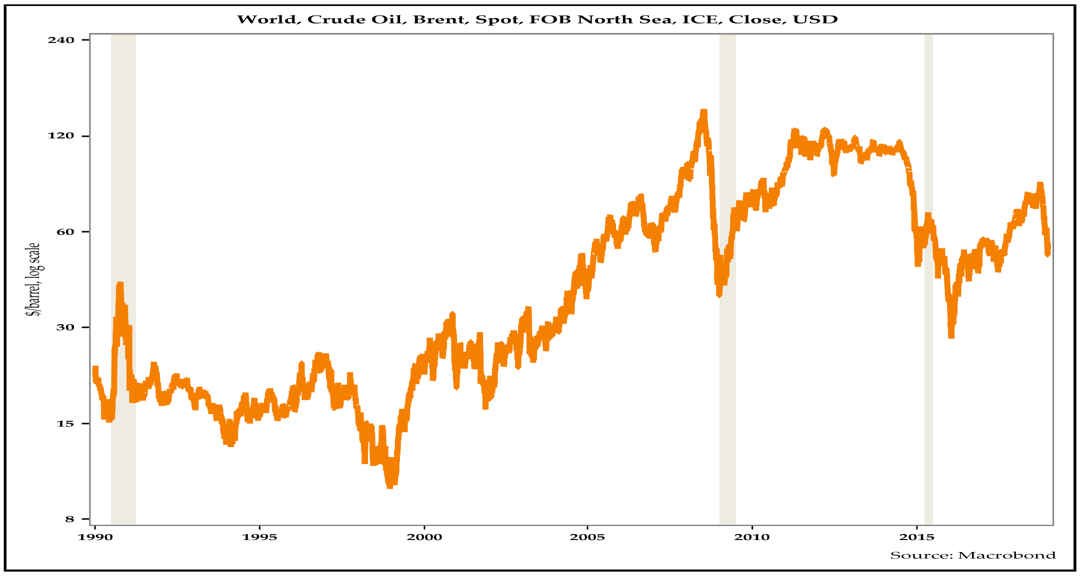
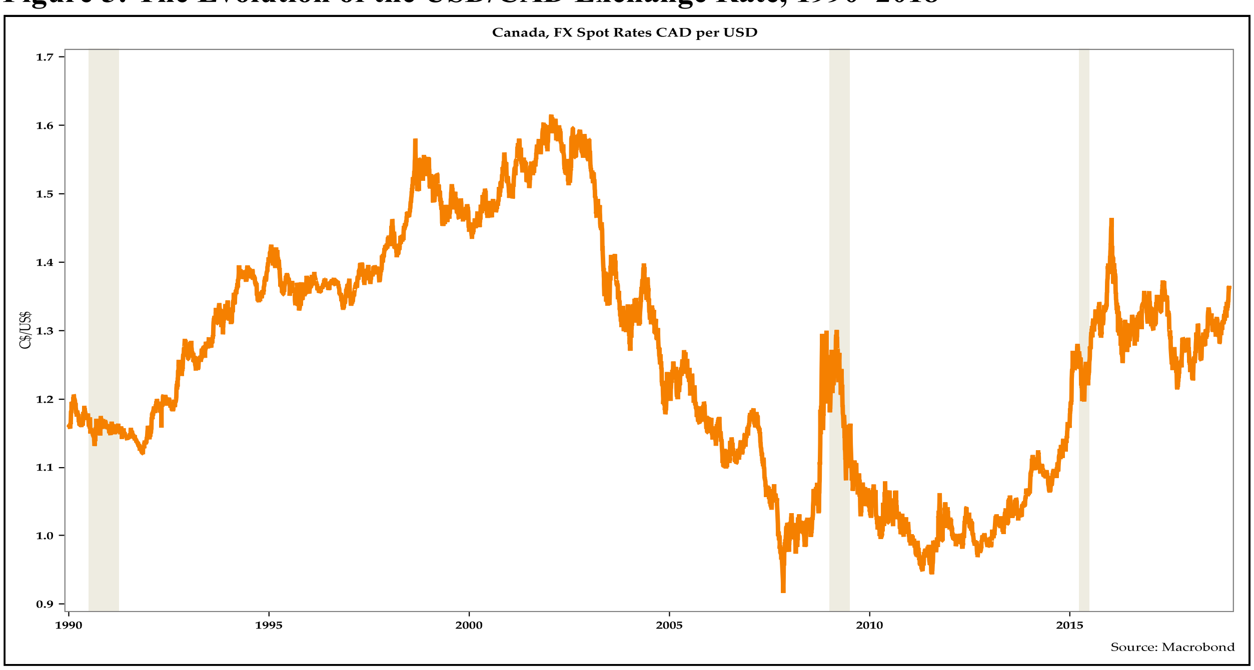


Figure 5 shows the evolution of the value of the Canadian dollar. The currency value can provide clues to investor confidence and sentiments, as well as to financial flows—all of which have a bearing on Canadian government securities yields. The USD/CAD exchange rate is expressed in terms of Canadian dollar per US dollar. This means that a rise in the exchange rate is a depreciation of the Canadian dollar with respect to the US dollar, while a decline in the exchange rate is an appreciation of the Canadian dollar with respect to the US dollar.

Figure 5: The Evolution of the USD/CAD Exchange Rate, 1990–2018



Figures 6 to 11 are scatterplots. They show the correlations between the yields of Canadian government securities of various maturity tenors and the yields of 3-month Treasury bills. These scatterplots reveal two clear and distinct patterns. First, there is a strong positive correlation between the yields of long-term Canadian government securities and Treasury bills. Second, the strong positive correlation between the yields of long-term Canadian government securities and Treasury bills declines as the maturity tenor of Canadian government securities increases.

Figure 6: Scatterplot of the Yields of 2-Year Canadian Government Securities and 3-Month Treasury Bills

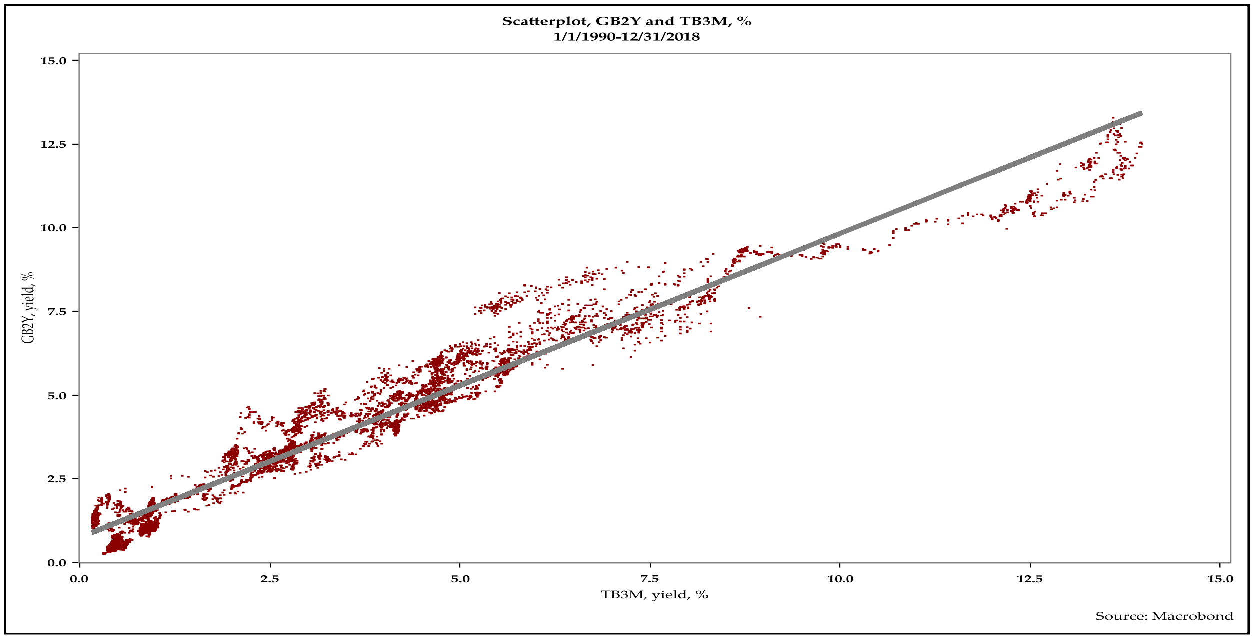


Figure 7: Scatterplot of the Yields of 3-Year Canadian Government Securities and 3-Month Treasury Bills

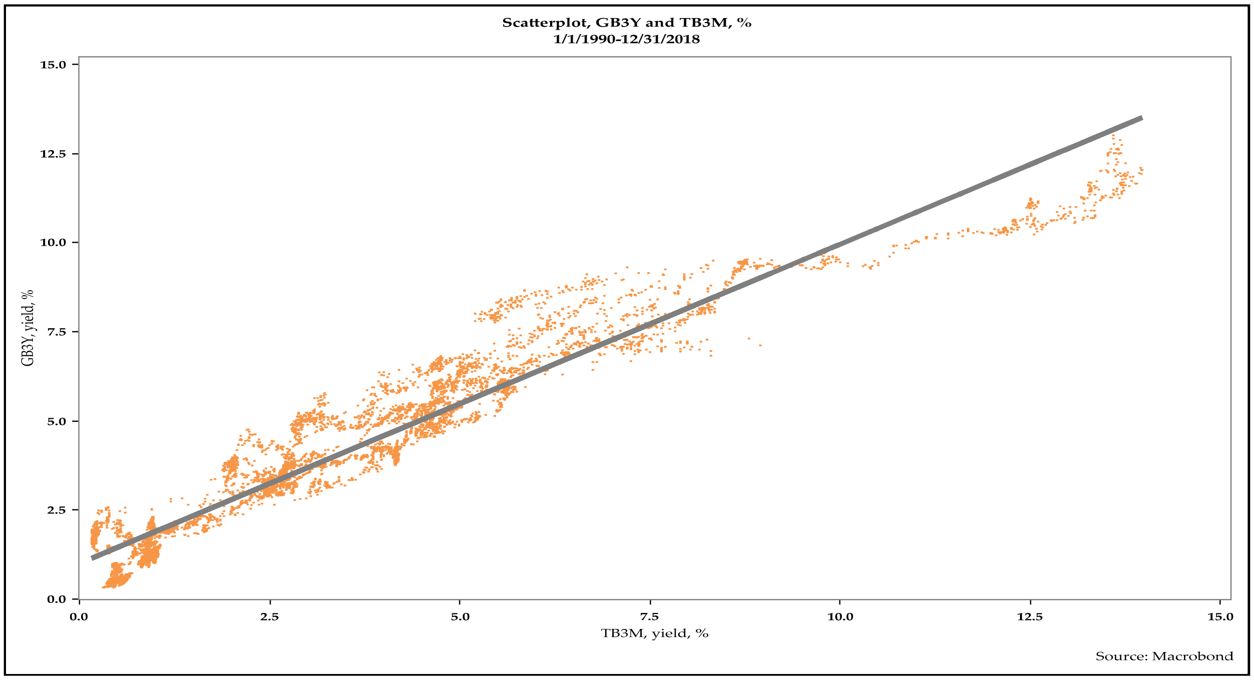


Figure 8: Scatterplot of the Yields of 5-Year Canadian Government Securities and 3-Month Treasury Bills

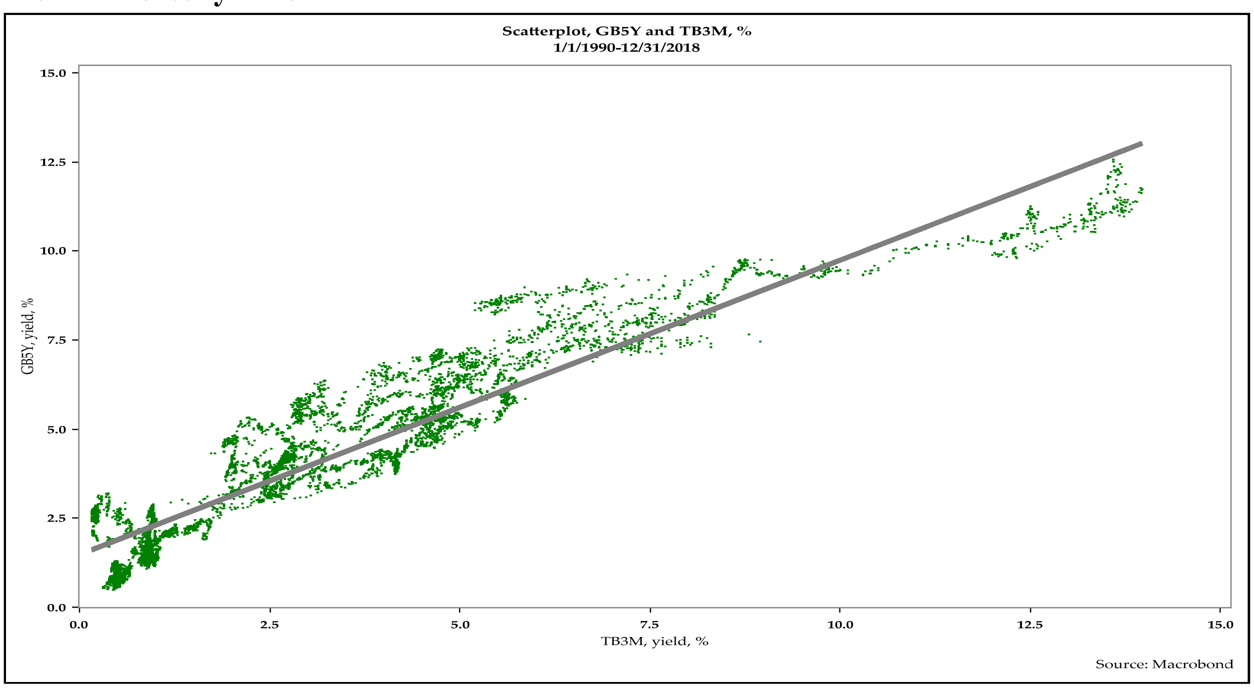


Figure 9: Scatterplot of the Yields of 7-Year Canadian Government Securities and 3-Month Treasury Bills

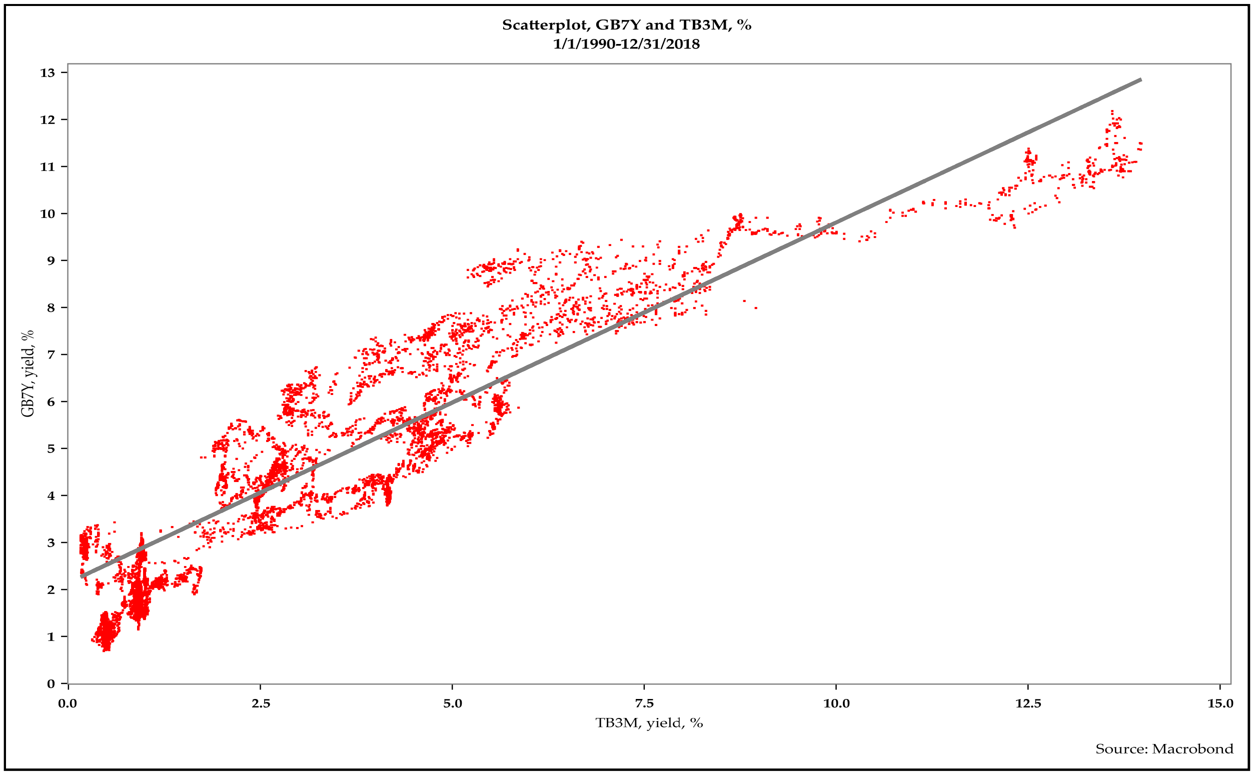


Figure 10: Scatterplot of the Yields of 10-Year Canadian Government Securities and 3-Month Treasury Bills

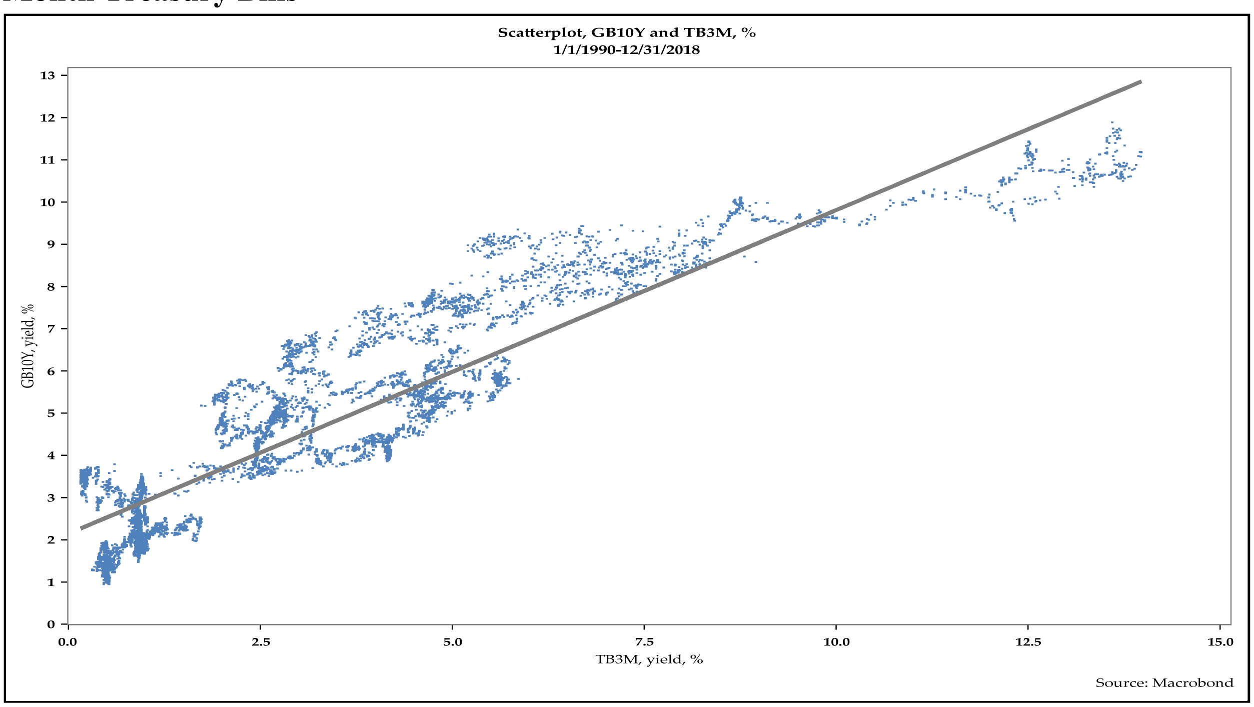
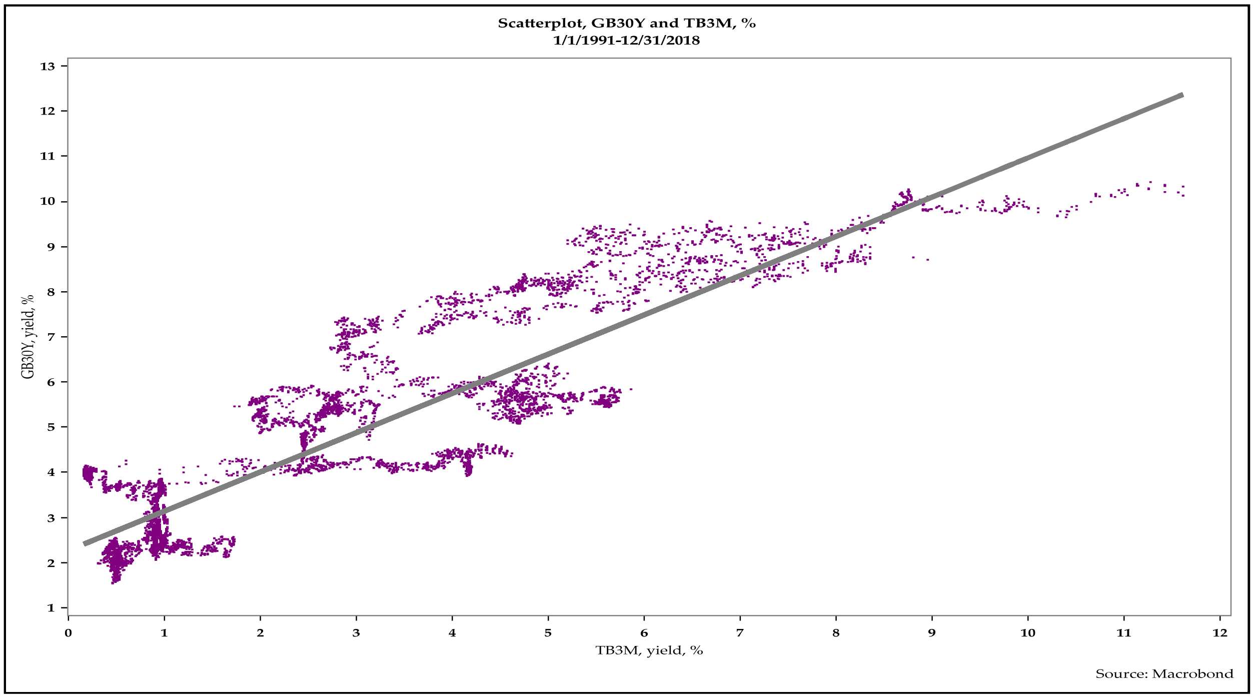


Figure 11: Scatterplot of the Yields of 30-Year Canadian Government Securities and 3-Month Treasury Bills



3. DATA, METHODS, AND EMPIRICAL FINDINGS

Data and Methods

Table 1, below, summarizes the data used in this paper. The first column lists the variables and their names. The second column describes the data and provides the data's date range. The third column displays the frequency of the data. The final column lists the data sources. The dataset begins on January 1, 1990 and ends on December 31, 2018.

Since the variables in the dataset are defined over a long period of time, it is important to identify whether these variables are stationary. If they are found to be nonstationary, applying the standard least squares technique would be an inappropriate approach. The order of variable integration is tested using both augmented Dickey-Fuller (ADF) (Dickey and Fuller 1979, 1981) and Phillips-Perron (PP) (Phillips and Perron 1988) techniques. Each of these techniques has three different versions: (1) with constant; (2) without constant; and (3) with constant and trend. This paper presents only the results produced from ADF and PP tests using constant and trend.

However, results from the other two versions of the unit root tests are similar to the third version. Findings from these tests are available upon request.

The standard Johansen (1991, 1995) cointegration technique can be applied if all the variables are integrated of order 1. This method serves as a basis for implementing a vector error-correction model (VECM). From this model, the long-run dynamics and the short-run causal relations among the variables can be identified. The Johansen test produces two likelihood ratio statistics, namely the trace and the maximum eigenvalue. Both statistics are reported in this paper. Schwarz's Bayesian information criterion criteria is used to select the optimal lag length. The next stage involves implementing the Granger causality testing within the VECM framework. Granger tests provide the short-run dynamics between changes in the short-term interest rate, Canadian government securities yields, and other control variables.

Table 1: Definition and Source of the Variables

Variable names	Data description	Frequency	Sources
Short-term interest rate			
TB3M	Treasury bills, 3-month, yield, %; 1/4/1990–12/31/2018	Daily	Bank of Canada; Macrobond
Canadian government security yields			
GB2Y	Canadian government securities, 2-year, yield, %; 1/1/1990–12/31/2018	Daily	Bank of Canada; Macrobond
GB3Y	Canadian government securities, 3-year, yield, %; 1/1/1990–12/31/2018	Daily	Bank of Canada; Macrobond
GB5Y	Canadian government securities, 5-year, yield, %; 1/1/1990–12/31/2018	Daily	Bank of Canada; Macrobond
GB7Y	Canadian government securities, 7-year, yield, %; 1/1/1990–12/31/2018	Daily	Bank of Canada; Macrobond
GB10Y	Canadian government securities, 10-year, yield, %; 1/1/1990–12/31/2018	Daily	Bank of Canada; Macrobond
GB30Y	Canadian government securities, 30-year, yield, %; 1/1/1991–12/31/2018	Daily	Bank of Canada; Macrobond
Equity			
TSX	S&P/TSX 60 equity index, price return, Canadian dollar, C$; 1/1/1990–12/31/2018	Daily	Toronto Stock Exchange; Macrobond
Energy prices			
BRENT	Crude oil, Brent Europe spot price, freight on board, US$; 1/1/1990–12/31/2018	Daily	Intercontinental Exchange; Macrobond
Currency			
CAD	FX spot rate, C$/US$; 1/1/1990–12/31/2018	Daily	Macrobond

Behavioral Equations

The following general equation is estimated to examine the relationship between the short-term interest rate and the long-term interest rate on Canadian government securities of various maturity tenors:

$$GB = F^{1}(STIR, FX, LN[EQUITY], LN[OIL]) \qquad (1)$$

where GB is the yield on Canadian government securities of different tenors, including 2-year (GB2Y), 3-year (GB3Y), 5-year (GB3Y), 7-year (GB7Y), 10-year (GB10Y), and 30-year (GB30Y) government bonds. The short-term interest rate (STIR) is the yield on Canadian 3-month Treasury bills (TB3M). The potential impact of foreign exchange (FX) is represented by the spot rate between the Canadian dollar and US dollar (CAD), measured as Canadian dollar (C$) per US dollar (US$). An increase (decrease) in CAD means that the Canadian dollar has depreciated (appreciated) with respect to the US dollar. Brent Europe spot price (BRENT) is used for oil prices (OIL). Standard and Poor's (S&P) and the Toronto Stock Exchange's (TSX) 60 equity index, which is an index of selected key stock prices listed in Canada's main stock exchange, is used for equity prices (EQUITY). The LN(.) represents the natural logarithmic transformation of selected variables.

Therefore, the behavioral equation estimated in this paper takes the following general form:

$$GB_{i} = F^{2}(TB3M, CAD, LN[TSX], LN[BRENT]) \qquad (2)$$

where i=2-year, 3-year, 5-year, 7-year, 10-year, and 30-year maturity tenors. Daily data on the relevant variables are used.

Empirical Findings

To examine the unit root properties of the variables, both ADF and PP unit root test results with a constant and trend term are reported in table 2. From this table, irrespective of the test used, the results show that the variables are nonstationary at levels, but stationary at first differences.

Table 2: Unit Root Tests

Variable	ADF	PP
GB2Y	-2.797	-2.698
ΔGB2Y	-80.173***	-80.018***
GB3Y	-2.913	-2.891
ΔGB3Y	-80.821***	-80.736***
GB5Y	-2.926	-2.824
ΔGB5Y	-83.339***	-83.339***
GB7Y	-2.915	-2.931
ΔGB7Y	-85.319***	-85.309***
GB10Y	-3.140	-3.003
ΔGB10Y	-84.059***	-84.019***
GB30Y	-3.444	-3.372
ΔGB30Y	-79.566***	-79.367***
TB3M	-2.808	-2.787
ΔTB3M	-25.530***	-83.569***
CAD	-1.642	-1.514
ΔCAD	-87.213***	-87.546***
LN[TSX]	-2.213	-2.062
ΔLN[TSX]	-41.190***	-87.319***
LN[BRENT]	-2.198	-2.145
ΔLN[BRENT]	-53.128***	-88.114***

Note: 1) *** indicates statistical significance at the 1 percent level. 2) The null hypothesis of both the ADF and PP tests is that the series contains unit roots.

Table 3 presents results from the Johansen cointegration tests, based on equation (2). The null hypothesis of no cointegration among variables in the model is rejected by both trace and maximum eigenvalue statistics in all six equations. The sequential testing for more than one cointegration equation by both statistics leads to the conclusion that there are two cointegrating vectors in each of the first four equations, and one cointegrating vector in each of the last two equations. Therefore, it is evident that there are long-run cointegrating relationships between government bond yields, short-term interest rates, and other relevant variables.

Table 3: Johansen Cointegration Tests (equation 2)

Hypothesized number of cointegrating equations	Eigenvalue	Trace statistic	Maximum eigenvalue statistics
	GB2Y=F²(TB3M, CAD, LN[TSX], LN[BRENT])		
None	0.007	108.295***	56.134***
At most 1	0.004	52.162**	33.319***
At most 2	0.002	18.844	11.771
At most 3	0.001	7.072	5.598
At most 4	0.000	1.474	1.474
	GB3Y=F²(TB3M, CAD, LN[TSX], LN[BRENT])		
None	0.006	96.320***	45.068***
At most 1	0.004	51.251**	32.103**
At most 2	0.002	19.148	11.912
At most 3	0.001	7.237	5.789
At most 4	0.000	1.447	1.447
	GB5Y=F²(TB3M, CAD, LN[TSX], LN[BRENT])		
None	0.005	87.195***	40.910***
At most 1	0.004	46.285*	27.481*
At most 2	0.002	18.805	11.712
At most 3	0.001	7.092	5.660
At most 4	0.000	1.432	1.432
	GB7Y=F²(TB3M, CAD, LN[TSX], LN[BRENT])		
None	0.005	84.364***	39.734***
At most 1	0.003	44.630*	25.317*
At most 2	0.002	19.313	11.943
At most 3	0.001	7.370	5.952
At most 4	0.000	1.418	1.418
	GB10Y=F²(TB3M, CAD, LN[TSX], LN[BRENT])		
None	0.005	84.988***	41.052**
At most 1	0.003	43.935	24.593
At most 2	0.002	19.342	12.088
At most 3	0.001	7.254	5.828
At most 4	0.000	1.426	1.426
	GB30Y=F²(TB3M, CAD, LN[TSX], LN[BRENT])		
None	0.006	85.942***	43.152***
At most 1	0.003	42.789	21.748
At most 2	0.002	21.042	14.355
At most 3	0.001	6.686	5.214
At most 4	0.000	1.472	1.472

Note: ***, **, and * indicate statistical significance at the 1 percent, 5 percent, and 10 percent levels, respectively.

To check whether or not the evidence of cointegration found is contingent in the above-mentioned model, additional models are constructed as follows:

$$GB_i = F^3(TB3M) \quad (3)$$

$$GB_i = F^4(TB3M, LN[TSX]) \quad (4)$$

$$GB_i = F^5(TB3M, CAD) \quad (5)$$

$$GB_i = F^6(TB3M, LN[BRENT]) \quad (6)$$

$$GB_i = F^7(TB3M, LN[TSX], LN[BRENT]) \quad (7)$$

$$GB_i = F^8(TB3M, LN[TSX], CAD) \quad (8)$$

$$GB_i = F^9(TB3M, LN[BRENT], CAD) \quad (9)$$

Cointegration tests are carried out for the models above and presented in tables 4–10. Table 4 shows that there is cointegration between government bond yields for all maturities and Treasury bill yields in models where the government bond yields are solely a function of Treasury bill yields. Table 5 shows evidence of cointegration between government bond yields for all maturity tenors, Treasury bill yields, and the log of equity prices. Table 6 shows that there is cointegration between Treasury bill yields, the Canadian dollar, and government bond yields, except for the 7-year and 10-year maturity tenors. In table 7, results are presented for the equation where government bond yields are a function of Treasury bills and the log of the crude oil price. Results show that there is evidence of cointegration between government bond yields and other variables, except for bonds of a 10-year maturity tenor. Table 8 presents results from estimating the cointegrating relationships between Treasury bill yields, government bond yields, the log of equity prices, and the log of crude oil prices. The cointegrating relationships are established irrespective of the specific maturity tenors of government bonds used in the equations. Table 9 shows that there is evidence of cointegration between Treasury bill yields, the log of equity prices, the Canadian dollar, and government bond yields of all maturity tenors, except for bonds of a 7-year maturity tenor. Finally, table 10 presents results from estimating the cointegrating relationships between Treasury bill yields, the log of crude oil prices, the Canadian dollar, and government bond yields. The cointegrating relationship is found in four of six equations.

These results, therefore, show that in most cases there is substantial evidence of cointegration in a wide range of models that include government bond yields of various maturity levels, Treasury bill yields, and a range of control variables, such as the log of equity prices, the log of crude oil prices, and the Canadian dollar.

Table 4: Johansen Cointegration Tests (equation 3)

Hypothesized number of cointegrating equations	Eigenvalue	Trace statistic	Maximum eigenvalue statistics
		GB2Y=F³(TB3M)	
None	0.004	37.586***	32.474***
At most 1	0.001	5.111**	5.111**
		GB3Y=F³(TB3M)	
None	0.003	30.607***	25.871***
At most 1	0.001	4.737**	4.737**
		GB5Y=F³(TB3M)	
None	0.003	22.908***	19.138***
At most 1	0.000	3.769*	3.769*
		GB7Y=F³(TB3M)	
None	0.002	19.678**	16.118**
At most 1	0.000	3.560*	3.560*
		GB10Y=F³(TB3M)	
None	0.002	18.465**	14.952**
At most 1	0.000	3.513*	3.513*
		GB30Y=F³(TB3M)	
None	0.002	22.033***	18.175**
At most 1	0.002	3.859**	3.859**

Note: ***, **, and * indicate statistical significance at the 1 percent, 5 percent, and 10 percent levels, respectively.

Table 5: Johansen Cointegration Tests (equation 4)

Hypothesized number of cointegrating equations	Eigenvalue	Trace statistic	Maximum eigenvalue statistics
		GB2Y=F⁴(TB3M, LN[TSX])	
None	0.007	64.098***	53.324***
At most 1	0.001	13.774*	10.488
At most 2	0.000	3.286*	3.286*
		GB3Y=F⁴(TB3M, LN[TSX])	
None	0.005	54.870***	40.983***
At most 1	0.001	13.887*	10.523
At most 2	0.000	3.364*	3.364*
		GB5Y=F⁴(TB3M, LN[TSX])	
None	0.004	45.778***	32.589***
At most 1	0.001	13.189	9.888
At most 2	0.000	3.301*	3.301*
		GB7Y=F⁴(TB3M, LN[TSX])	
None	0.004	42.409***	28.898***
At most 1	0.001	13.510*	10.041
At most 2	0.000	3.469*	3.469*
		GB10Y=F⁴(TB3M, LN[TSX])	
None	0.004	42.153***	28.266***
At most 1	0.001	13.887*	10.332
At most 2	0.000	3.555*	3.555*
		GB30Y=F⁴(TB3M, LN[TSX])	
None	0.003	40.297***	24.098**
At most 1	0.002	16.1999**	11.816
At most 2	0.001	4.383**	4.383**

Note: ***, **, and * indicate statistical significance at the 1 percent, 5 percent, and 10 percent levels, respectively.

Table 6: Johansen Cointegration Tests (equation 5)

Hypothesized number of cointegrating equations	Eigenvalue	Trace statistic	Maximum eigenvalue statistics
	GB2Y=F⁵(TB3M, CAD)		
None	0.005	48.130***	38.714***
At most 1	0.001	9.416	7.620
At most 2	0.000	1.796	1.796
	GB3Y=F⁵(TB3M, CAD)		
None	0.003	35.544***	26.224****
At most 1	0.001	9.320	7.615
At most 2	0.000	1.705	1.705
	GB5Y=F⁵(TB3M, CAD)		
None	0.003	27.404*	49.271*
At most 1	0.001	8.133	6.580
At most 2	0.000	1.553	1.553
	GB7Y=F⁵(TB3M, CAD)		
None	0.002	24.132	16.279
At most 1	0.001	7.853	6.382
At most 2	0.000	1.471	1.471
	GB10Y=F⁵(TB3M, CAD)		
None	0.002	22.822	15.319
At most 1	0.001	7.502	6.031
At most 2	0.000	1.472	1.472
	GB30Y=F⁵(TB3M, CAD)		
None	0.003	28.386*	21.448**
At most 1	0.001	6.937	5.472
At most 2	0.000	1.466	1.466

Note: ***, **, and * indicate statistical significance at the 1 percent, 5 percent, and 10 percent level, respectively.

Table 7: Johansen Cointegration Tests (equation 6)

Hypothesized number of cointegrating equations	Eigenvalue	Trace statistic	Maximum eigenvalue statistics
	GB2Y=F⁶(TB3M, LN[BRENT])		
None	0.006	55.971***	43.341***
At most 1	0.001	12.630	10.547
At most 2	0.000	2.083	2.083
	GB3Y=F⁶(TB3M, LN[BRENT])		
None	0.004	41.841***	28.904***
At most 1	0.001	12.938	10.892
At most 2	0.000	2.046	2.046
	GB5Y=F⁶(TB3M, LN[BRENT])		
None	0.003	31.948**	20.087*
At most 1	0.001	11.861	9.887
At most 2	0.000	1.974	1.974
	GB7Y=F⁶(TB3M, LN[BRENT])		
None	0.002	28.088*	16.333
At most 1	0.001	11.754	9.822
At most 2	0.000	1.932	1.932
	GB10Y=F⁶(TB3M, LN[BRENT])		
None	0.002	26.112	14.985
At most 1	0.001	11.128	9.211
At most 2	0.000	1.917	1.917
	GB30Y=F⁶(TB3M, LN[BRENT])		
None	0.003	31.738**	22.902**
At most 1	0.001	8.836	6.673
At most 2	0.000	2.163	2.163

Note: ***, **, and * indicate statistical significance at the 1 percent, 5 percent, and 10 percent level, respectively.

Table 8: Johansen Cointegration Tests (equation 7)

Hypothesized number of cointegrating equations	Eigenvalue	Trace statistic	Maximum eigenvalue statistics
	GB2Y=F[7](TB3M, LN[TSX], LN[BRENT])		
None	0.007	79.456***	54.729***
At most 1	0.002	24.727	14.521
At most 2	0.001	10.206	7.695
At most 3	0.000	2.511	2.511
	GB3Y=F[7](TB3M, LN[TSX], LN[BRENT])		
None	0.006	67.534***	42.483***
At most 1	0.002	25.051	14.964
At most 2	0.001	10.087	7.548
At most 3	0.000	2.538	2.538
	GB5Y=F[7](TB3M, LN[TSX], LN[BRENT])		
None	0.005	58.562***	34.917***
At most 1	0.002	23.645	13.641
At most 2	0.001	10.003	7.414
At most 3	0.000	2.589	2.589
	GB7Y=F[7](TB3M, LN[TSX], LN[BRENT])		
None	0.004	55.789***	31.735**
At most 1	0.002	24.053	13.862
At most 2	0.001	10.192	7.541
At most 3	0.000	2.650	2.650
	GB10Y=F[7](TB3M, LN[TSX], LN[BRENT])		
None	0.004	56.240***	32.425**
At most 1	0.002	23.815	13.820
At most 2	0.001	9.995	7.297
At most 3	0.000	2.698	2.698
	GB30Y=F[7](TB3M, LN[TSX], LN[BRENT])		
None	0.005	58.076***	34.058***
At most 1	0.002	24.018	15.393
At most 2	0.001	8.624	5.207
At most 3	0.000	3.418*	3.418*

Note: ***, **, and * indicate statistical significance at the 1 percent, 5 percent, and 10 percent level respectively.

Table 9: Johansen Cointegration Tests (equation 8)

Hypothesized number of cointegrating equations	Eigenvalue	Trace statistic	Maximum eigenvalue statistics
		GB2Y=F⁸(TB3M, LN[TSX], CAD)	
None	0.007	73.943***	54.753***
At most 1	0.002	19.190	11.859
At most 2	0.001	7.331	5.709
At most 3	0.000	1.622	1.622
		GB3Y=F⁸(TB3M, LN[TSX], CAD)	
None	0.005	61.058***	41.627***
At most 1	0.002	19.431	11.907
At most 2	0.001	7.524	5.943
At most 3	0.000	1.581	1.581
		GB5Y=F⁸(TB3M, LN[TSX], CAD)	
None	0.004	51.951**	33.346***
At most 1	0.001	18.605	11.222
At most 2	0.001	7.383	5.863
At most 3	0.000	1.520	1.520
		GB7Y=F⁸(TB3M, LN[TSX], CAD)	
None	0.004	48.405	29.472
At most 1	0.001	18.933	11.290
At most 2	0.001	7.643	6.144
At most 3	0.000	1.499	1.499
		GB10Y=F⁸(TB3M, LN[TSX], CAD)	
None	0.004	48.388**	29.311**
At most 1	0.002	19.077	11.542
At most 2	0.001	7.535	6.019
At most 3	0.000	1.516	1.516
		GB30Y=F⁸(TB3M, LN[TSX], CAD)	
None	0.004	48.445**	27.775**
At most 1	0.002	20.670	13.203
At most 2	0.001	7.467	5.891
At most 3	0.000	1.575	1.575

Note: ***, **, and * indicate statistical significance at the 1 percent, 5 percent, and 10 percent level, respectively.

Table 10: Johansen Cointegration Tests (equation 9)

Hypothesized number of cointegrating equations	Eigenvalue	Trace statistic	Maximum eigenvalue statistics
	GB2Y=F⁹(TB3M, LN[BRENT], CAD)		
None	0.006	70.862***	46.764***
At most 1	0.002	24.098	15.220
At most 2	0.001	8.878	7.5217
At most 3	0.000	1.356	1.356
	GB3Y=F⁹(TB3M, LN[BRENT], CAD)		
None	0.004	57.167***	32.927***
At most 1	0.002	24.239	15.315
At most 2	0.001	8.924	7.608
At most 3	0.000	1.316	1.316
	GB5Y=F⁹(TB3M, LN[BRENT], CAD)		
None	0.003	47.365*	24.898
At most 1	0.002	22.467	14.383
At most 2	0.001	8.083	6.810
At most 3	0.000	1.273	1.273
	GB7Y=F⁹(TB3M, LN[BRENT], CAD)		
None	0.003	43.656	21.717
At most 1	0.002	21.940	14.072
At most 2	0.001	7.867	6.633
At most 3	0.000	1.234	1.234
	GB10Y=F⁹(TB3M, LN[BRENT], CAD)		
None	0.003	41.934	20.585
At most 1	0.002	21.350	13.891
At most 2	0.001	7.459	6.233
At most 3	0.000	1.226	1.226
	GB30Y=F⁹(TB3M, LN[BRENT], CAD)		
None	0.003	47.914**	23.979
At most 1	0.002	23.935	17.038
At most 2	0.001	6.897	5.639
At most 3	0.000	1.258	1.258

Note: ***, **, and * indicate statistical significance at the 1 percent, 5 percent, and 10 percent level, respectively.

The next set of tables (11–16) present causality results based on the first model (equation 2). The causal ordering is established by applying the Granger causality test (that is, block exogeneity Wald test) within the VECM framework. These results indicate that in the short run, ΔTB3M Granger causes ΔGB2Y, ΔGB3Y, ΔGB5Y, and ΔGB7Y. This effect dissipates when ΔGB10Y and ΔGB30Y are used as dependent variables. ΔGBs consistently Granger causes ΔTB3M when the latter is included in the equation as the dependent variable. Therefore, it can be argued that the daily change in the short-term interest rate generally has an impact on the daily change in long-term bond yields with a maturity of 7 years or less. However, this impact disappears when the tenor of long-term bonds is 10 years and above. Long-term government bonds, however, consistently Granger cause the short-term interest rate. Therefore, in most cases, there is evidence of bidirectional causality between long-term bond yields and the short-term interest rate. Among other variables, there is evidence of bidirectional Granger causality between ΔGBs and ΔLN[TSX], unidirectional causality from ΔGBs to ΔCAD, and no Granger causality (with the exception of the GB10Y equation) between ΔGBs and ΔLN[BRENT].

Table 11: Granger Causality within the VECM Framework (equation 2: ΔGB2Y)

	Dependent Variable				
Excluded	ΔGB2Y	ΔTB3M	ΔCAD	ΔLN[TSX]	ΔLN[BRENT]
ΔGB2Y	-	610.904***	10.947***	14.375***	2.861
ΔTB3M	12.927***	-	3.238	7.404**	3.456
ΔCAD	0.932	5.929*	-	19.603***	96.721***
ΔLN[TSX]	9.409***	9.344***	180.028***	-	64.928***
ΔLN[BRENT]	0.021	1.808	9.204**	0.750	-
All	23.898***	627.197***	208.116***	53.073***	221.483

Note: *** and ** indicate statistical significance at the 1 percent and 5 percent levels, respectively.

Table 12: Granger Causality within the VECM Framework (equation 2: ΔGB3Y)

	Dependent Variable				
Excluded	ΔGB3Y	ΔTB3M	ΔCAD	ΔLN[TSX]	ΔLN[BRENT]
ΔGB3Y	-	437.504***	11.798***	14.428***	1.659
ΔTB3M	6.683**	-	2.456	9.077**	2.925
ΔCAD	3.008	6.584**	-	19.684***	95.851***
ΔLN[TSX]	10.000***	8.395**	178.159***	-	65.047***
ΔLN[BRENT]	0.851	1.525	9.122**	0.810	-
All	20.516***	453.419***	208.918***	53.163***	220.136***

Note: *** and ** indicate statistical significance at the 1 percent and 5 percent levels, respectively.

Table 13: Granger Causality within the VECM Framework (equation 2: ΔGB5Y)

	Dependent Variable				
Excluded	ΔGB5Y	ΔTB3M	ΔCAD	ΔLN[TSX]	ΔLN[BRENT]
ΔGB5Y	-	383.964***	14.492***	18.377***	3.033
ΔTB3M	9.028**	-	2.477	9.545***	4.189
ΔCAD	0.547	7.966**	-	20.018***	96.361***
ΔLN[TSX]	12.740***	10.371***	175.695***	-	64.494***
ΔLN[BRENT]	0.318	1.406	8.692**	0.881	-
All	22.874***	400.211***	211.592***	57.236***	220.680***

Note: *** and ** indicate statistical significance at the 1 percent and 5 percent levels, respectively.

Table 14: Granger Causality within the VECM Framework (equation 2: ΔGB7Y)

	Dependent Variable				
Excluded	ΔGB7Y	ΔTB3M	ΔCAD	ΔLN[TSX]	ΔLN[BRENT]
ΔGB7Y	-	280.816***	9.255***	22.406***	3.591
ΔTB3M	8.363**	-	1.850	9.344***	4.102
ΔCAD	0.024	9.398***	-	20.981***	95.745***
ΔLN[TSX]	14.625***	8.873**	176.300***	-	64.232***
ΔLN[BRENT]	0.430	1.422	8.804**	0.988	-
All	24.130***	296.900***	206.301***	61.284***	220.832***

Note: *** and ** indicate statistical significance at the 1 percent and 5 percent levels, respectively.

Table 15: Granger Causality within the VECM Framework (equation 2: ΔGB10Y)

	Dependent Variable				
Excluded	ΔGB10Y	ΔTB3M	ΔCAD	ΔLN[TSX]	ΔLN[BRENT]
ΔGB10Y	-	234.761***	13.624***	24.447***	10.420***
ΔTB3M	2.604	-	1.635	10.477***	3.757
ΔCAD	0.397	8.260**	-	20.861***	96.648***
ΔLN[TSX]	11.791***	8.900**	174.307***	-	63.650***
ΔLN[BRENT]	0.642	1.793	8.844**	0.788	-
All	15.519**	251.726***	210.790***	63.845***	228.373***

Note: *** and ** indicate statistical significance at the 1 percent and 5 percent levels, respectively.

Table 16: Granger Causality within the VECM Framework (equation 2: ΔGB30Y)

	Dependent Variable				
Excluded	ΔGB30Y	ΔTB3M	ΔCAD	ΔLN[TSX]	ΔLN[BRENT]
ΔGB30Y	-	193.012***	6.120**	15.873***	4.488
ΔTB3M	2.077	-	1.860	12.102***	4.772*
ΔCAD	0.040	7.820**	-	21.309***	103.504***
ΔLN[TSX]	4.901*	7.454**	175.774***	-	74.553***
ΔLN[BRENT]	1.915	3.307	10.921***	1.636	-
All	9.213	209.095***	200.156***	55.550***	249.446***

Note: ***, **, and * indicate statistical significance at the 1 percent, 5 percent, and 10 percent levels, respectively.

The last set of tables (17–23) present Granger causality results from estimating equations 3–9. These results reinforce the earlier findings from equation 2. In most cases, ΔTB3M Granger causes ΔGB_i; in all cases ΔGB_i Granger causes ΔTB3M. Therefore, there is general evidence of bidirectional Granger causality between ΔTB3M and ΔGB_i. The other bidirectional Granger relationship that is generally evident in these findings is the connection between the change in bond yields and the change in the log of the equity index: ΔGB_i and ΔLN[TSX]. It should be recognized that the findings related to Granger causality merely establish statistical evidence of temporal precedence, not definitive provenience.

Table 17: Granger Causality within the VECM Framework (equation 3)

	Dependent Variable	
Excluded	ΔGB2Y	ΔTB3M
ΔGB2Y	-	631.692***
ΔTB3M	15.921***	-
All	15.921***	631.692***
	ΔGB3Y	ΔTB3M
ΔGB3Y	-	438.330***
ΔTB3M	6.504**	-
All	6.504**	438.330***
	ΔGB5Y	ΔTB3M
ΔGB5Y	-	380.662***
ΔTB5M	8.858**	-
All	8.858**	380.662***
	ΔGB7Y	ΔTB3M
ΔGB7Y	-	277.319***
ΔTB7M	8.538**	-
All	8.538**	277.319***
	ΔGB10Y	ΔTB3M
ΔGB10Y	-	230.690***
ΔTB10M	2.732	-
All	2.732	230.690***
	ΔGB30Y	ΔTB3M
ΔGB30Y	-	188.261***
ΔTB10M	2.119	-
All	2.119	188.261***

Note: *** and ** indicate statistical significance at the 1 percent and 5 percent levels, respectively.

Table 18: Granger Causality within the VECM Framework (equation 4)

	Dependent Variable		
Excluded	ΔGB2Y	ΔTB3M	ΔLN[TSX]
ΔGB2Y	-	613.937***	14.987***
ΔTB3M	12.616***	-	7.877***
ΔLN[TSX]	9.459***	12.794***	-
All	22.345***	619.851***	32.988***
	ΔGB3Y	ΔTB3M	ΔLN[TSX]
ΔGB3Y	-	439.634***	14.857***
ΔTB3M	6.169**	-	9.606***
ΔLN[TSX]	9.231***	11.732***	-
All	15.735***	445.363***	32.881***
	ΔGB5Y	ΔTB3M	ΔLN[TSX]
ΔGB5Y	-	384.834***	18.449***
ΔTB3M	8.715**	-	10.161***
ΔLN[TSX]	12.580***	14.442***	-
All	21.575***	390.665***	36.621***
	ΔGB7Y	ΔTB3M	ΔLN[TSX]
ΔGB7Y	-	280.232***	21.476***
ΔTB3M	8.357**	-	9.949***
ΔLN[TSX]	14.588***	12.970***	-
All	23.281***	285.978***	39.638***
	ΔGB10Y	ΔTB3M	ΔLN[TSX]
ΔGB10Y	-	233.853***	23.001***
ΔTB3M	2.585	-	10.877***
ΔLN[TSX]	11.540***	12.367***	-
All	14.397***	239.559***	41.117***
	ΔGB30Y	ΔTB3M	ΔLN[TSX]
ΔGB30Y	-	190.878***	14.111***
ΔTB3M	2.103	-	12.712***
ΔLN[TSX]	5.087*	10.222***	-
All	7.215	196.359***	31.935***

Note: ***, **, and * indicate statistical significance at the 1 percent, 5 percent, and 10 percent levels, respectively.

Table 19: Granger Causality within the VECM Framework (equation 5)

	Dependent Variable		
Excluded	ΔGB2Y	ΔTB3M	ΔCAD
ΔGB2Y	-	611.191***	13.465***
ΔTB3M	13.072***	-	4.241
ΔCAD	1.457	8.676**	-
All	14.364***	621.348***	16.683***
	ΔGB3Y	ΔTB3M	ΔCAD
ΔGB3Y	-	438.127***	16.021***
ΔTB3M	6.873**	-	3.459
ΔCAD	3.297	9.240***	-
All	9.797**	447.704***	19.239***
	ΔGB5Y	ΔTB3M	ΔCAD
ΔGB5Y	-	383.026***	21.687***
ΔTB3M	9.130**	-	3.583
ΔCAD	0.799	11.307***	-
All	9.656**	392.438***	24.913***
	ΔGB7Y	ΔTB3M	ΔCAD
ΔGB7Y	-	281.376***	15.738***
ΔTB3M	8.403**	-	2.967
ΔCAD	0.341	12.696***	-
All	8.880*	290.534***	18.961***
	ΔGB10Y	ΔTB3M	ΔCAD
ΔGB10Y	-	233.051***	21.596***
ΔTB3M	2.821	-	2.795
ΔCAD	0.422	11.011***	-
All	3.156	242.100***	24.823***
	ΔGB30Y	ΔTB3M	ΔCAD
ΔGB30Y	-	191.280***	7.221**
ΔTB3M	2.003	-	3.221
ΔCAD	0.165	10.107***	-
All	2.253	198.577***	10.239**

Note: ***, **, and * indicate statistical significance at the 1 percent, 5 percent, and 10 percent levels, respectively.

Table 20: Granger Causality within the VECM Framework (equation 6)

	Dependent Variable		
Excluded	ΔGB2Y	ΔTB3M	ΔLN[BRENT]
ΔGB2Y	-	609.924***	2.981
ΔTB3M	12.840***	-	0.667
ΔLN[BRENT]	0.014	0.968	-
All	12.849**	611.012	4.663
	ΔGB3Y	ΔTB3M	ΔLN[BRENT]
ΔGB3Y	-	436.563***	2.929
ΔTB3M	6.454**	-	0.378
ΔLN[BRENT]	0.787	0.761	-
All	7.225	437.766***	4.618
	ΔGB5Y	ΔTB3M	ΔLN[BRENT]
ΔGB5Y	-	379.441***	3.844
ΔTB3M	8.825**	-	0.761
ΔLN[BRENT]	0.292	0.871	-
All	9.113*	380.776***	5.530
	ΔGB7Y	ΔTB3M	ΔLN[BRENT]
ΔGB7Y	-	276.406***	5.295*
ΔTB3M	8.530**	-	0.789
ΔLN[BRENT]	0.326	0.894	-
All	8.843*	277.832***	6.981
	ΔGB10Y	ΔTB3M	ΔLN[BRENT]
ΔGB10Y	-	230.212***	12.828***
ΔTB3M	2.731	-	0.867
ΔLN[BRENT]	0.563	1.050	-
All	3.295	231.714***	14.513***
	ΔGB30Y	ΔTB3M	ΔLN[BRENT]
ΔGB30Y	-	188.701***	7.986**
ΔTB3M	2.033	-	1.167
ΔLN[BRENT]	1.999	2.157	-
All	4.072	190.931***	10.098**

Note: ***, **, and * indicate statistical significance at the 1 percent, 5 percent, and 10 percent levels, respectively.

Table 21: Granger Causality within the VECM Framework (equation 7)

	Dependent Variable			
Excluded	ΔGB2Y	ΔTB3M	ΔLN[TSX]	ΔLN[BRENT]
ΔGB2Y	-	613.400***	14.931***	1.878
ΔTB3M	12.630***	-	7.867**	1.856
ΔLN[TSX]	9.456***	12.901***	-	127.533***
ΔLN[BRENT]	0.021	1.151	0.451	-
All	22.366***	620.716***	33.256***	131.727
	ΔGB3Y	ΔTB3M	ΔLN[TSX]	ΔLN[BRENT]
ΔGB3Y	-	438.995***	14.884***	1.617
ΔTB3M	6.180**	-	9.571***	1.363
ΔLN[TSX]	9.399***	11.929***	-	127.166***
ΔLN[BRENT]	0.962	0.899	0.489	-
All	16.696**	446.314	33.246***	131.179***
	ΔGB5Y	ΔTB3M	ΔLN[TSX]	ΔLN[BRENT]
ΔGB5Y	-	383.982***	18.588***	2.189
ΔTB3M	8.705**	-	10.090***	2.200
ΔLN[TSX]	12.622***	14.668***	-	126.065***
ΔLN[BRENT]	0.339	0.882	0.548	-
All	21.914***	391.631***	37.094***	131.031
	ΔGB7Y	ΔTB3M	ΔLN[TSX]	ΔLN[BRENT]
ΔGB7Y	-	279.292***	21.697***	3.327
ΔTB3M	8.372**	-	9.860***	2.184
ΔLN[TSX]	14.660***	13.236***	-	125.466***
ΔLN[BRENT]	0.402	0.885	0.632	-
All	23.697***	286.895***	40.180***	131.853
	ΔGB10Y	ΔTB3M	ΔLN[TSX]	ΔLN[BRENT]
ΔGB10Y	-	232.867***	23.325	9.772***
ΔTB3M	2.593	-	10.741	2.192
ΔLN[TSX]	11.659***	12.740***	-	124.047***
ΔLN[BRENT]	0.681	1.002	0.701	
All	15.095**	240.536***	41/752***	137/837***
	ΔGB30Y	ΔTB3M	ΔLN[TSX]	ΔLN[BRENT]
ΔGB30Y	-	190.107***	14.547***	5.262*
ΔTB3M	2.086	-	12.575***	2.801
ΔLN[TSX]	4.988*	11.184***	-	-
ΔLN[BRENT]	1.871	2.272	1.553	141.503***
All	9.142	198.617***	33.502***	151.020***

Note: ***, **, and * indicate statistical significance at the 1 percent, 5 percent, and 10 percent levels, respectively.

Table 22: Granger Causality within the VECM Framework (equation 8)

	Dependent Variable			
Excluded	ΔGB2Y	ΔTB3M	ΔLN[TSX]	ΔCAD
ΔGB2Y	-	611.594***	14.316***	11.397***
ΔTB3M	12.882***	-	7.398**	3.076
ΔLN[TSX]	9.001**	8.985**	-	179.009***
ΔCAD	1.024	5.273*	19.647***	-
All	23.368***	624.978***	52.610***	195.773***
	ΔGB3Y	ΔTB3M	ΔLN[TSX]	ΔCAD
ΔGB3Y	-	438.712***	14.301***	12.179***
ΔTB3M	6.651**	-	9.090**	2.314
ΔLN[TSX]	9.241***	7.979**	-	177.142***
ΔCAD	3.352	5.895*	19.724***	-
All	19.085***	451.439***	52.611***	196.608
	ΔGB5Y	ΔTB3M	ΔLN[TSX]	ΔCAD
ΔGB5Y	-	385.474***	18.215***	15.364***
ΔTB3M	9.029**	-	9.557***	2.368
ΔLN[TSX]	12.404***	9.873***	-	174.480***
ΔCAD	0.671	7.269**	20.049***	-
All	22.251***	398.219***	56.661***	199.694***
	ΔGB7Y	ΔTB3M	ΔLN[TSX]	ΔCAD
ΔGB7Y	-	282.386***	22.203***	10.078***
ΔTB3M	8.370**	-	9.346***	1.754
ΔLN[TSX]	14.281***	8.359**	-	175.072***
ΔCAD	0.037	8.647**	21.008***	-
All	23.328***	294.871***	60.648***	194.318***
	ΔGB10Y	ΔTB3M	ΔLN[TSX]	ΔCAD
ΔGB10Y	-	234.641***	23.421***	14.449***
ΔTB3M	2.750	-	10.244***	1.644
ΔLN[TSX]	11.555***	8.144**	-	173.507***
ΔCAD	0.463	7.347**	20.690***	-
All	14.891**	247.020***	61.786***	198.635***
	ΔGB30Y	ΔTB3M	ΔLN[TSX]	ΔCAD
ΔGB30Y	-	192.317***	15.086***	6.659**
ΔTB3M	2.095	-	11.811***	1.766
ΔLN[TSX]	4.973*	6.391**	-	175.939***
ΔCAD	0.007	6.722**	21.089***	-
All	7.286	202.868***	52.975***	186.268***

Note: ***, **, and * indicate statistical significance at the 1 percent, 5 percent, and 10 percent levels, respectively.

Table 23: Granger Causality within the VECM Framework (equation 9)

	Dependent Variable			
Excluded	ΔGB2Y	ΔTB3M	ΔLN[BRENT]	ΔCAD
ΔGB2Y	-	609.291***	4.157	12.206***
ΔTB3M	13.036***	-	2.631	4.401
ΔLN[BRENT]	0.001	1.401	-	11.567***
ΔCAD	1.461	8.765**	157.339***	-
All	14.344**	620.585***	162.445***	27.605***
	ΔGB3Y	ΔTB3M	ΔLN[BRENT]	ΔCAD
ΔGB3Y	-	437.173***	2.908	14.672***
ΔTB3M	6.853***	-	2.227	3.613
ΔLN[BRENT]	0.681	1.135	-	11.369***
ΔCAD	3.194	9.255***	155.634***	-
All	10.480	447.987***	160.816***	30.023***
	ΔGB5Y	ΔTB3M	ΔLN[BRENT]	ΔCAD
ΔGB5Y	-	382.228***	4.328	19.794***
ΔTB3M	9.205**	-	3.292	3.771
ΔLN[BRENT]	0.252	1.128	-	10.603***
ΔCAD	0.734	11.198***	155.767***	-
All	10.005	393.091***	161.875***	34.949***
	ΔGB7Y	ΔTB3M	ΔLN[BRENT]	ΔCAD
ΔGB7Y	-	280.647***	5.006*	14.125***
ΔTB3M	8.485**	-	3.330	3.154
ΔLN[BRENT]	0.357	1.172	-	10.567***
ΔCAD	0.386	12.678***	154.158***	-
All	9.383	291.454***	161.786***	29.152***
	ΔGB10Y	ΔTB3M	ΔLN[BRENT]	ΔCAD
ΔGB10Y	-	232.352***	13.203***	19.419***
ΔTB3M	2.876	-	3.191	2.957
ΔLN[BRENT]	0.550	1.308	-	9.932***
ΔCAD	0.389	11.095***	154.306***	-
All	3.782	243.231***	169.743***	34.353***
	ΔGB30Y	ΔTB3M	ΔLN[BRENT]	ΔCAD
ΔGB30Y	-	192.172***	6.255**	6.067**
ΔTB3M	1.996	-	3.819	3.386
ΔLN[BRENT]	1.898	3.169	-	13.277***
ΔCAD	0.163	11.504***	169.076***	-
All	4.170	203.572***	179.438***	23.568***

Note: ***, **, and * indicate statistical significance at the 1 percent, 5 percent, and 10 percent levels, respectively.

4. CONCLUSION

This paper examines the Keynesian perspective on the relationship between Canadian government securities yields and the short-term interest rate by examining their long-run and short-run dynamics. A set of macroeconomic and financial variables are included in the regressions to control for the variables' potential impacts on government securities yields. There is ample evidence that the short-term and long-term interest rates are cointegrated in a wide range of models.

The empirical findings reported in the paper have implications for both economic theory and public policy, as well as for the ongoing debate in macroeconomics as reflected in the discussions pertaining to the implementation of monetary policy and central bank operations (Bindseil 2004; Fullwiler 2008 [2017]; Kregel 2014; Lavoie 2014), the fiscal theory of price (Sims 2013), and other issues on monetary policy (Davidson 2015; Wray 2012).

The results obtained show evidence of short-run and long-run dynamics between Canadian government bond yields, the short-term interest rate, and other macroeconomic and financial variables, such as the exchange rate of the Canadian dollar, the log of the price of crude oil, and the log of the equity index. Therefore, one can argue that the Bank of Canada's actions affect Canadian government bond yields primarily through the short-term interest rate on Treasury bills. This supports Keynes's view that the central bank's actions influence the long-term interest rate on government bonds mainly through the effects of its actions on the short-term interest rate. The findings also show that while the short-term interest rate is an important driver of the long-term interest rate on Canadian government securities yields, other factors—such as the exchange rate of the Canadian dollar, the price of crude oil, and the equity index—also matter.

These findings are relevant for policy issues in Canada and elsewhere. The findings can inform the Bank of Canada in formulating its monetary policy, it can provide useful information to the monetary authorities in assessing and evaluating the monetary transmission, and it can also be useful for fiscal policy. The findings can inform private investors' portfolio decisions and views, as well as the Treasury's management of government debt. It also can help policymakers in making decisions concerning fiscal policy, and in assessing the impact of fiscal stimulus and contraction on long-term interest rates on Canadian government bonds. It can be of particular interest if these findings are supplemented with additional results obtained from macroeconomic models that incorporate quarterly macroeconomic data, especially concerning ratios of government debt and fiscal deficit as a share of national income.

REFERENCES

Akram, T. 2014. "The Economics of Japan's Stagnation." *Business Economics* 49(3): 156–75.

Akram, T., and A. Das. 2014. "Understanding the Low Yields of the Long-Term Japanese Sovereign Debt." *Journal of Economic Issues* 48(2): 331–40.

———. 2015. "A Keynesian Explanation of Indian Government Bond Yields." *Journal of Post Keynesian Economics* 38(4): 565–87.

———. 2017. "The Dynamics of Government Bond Yields in the Eurozone." *Annals of Financial Economics* 12(3): 1750011-1-1750011-18.

———. 2019a. "The Long-Run Determinants of Indian Government Bond Yields." *Asian Development Review* 36(1): 168–205.

———. 2019b. "An Analysis of the Daily Changes in US Treasury Security Yields." Levy Institute Working Papers No. 934. Annandale-on-Hudson, NY: Levy Economics Institute of Bard College.

———. Forthcoming. "Australian Government Bonds Nominal Yields: A Keynesian Perspective." *Annals of Financial Economics*.

Akram, T., and H. Li. 2016. "The Empirics of Long-Term US Interest Rates." Levy Economics Working Papers No. 863. Annandale-on-Hudson, NY: Levy Economics Institute of Bard College.

———. 2017. "What Keeps Long-Term U.S. Interest Rates So Low?" *Economic Modelling* 60: 380–90.

———. 2018. "The Dynamics of JGBs Nominal Yields." Levy Institute Working Paper No. 906. Annandale-on-Hudson, NY: Levy Economics Institute of Bard College.

———. 2019a. "The Impact of the Bank of Japan's Monetary Policy on Japanese Government Bonds' Low Nominal Yields." Levy Economics Institute Working Paper No. 938. Annandale-on-Hudson, NY: Levy Economics Institute of Bard College.

———. 2019b. "An Inquiry Concerning Long-Term U.S. Interest Rates Using Monthly Data." *Applied Economics*. Available at: https://doi.org/10.1080/00036846.2019.1693696

Ardagna, S., F. Caselli, and T. Lane. 2007. "Fiscal Discipline and the Cost of Public Debt Service: Some Estimates for OECD Countries." *The B. E. Journal of Macroeconomics* 7(1): 1–33.

Baldacci, E., and M. Kumar. 2010. "Fiscal Deficits, Public Debt, and Sovereign Bond Yields." IMF Working Paper No. 10/184. Washington, DC: International Monetary Fund.

Bindseil, U. 2004. *Monetary Policy Implementation: Theory, Past and Present*. Oxford and New York: Oxford University Press.

Bollerslev, T., J. Cai, and F. M. Song. 2000. "Intraday Periodicity, Long Memory Volatility, and Macroeconomic Announcement Effects in the US Treasury Bond Market." *Journal of Empirical Finance* 7(1): 37–55.

Davidson, P. 2015. *Post Keynesian Theory and Policy: A Realistic Analysis of the Market-Oriented Capitalist Economy*. Cheltenham, UK and Northampton, MA: Edward Elgar.

Dickey, D. A., and W. A. Fuller. 1979. "Distribution of the Estimators for Autoregressive Time Series with a Unit Root." *Journal of the American Statistical Association* 74(366): 427–31.

———. 1981. "Likelihood Ratio Statistics for Autoregressive Time Series with a Unit Root." *Econometrica* 49(4): 1057–72.

Doi, T., T. Hoshi, and T. Okimoto. 2011. "Japanese Government Debt and Sustainability of Fiscal Policy." *Journal of the Japanese and International Economies* 25(4): 414–33.

Elmendorf, D. W., and N. G. Mankiw. 1998. "Government Debt." NBER Working Paper No. 6470. Cambridge, MA: National Bureau of Economic Research.

Fullwiler, S. T. 2008 [2017]. "Modern Central Bank Operations: The General Principles." In Louis-Philippe Rochon and Sergio Rossi (eds.), *Advances in Endogenous Money Analysis*. Northampton, MA: Edward Elgar.

Gruber, J. W., and S. B. Kamin. 2012. "Fiscal Positions and Government Bond Yields in OECD Countries." *Journal of Money, Credit, and Banking* 44(8): 1563–87.

Gürkaynak, R. S., B. Sack, and J. H. Wright. 2007. "The US Treasury Yield Curve: 1961 to the Present." *Journal of Monetary Economics* 54(8): 2291–304.

Hansen, G., and S. İmrohoroğlu. 2013. "Fiscal Reform and Government Debt in Japan: A Neoclassical Perspective." *Review of Economic Dynamics* 21: 201–24.

Horioka, C. Y., T. Nomoto, and A. Terada-Hagiwara. 2014. "Why Has Japan's Massive Government Debt Not Wreaked Havoc (Yet)?" *The Japanese Political Economy* 40(2): 3–23.

Hoshi, T., and T. Ito. 2013. "Is the Sky the Limit? Can Japanese Government Bonds Continue to Defy Gravity?" *Asian Economic Policy Review* 8(2): 218–47.

———. 2014. "Defying Gravity: Can Japanese Sovereign Debt Continue to Increase Without a Crisis?" *Economic Policy* 29(77): 5–44.

Johansen, S. 1991. "Estimation and Hypothesis Testing of Cointegration Vectors in Gaussian Vector Autoregressive Models." *Econometrica* 59(6): 1551–80.

———. 1995. *Likelihood-Based Inference in Cointegrated Vector Autoregressive Models*. Oxford: Oxford University Press.

Keynes, J. M. 1930. *A Treatise on Money, Vol. II: The Applied Theory of Money*. London: Macmillan.

———. [1936] 2007. *The General Theory of Employment, Interest, and Money*. New York: Palgrave Macmillan.

Kregel, J. 2011. "Was Keynes' Monetary Policy *À Outrance* in the *Treatise*, A Forerunner of ZIRP and QE? Did He Change His Mind in the *General Theory*?" Levy Institute Policy Note No. 2011/4. Annandale-on-Hudson, NY: Levy Economics Institute of Bard College.

Lam, R. W., and K. Tokuoka. 2011. "Assessing the Risks to the Japanese Government Bond (JGB) Market." *Journal of International Commerce, Economics and Policy* 4(1): 1350002-1-1350002-15.

Lavoie, M. 2014. *Post-Keynesian Economics: New Foundations.* Cheltenham, UK and Northampton, MA: Edward Elgar.

Lerner, A. P. 1947. "Money as a Creature of the State." *American Economic Review* 37(2): 312–17.

Levrero, E. S., and M. Deleidi. 2019. "The Causal Relationship Between Short- and Long-Term Interest Rates: An Empirical Assessment of the United States." MPRA Paper No. 93608. Available at: https://mpra.ub.uni-muenchen.de/93608/

Macrobond. Various years. Macrobond subscription services (accessed October 20, 2019).

Paccagnini, A. 2016. "The Macroeconomic Determinants of the US Term Structure during the Great Moderation." *Economic Modelling* 52(A): 216–25.

Phillips, P. C. B., and P. Perron. 1988. "Testing for a Unit Root in Time Series Regression." *Biometrika* 75(2): 335–46.

Poghosyan, T. 2014. "Long-Run and Short-Run Determinants of Sovereign Bond Yields in Advanced Economies." *Economic Systems* 38(1): 100–14.

Reinhart, C. M., and K. S. Rogoff. 2009. *This Time Is Different: Eight Centuries of Financial Folly*. Princeton, NJ: Princeton University Press.

Riefler, W. W. 1930. *Money Rates and Money Markets in the United States*. New York and London: Harper & Brothers.

Simoski, S. 2019. "A Keynesian Exploration of the Determinants of Government Bond Yields for Brazil, Colombia, and Mexico." Master of Science thesis, Levy Economics Institute of Bard College. Annandale-on-Hudson, NY: Levy Economics Institute of Bard College.

Sims, C. A. 2013. "Paper Money." *American Economic Review* 103(2): 563–84.

Tkačevs, O., and K. Vilerts. 2019. "The Impact of Government Borrowing Costs on Fiscal Discipline." *Kyklos* 729(3): 446–71.

Tokuoka, K. 2012. "Intergenerational Implications of Fiscal Consolidation in Japan." IMF Working Paper No. 12/197. Washington, DC: International Monetary Fund.

Wray, L. R. 2012. *Modern Money Theory: A Primer on Macroeconomics for Sovereign Monetary Systems*. New York: Palgrave Macmillan.