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The Sraffian Supermultiplier and Cycles: Theory and Empirics

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ABSTRACT

This paper provides a theoretical and empirical reassessment of supermultiplier theory. First, we show that, as a result of the passive role it assigns to investment, the Sraffian supermultiplier (SSM) predicts that the rate of utilization leads the investment share in a dampened cycle or, equivalently, that a convergent cyclical motion in the utilization-investment share plane would be counterclockwise. Second, impulse response functions from standard recursive vector autoregressions (VAR) for postwar US samples strongly indicate that the investment share leads the rate of utilization, or that these cycles are clockwise. These results raise questions about the key mechanism underlying supermultiplier theory.

KEYWORDS: Sraffian Supermultiplier; Cyclical Growth; (Induced) Investment; Desired Rate of Capacity Utilization.

JEL CLASSIFICATION: E12; E24; E25; E32.

1 INTRODUCTION

The theory of the Sraffian supermultiplier (SSM) presents a vision of classical–Keynesian demand-driven growth. Initially conceptualized by Serrano (1995) and Bortis (1997), it proposes a macroeconomic adjustment driven by the growth rate of non capacity generating autonomous spending.[1] This adjustment allows for a demand-driven closure, combined with classical theory of distribution and the attainment of a constant exogenous-to-demand normal rate of utilization in the long run. The convergence of the system to an exogenous-to-demand normal rate of utilization addresses what some consider a key weakness of neo-Kaleckian models, which also combine a demand-led closure with a classical theory of distribution.

In this literature, Freitas and Serrano (2015) and Allain (2015) are two important contributions because they were the first to propose an adjustment mechanism which brings the system to its long-run steady state. The main idea is that accumulation—either the warranted growth rate in the case of Allain or investment-to-output ratio in the case of Freitas and Serrano—adjusts in response to deviations of utilization from its normal rate. Subsequent contributions such as Lavoie (2016) and Allain (2019, 2021) provide further discussion and extensions of this baseline model. One issue that arises in all these contributions is Harrodian instability, as the system becomes unstable if the adjustment of accumulation is "too fast."[2]

In this paper, we neither review the utilization controversy in all its detail, nor discuss Harrodian issues. Our focus instead lies on a particular aspect of SSM theory that has to our knowledge heretofore not been investigated: its inherent prediction that investment *lags* the cycle. This

[1] Examples of autonomous and non capacity generating expenditures considered in SSM literature include exports, (capitalist) consumption and residential investment financed by credit, firms' research and development expenditures, and (portions of) government spending.

[2] A critique of the SSM model in relation to the issue of Harrodian instability is provided by Skott (2019).

contrasts sharply with any other classical or Keynesian theory of cycles or growth, and, as we will argue, is rejected by a simple empirical test.

This feature of the SSM model is not accidental. The ability of the SSM model to combine a demand-led closure, the classical theory of distribution, and an exogenous-to-demand rate of utilization comes at a cost: unlike most classical and Keynesian theory it assigns only a passive role to investment. The cycles are the result of over- and under-shooting of accumulation as it adjusts to bring utilization to its normal level. As a result, utilization leads accumulation over the cycle.

Empirical investigations of SSM literature are still limited in number. Works such as Girardi and Pariboni (2016), Pérez-Montiel and Erbina (2020), and Haluska, Braga, and Summa (2021) aim to estimate the linkages between autonomous non capacity-generating expenditures and output growth, as well as to estimate the parameters of investment functions connecting supermultiplier effects with long-run growth and distribution. The first and third study apply time-series techniques to the US economy, while the second applies panel data methods to sixteen European countries. These models focus on "Granger causality," finding evidence of such linkages from autonomous demand components to investment. Furthermore, Haluska, Braga, and Summa (2021) estimate an SSM investment function that shows a slow speed of adjustment of capacity utilization to converge to its normal level, as well as inertial behavior of the investment share. These results are consonant with SSM predictions. However, none of these papers examine the direction of the cycle.

This paper provides an empirical reassessment of supermultiplier theory that does exactly that. First, as explained above, we show that Sraffian supermultiplier theory predicts that the rate of utilization leads the investment share in a dampened cycle, or equivalently that a convergent cyclical motion in the utilization-investment share-plane would be counter clockwise. Second, impulse response functions (IRFs) from standard recursive vector autoregressions (VARs) for postwar US samples strongly indicate that the investment share leads the rate of utilization, or that these cycles are clockwise. These results raise questions about the key mechanism underlying supermultiplier theory.

The remainder of the paper is organized as follows. The next section lays out a baseline supermultiplier model. It closely follows Freitas and Serrano (2015), as this paper provides an excellent description of the main theoretical building blocks of the SSM; Allain (2015) implies analog narratives. Section 3 contextualizes our recasting of the SSM. This critical discussion provides a foundation for subsequent empirical tests. In section 4, we discuss methodology, data and results: in short, IRFs from the VARs show that the investment share tends to lead the rate of utilization. This result is consistent with standard Keynesian theories, but contradicts a key SSM prediction. Section 5 concludes.

2 A BASELINE SUPERMULTIPLIER MODEL

Here we summarize SSM theory as formalized in Freitas and Serrano (2015), and show that the centrally important prediction of the theory is that the rate of utilization leads the investment share. We limit our exposition of the formal modeling in this literature to this paper since it presents a simple and tractable version. However, we also emphasize that other contributions in this vein imply analog predictions, and that therefore our empirical results pertain to SSM theory more broadly.[3] All relevant derivations are presented in Freitas and Serrano (2015), so that our summary will be concise.

[3] For an important example, see Allain (2015, fig. B1). In this model, the utilization rate adjusts rapidly and therefore is not included as a state variable in the dynamic system. The system is set up in the ratio of *autonomous and growing* government expenditures relative to the capital stock (λ) and accumulation rate (γ), and the dynamics of the model predict a counter clockwise cycle in the $< \lambda, \gamma >$ plane, or equivalently that λ leads γ. Further, and crucially, there is a positive relationship at a point in time between the rate of utilization and government expenditures—so that the model also implies that utilization leads investment.

To begin, let us define our notation. The rate of utilization is $U \equiv Y/Y^*$, where Y is the observed level real GDP and Y^* is the level of real GDP consistent with full capacity utilization: $Y^* \equiv \sigma K$, where K in turn is the stock of (non depreciating) capital. Further, $h \equiv I/Y$ is the share of real investment in real GDP. The rate of accumulation follows as $g \equiv I/K = h\sigma U$. The exogenously given and constant desired rate of utilization is $U^* = \mu$; the growth rate of non capacity-creating autonomous expenditures (i.e., consumption) is z.

In the short run, productive capacity is fixed. Macroeconomic equilibrium—where investment and savings equilibrate—is attained through variations in the rate of utilization, U. This implies two steady states; $U_1^* = 0$ and $U_2^* = z/(h\sigma)$, where the latter is non trivial and locally stable if $z > 0$. In the long run, productive capacity becomes endogenous and the model becomes two dimensional through endogenization of the investment share, h. Specifically, the time rate of change in the rate of utilization follows from its law of motion as the difference between the growth rates of output and capital stock; the time rate of change in the investment share responds positively to deviations of the rate of utilization from its desired long-run equilibrium, μ. In other words, capacity-creating expenditures feed into the capital stock so as to return U to its desired rate.

The following dynamic system—equivalent to equations (8) and (11) in Freitas and Serrano (2015, 267)—results in:

$$\dot{U} = U\left(z + \frac{\dot{h}}{s-h} - h\sigma U\right) \tag{1}$$

$$\dot{h} = h\gamma(U - \mu) \tag{2}$$

where $\gamma > 0$ is a speed of adjustment coefficient. Following Freitas and Serrano (2015), we further assume $z > 0$ at all times and also that $s > h$.

The non trivial steady state of this nonlinear system of differential equations is $U^* = \mu, h^* = z/(\sigma\mu)$. The Jacobian matrix of partial derivatives at the non trivial steady state is:

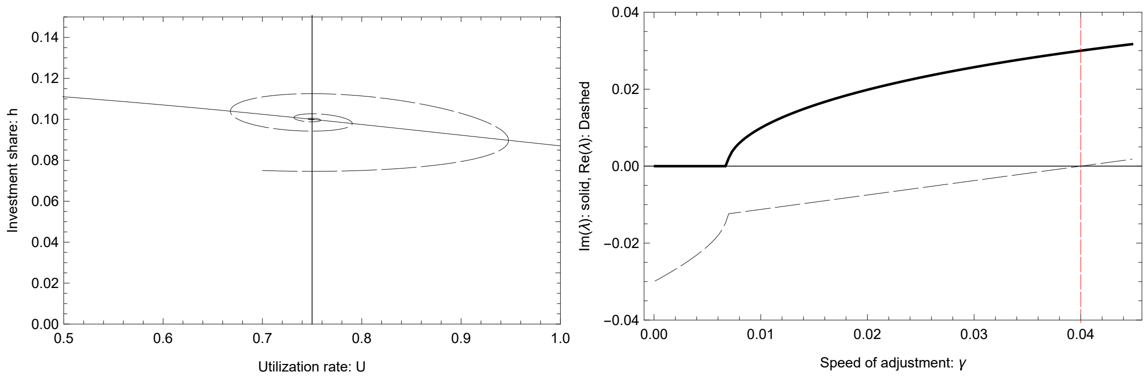
$$J^* = \begin{pmatrix} \left(\frac{h\gamma}{s-h} - \sigma h\right)\mu & -\sigma\mu^2 \\ \frac{z\gamma}{\sigma\mu} & 0 \end{pmatrix} \tag{3}$$

The sign pattern of the Jacobian is:

$$J^* = \begin{pmatrix} - & - \\ + & 0 \end{pmatrix}, \tag{4}$$

Off-diagonal and lower-right entries are signed unambiguously given assumptions made. The upper left entry is negative as long as $0 < \gamma < (s-h)\sigma$. Assuming this inequality holds, $Tr(J) < 0$. Further, $|J^*| = z\gamma\mu > 0$. Hence, both eigenvalues will be negative if real or will have negative real parts if complex, thus ensuring asymptotic stability. The oppositely signed off-diagonal entries make cycles likely: as the subsequent discussion shows, eigenvalues are very likely to be complex.

Figure 1: Dynamic System and its Eigenvalues.



(a) Phase diagram **(b)** Eigenvalues

Note: The left panel shows a phase diagram of the system discussed in section 2. The right panel shows real and imaginary parts of the pair of eigenvalues as a function of the speed of adjustment parameter γ. Numerical values used for this illustration are $z = 0.03, \sigma = Y^*/K = 0.4, s = \partial S/\partial Y = 0.2, \mu = U^* = 0.75, \gamma = 0.025$, which imply a steady state investment share of $h^* = z/(\sigma\mu) = 0.03/(0.4 * 0.75) = 0.1 = S/Y$ and a warranted growth rate of $g^* = h\sigma\mu = 0.1 * 0.4 * 0.75 = 0.03 = z$.

Figure 1 illustrates the dynamical system. The left panel shows a phase diagram and an exemplary trajectory. First, note that the isocline for the investment share is independent of the investment share itself. It responds only to the rate of utilization and hence is a vertical line at $U^* = \mu$. Deviations of U imply a change of h in the same direction: to the right (left) of $\dot{h} = 0$, h increases (decreases). The isocline for the rate of utilization is nonlinear, but downward sloping in the vicinity of the steady state. It is also own-stable, and deviations of h imply a change of U in the opposite direction: below (above) $\dot{U} = 0$, U increases (decreases). These dynamics are contained in the Jacobian matrix and are here further illustrated with an exemplary trajectory that clearly exhibits the counter clockwise pattern.[4]

Indeed, SSM theory predicts that in the trough of the recession, h is falling—since investment is depressed but non capacity-generating expenditures (i.e., the autonomous component of

[4] It would be straightforward to change the model to produce limit cycles as opposed to the dampened cycles presented in figure 1. One way would be that the adjustment of the investment share is not linear, but s-shaped in u with a Harrod-Kaldor type motivation: firms try to adjust capacity a lot more vigorously near normal utilization, μ, than further away from it.

consumption) grow at the constant rate $z > 0$. Therefore, U rises. As the recovery picks up steam, investment is induced and h increases, further increasing U. Ultimately, h overshoots and U declines through "excessive capacity creation." In short, autonomous expenditure leads the cycle and triggers the increase in utilization that then induces investment and capacity creation.

The right panel of figure 1 provides additional detail. The key issue here is that SSM theory as presented in Freitas and Serrano (2015) implies the prediction that cycles of investment share and the rate of utilization are very likely. First, note that oppositely signed off-diagonal entries in the Jacobian matrix are a necessary condition for the pair of eigenvalues of a linearized two-dimensional system of differential equations to be complex. A pair of complex eigenvalues with negative real parts indicates a stable focus: the model is asymptotically stable, but converges to the steady state in a cyclical motion. The right panel shows real and complex parts of the eigenvalues for a plausible calibration (discussed in the caption of the figure) as a function of the speed of adjustment coefficient. Over the relevant range of asymptotic stability, the eigenvalues are almost certainly complex. Hence, SSM theory contains the prediction that deviations from the steady state are corrected in a *cyclical and counter clockwise* fashion.

In the next section, we critically discuss these and other issues in order to motivate our subsequent empirical tests.

3 THEORY

Models of growth and distribution proposed by various economic traditions are defined by their closure with respect to accumulation and income distribution, as well as the "normal" long-run rate of capacity utilization. We can understand this triangle between accumulation, distribution, and utilization with reference to the Cambridge equation:

$$g = s_\pi \pi \sigma U = s_\pi r \tag{5}$$

where g is the accumulation rate, s_π is the saving rate of capitalists, and r is the rate of profit. If we treat s and σ as constant (assuming that saving is determined by slowly evolving norms and there is not much space for substitution), three variables remain that can be potentially determined within the system: the accumulation rate, income distribution, and the utilization rate.[5]

Starting with the determination of the accumulation rate, two main answers have been given. According to classical political economy and neoclassical economics, accumulation is determined by available savings. In other words, according to classical and neoclassical theory, as soon as we are able to specify the expenditure-saving decision, we automatically also know investment. This implies that the causality runs from the right- to the left-hand side of equation (5). Hence, the accumulation rate, g, is endogenous.

The other answer has been given by the Keynesian-Kaleckian theory of effective demand, where investment is posited to have an autonomous role, since saving and investment are distinct decisions. Kaldor (1955, 95) called this the "hypothesis that investment ... can be treated as an independent variable" the "Keynesian hypothesis." In terms of equation (5), this means that g on the left-hand side is exogenous and (some of) the variables on the right need to bear the adjustment. When it comes to the theory of distribution, classical political economy postulates that distribution is determined outside the economic sphere based on institutions and social norms. In terms of equation (5) this implies that distribution on the right-hand side is exogenous.

[5] A more detailed discussion of the relation between accumulation, distribution, and utilization can be found in Nikiforos (2020, sec. 2).

Another answer is provided by the neo-Keynesian theory. Here, distribution is endogenous and adjusts so that enough savings are generated to meet the exogenous investment demand. Different versions of this theory were suggested by Kaldor (1955), Robinson (1962), and Pasinetti (1962), among others. The main idea is that given that capitalists save more than workers, an increase in the profit share leads—all other things equal—to an increase in savings. Thus an increase in investment demand (on the left-hand side of equation [5]) leads to an increase in savings through an increase in the profit share (on the right-hand side). For neo-Keynesians it is the nominal wages that are determined outside of the economy based on bargaining, institutions, and social norms. Thus an increase in investment demand leads to an increase in prices, which—given constant nominal wages—leads to a decrease in the real wage and an increase in the profit share.

The adjustment through endogenous changes in distribution is imperative in neo-Keynesian models because the rate of utilization is considered to be exogenous. For Kaldor (1957) this was one of his stylized facts, while Robinson (1962) provided theoretical reasons to justify it. In terms of equation (5), if utilization and accumulation are both exogenous, adjustment *has to* come through changes in distribution. Garegnani (1992) called this closure with autonomous investment (exogenous utilization) and endogenous distribution the "First Keynesian Position."

Based on this logic, it is straightforward that the classical theory of distribution cannot be combined with the Keynesian hypothesis *and* exogenous-to-demand utilization. The combination of an autonomous role for demand and the classical theory of distribution requires utilization to become endogenous. This is how the system closes in the so-called Kaleckian model of growth and distribution, but also in a large part of the macroeconomic approach of neo-Sraffians. Garegnani (1992) called this kind of closure the "Second Keynesian Position."

Seen from this point of view, an important theoretical achievement of the SSM model is the combination of the Second Keynesian Position with an exogenous-to-demand rate of utilization.

As explained in the previous section, this result is obtained by giving autonomous expenditures the most prominent role. The addition of autonomous expenditure leads to the transformation of the Cambridge equation into:

$$
\begin{aligned}
g + g_z = s_\pi \pi \sigma U \iff \\
g_z = s_\pi \pi \sigma U - g
\end{aligned}
\tag{6}
$$

where g_z is autonomous expenditure normalized for capital stock. The new term allows for the role of demand, exogenous distribution, and exogenous utilization to be maintained (captured as exogenous g_z, π, and U, respectively) by making the accumulation rate endogenous.

However, this achievement of SSM theory comes at a considerable cost. A key shortcoming is that the concept of autonomous expenditure itself is not very convincing. Some categories of expenditure could potentially be exogenous in the short run, though that also is debatable: consumer durables and residential investment are more strongly interest elastic than other expenditure components. Importantly, it is unclear how these expenditures could be independent of the level of economic activity and other economic variables in the medium and long run. As discussed in detail in Nikiforos (2018, sec. 5), such a treatment disregards the implications of these kinds of expenditures on the related stocks of financial assets and liabilities.

For example, debt-financed consumption or residential investment can indeed be independent of the current level of income. However, these expenditure flows have important stock implications (e.g. accumulation of household debt). Saying that these expenditures are autonomous in the long run is tantamount to saying that the accumulation of debt does not matter, and that an increasing debt–income ratio of households does not matter. The literature on stock-flow consistent models (e.g., Godley and Lavoie 2007; Nikiforos and Zezza 2017) and the experience of the last several decades has shown that this is not a plausible way to look at modern capitalist economies.

A further and arguably more relevant issue for the present paper is that SSM theory assigns only a rudimentary and passive role to investment. Following Kaldor, the Keynesian hypothesis can be stated in two different ways: i) “demand matters” and ii) “investment ... can be treated as an independent variable.” In most settings these two definitions can be used interchangeably. However, this is not the case with SSM theory: here, only the first definition of the Keynesian hypothesis applies. Investment is not an independent variable. Rather, it passively adjusts to changes in the level of income and utilization.

This is important in light of our empirical results further below. The fact that in the SSM theory investment does not lead the cycle—as in most classical and Keynesian theories of economics fluctuations—is not accidental. It is a basic and built-in characteristic of the model. Given the primacy of autonomous expenditure for the behavior of the SSM system, shocks to the growth rate of autonomous expenditure cause changes in the level of utilization and then investment adjusts to bring the rate of utilization to its exogenous normal rate. In terms of equation (6), causality runs from g_z to g. Indeed, the growth rate of autonomous expenditure acquires a status similar to the natural growth rate of the Kaldorian First Keynesian position and the neoclassical growth model.

Two final issues have to do with the forces that drive the business cycle and their relation to long-run growth within the SSM model. First, due to the passive role of capacity-generating investment, any kind of cycle that is driven by cyclical fluctuations of investment is precluded. The usual investment driven cycles à la Marx, Keynes, Kalecki, Goodwin, and Minsky (and others) are incompatible with the SSM model.

Another candidate for the force that drive the cycle could be autonomous expenditure. However, because of the autonomy of this kind of expenditure, it is impossible to have *endogenous* cycles driven by it. For example, assume that debt-financed consumption or residential investment increases in the way it did in the United States in the decades before the 2007-09 crisis. In the

SSM model, the system would converge to a higher steady state; the dynamics that led to the crisis cannot be explained.

Therefore, and as was explained in the previous section, cyclical fluctuations are produced as the system adjusts towards the steady state, the result of the exogenous growth of autonomous expenditure and the changes of the investment share in order to bring utilization to its normal exogenous level. This is a rudimentary cycle, as the investment share over- and under-shoots the level that would be bring utilization to its proper level.

Another issue that arises here, related to the above, is that the forces that drive long-run growth in the SSM model are independent from the forces that drive the cycle. Cyclical fluctuations have no effect on the behavior of the system in the long run. This is a common issue in models that are driven in the long run by "natural growth rates," such as the neoclassical model, or neo-Kaldorian models that follow the neo-Keynesian closure described above. In the latter case, a possible way out is the endogenization of the natural growth rate. One could argue, for example, that demand, distribution or the state of the labor and product markets can have an effect on the population growth rate or the rate of technical change (e.g., along the lines of Kaldor-Verdoorn or induced technical change hypotheses). In this way it can be theorized that the forces that drive the system over the cycle have an effect on its natural growth rate, and thus the trajectory of the system in the long run (see [Barrales et al. (2021, sec. 5)] and references therein). However, such an endogenenization is impossible in the case of the SSM model because of the autonomy of autonomous expenditures.

Probably the only way to have a consistent story of the cycle with the long-run theory of growth as proposed by SSM theorists—where autonomous expenditure would drive both the cyclical fluctuations and long-run growth—would be to resort to random shocks to autonomous expenditure. The similarity to the Real Business Cycle (RBC) theory is striking but again not accidental. As mentioned in the previous paragraphs, the growth rate of autonomous expenditure in the SSM model plays technically the same role as the natural growth rate in the neoclassical model. Both are exogenous and the rest of the system in the respective models adjusts to them. Thus, in the same way that the RBC model provides the only consistent theory of the cycle and long-run growth within the neoclassical system, random shocks to autonomous expenditure could do the trick for the SSM model.

The dynamics described in the previous section could be understood along these lines. We can think of the system being in steady state; there is a shock to autonomous expenditure, and the system converges cyclically to the new steady state, with utilization adjusting first and investment following. If autonomous expenditure was modeled as a stochastic process with an error term, the resulting model would produce cyclical fluctuations around its long-run steady state. Of course, like in the case of the RBC theory, it is questionable whether such a theory of the cycle is really convincing. Is it really convincing that the downturn of 2007–9 was caused by a random exogenous decrease in credit-financed consumption and residential investment? Besides that, as we show in the next section the dynamics predicted in such an adjustment—with utilization leading and investment following—run contrary to the patterns that have been observed in the United States over the last decades.

4 EMPIRICS

This section lays out our empirical test(s). We briefly circumscribe (the quite standard) methodology. Section 4.2 summarizes our data sources and filtering. Subsequently, we present results.

4.1 Methodology

An empirical assessment about the theoretical assumptions presented in the previous section(s) can be implemented with Vector Autoregressive (VAR) models (Sims 1980). Put simply, this technique uses the Ordinary Least Squares (OLS) estimator to evaluate how one variable within a system is affected by lagged values of itself and of other variable(s). In our model, we have only two variables and thus we apply this technique to a two-equation system: the investment share is regressed on its own lagged values as well as on lagged values of the utilization rate. The same applies to the converse scenario, with U_t on the regression's left-hand side.

A reduced-form VAR(p) model can be represented as follows:

$$\mathbf{y_t} = \beta + \sum_{i=1}^{p} \mathbf{B_i}\, \mathbf{z_{t-i}} + \varepsilon_t \tag{7}$$

where $\mathbf{y_t}$ is $(h_t, U_t)'$, that is, a vector containing the model variables; β is a vector of intercept coefficients; $\mathbf{B_i}$ is a vector of reduced-form slope coefficients; ε_t is a vector of reduced-form residuals that are not mutually uncorrelated; finally, p is the lag length (order) of the model, i.e., the number of lags of the variables included on the right-hand side. The latter component is of crucial relevance to this technique, since including the appropriate number of lags provides a correct specification of the model (see next section for additional details).

From the VAR estimates, we are able to evaluate how its components react to shocks (i.e., deviations from the mean) to themselves and to other system variables. These impulse-response functions (IRFs) project how a one-standard deviation disturbance in one covariate affects other endogenous variables over a specified time horizon. Whenever these projections and their respective confidence intervals do not include zero, we may consider these effects as statistically significant, thus denoting an empirically compelling dynamic association between the involved variables.

After conducting the appropriate tests guaranteeing a correct specification of the VAR model, the final step is conveying the causal ordering between the endogenous variables of the system. For this study, we evaluate the empirical linkages between investment shares and rates of utilization for the US economy through a Cholesky decomposition (Sims 1980). This requires one to specify, on the basis of theory, which variable(s) is (are) contemporaneously affected by the other(s). In other words, according to theory, does the aggregate investment share precede the utilization rate or vice-versa?

As already noted in previous sections, the SSM model adopts a particular theoretical stance regarding the linkages between investment and rates of utilization. It assumes that autonomous expenditure takes central stage and accumulation follows. Investment is considered "quasi endogenous," and it adjusts to move the rate of utilization in the present period to its exogenous equilibrium level. These interactions produce the counter clockwise cycle in the U-h plane discussed above. In standard classical and Keynesian theories of cyclical growth, it is investment that leads the cycle. The primacy of investment implies that the direction of the cycle is opposite: clockwise in U,h-plane.

These contrasting predictions can be empirically scrutinized; we do this in subsection 4.3 after discussing data sources.

4.2 Data

We use US quarterly data for the 1949Q1–2020Q4 period, with our baseline VAR specification comprising the following variables:

- Investment share (h_t): ratio between the natural logarithms of real gross private domestic investment (FRED's GPDIC1 series) and real gross domestic product (FRED's GDPC1 series).

- Rate of utilization (U_t): log ratio of real potential GDP (measured by Congressional Budget Office) and real GDP.

For robustness purposes, we also estimate the same VAR models with alternative variable specifications:

- Investment share (h_t^*): ratio between the natural logarithms of real gross private domestic investment and real gross domestic product (index, 2012=100) from the National Income and Product Accounts (NIPA), table 1.1.3.
- Rate of utilization (U_t^*): capacity utilization total index (FRED's TCU series), constructed as the percentage of resources used by 71 manufacturing, 16 mining, and 2 electric and gas utilities industries. Given that data availability for the TCU series only starts at 1967, the sample period for the robustness models is 1967Q1–2020Q4.

Both for baseline and robustness estimations, we carry out our models with variables detrended by the Hamilton filter, which is a currently favored one-sided detrending option (Hamilton 2018). This procedure defines the cyclical component of a time series y as how different would its value be at period $t+h$ from the value that one would have expected to observe based on its behavior at t. In other words, it proposes using forecasts to remove cyclical trajectories from a series. On the other hand, a time series' trend component would be defined as the smoothed estimate of y_{t+h} from a regression model containing the fitted values from a regression of y_{t+h} on an intercept and the s most-recent values of y, as of the present period t. In econometric terms,

$$y_{t+h} = \gamma_0 + \gamma_1 y_t + \gamma_2 y_{t-1} + \cdots + \gamma_s y_{t-s} + e_{t+h} \tag{8}$$

where the estimated residual term, $\hat{e}_{t+h}$, is the cyclical component of y. For the present study, we adopt $s = 16$ lags, implying four (4) years of previous data for a forecast horizon of $h = 8$ quarters, or two (2) years.

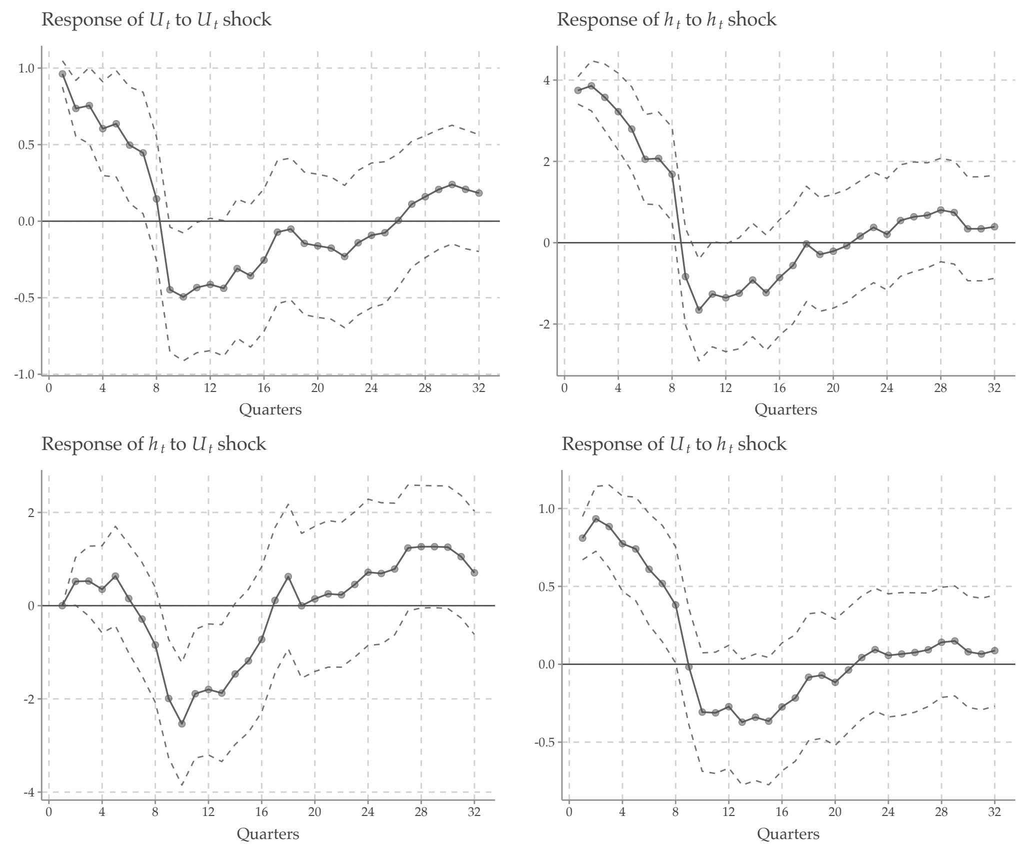
4.3 Results: Impulse Response Functions

Next, we proceed to the impulse-response analysis. We begin with the causal ordering consistent with standard Keynesian theory, in which investment is causally prior to the rate of capacity utilization. Specifically, the former is not contemporaneously affected by the latter, whereas investment does affect utilization contemporaneously. We label this as our "preferred" ordering: $h_t \to U_t$. Figure 2 illustrates the IRFs derived from a VAR with the baseline variables. This model is estimated with 17 lags, which provides a well-specified equation with neither serial correlation nor heteroskedasticity within its residuals, according to LM- and White-type tests at a significance level of 5 percent. The solid black lines designate the estimated responses to shocks, while dashed lines are 95 percent confidence intervals, derived via 5,000 Monte Carlo replications. Finally, our analysis includes an estimation horizon of thirty two (32) periods ahead, equivalent to eight (8) years.

The top row contains the responses of U_t and h_t to their own-shocks, respectively, while the bottom row contains the key elements of our analysis.[6] On the left, the response of the investment share to a shock in the rate of utilization and, on the right, the response of the rate of utilization to an investment share shock. The plots clearly show that the investment share reacts negatively to a utilization shock between the 9th and 13th quarters, while the rate of utilization responds positively to a shock to the investment share for the first eight (8) quarters (considering only the statistically significant portions of the figures).

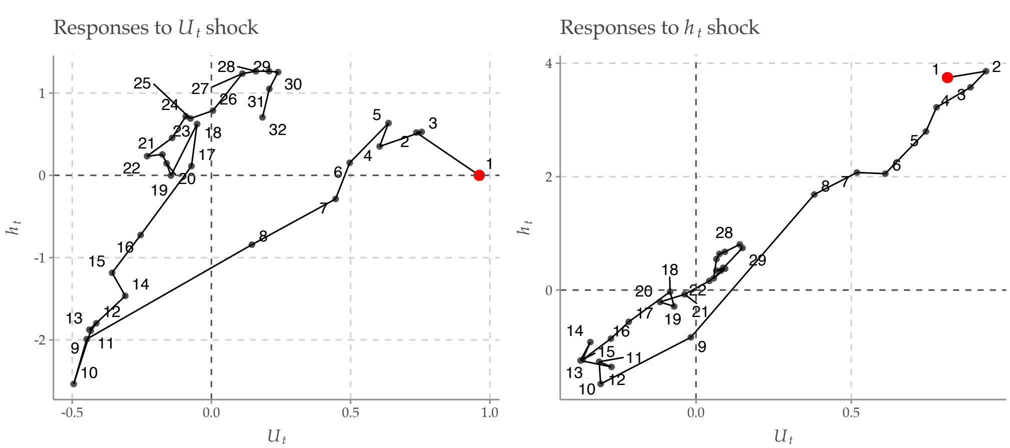
[6] IRFs are often reported with own-shocks along the main diagonal. Since we are reporting robustness results for a second ordering, we maintain the same ordering of IRF panels across all results in the hope that this eases readability.

Figure 2: IRFs from the Preferred Ordering's VAR Model



Note: Solid lines indicate estimated responses, while dashed lines are 95% bootstrapped confidence intervals obtained via 5,000 Monte Carlo replications.

Figure 3: Cyclical Trajectories Derived from Preferred Ordering's VAR Model



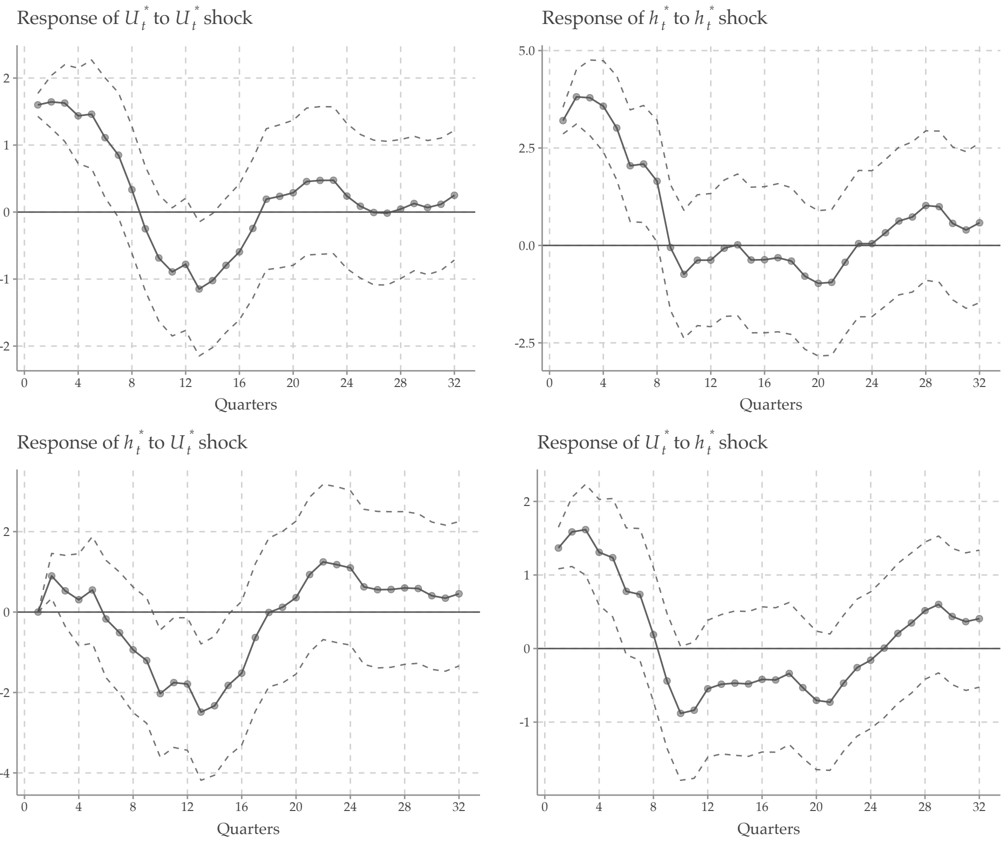
Note: Red dots denote the starting point, and labels represent 1–32 quarters ahead.

These results suggest a clockwise cycle in the U-h plane, in contrast to the one predicted by Freitas and Serrano (2015). While an exogenous shock to the rate of utilization negatively impacts the investment share, an exogenous disturbance in the latter positively affects the former. To further investigate this point, figure 3 combines the estimates from the above IRFs according to each exogenous shock: the left panel relates responses of U_t and h_t to a rate of utilization shock, while the right panel combines their responses to an investment share shock. Put differently, the left panel of figure 3 combines the left column of panels of figure 2, etc. We connect the data points in the scatter plot from the first until the thirty second period ahead, resulting in cyclical trajectories in the $< U, h >$ plane. Our inferences from the IRFs are confirmed: the variables relate in a clockwise manner.[7]

We then analyze our robustness specification in figure 4, still following the same causal ordering. This VAR model was estimated with a lag length of 18 quarters, and also shows neither serial correlation nor heteroskedasticity problems. We observe the same sign pattern on the bottom row as in the previous figure. Further, the investment share shows a brief significant positive response to a utilization rate exogenous shock in the second quarter. Figure 5 shows a clockwise cyclical trajectory on the left panel, and for the last 10 quarters, there is a brief counter-clockwise pattern. The right-hand panel, however, basically shows no cyclical trajectory at all.

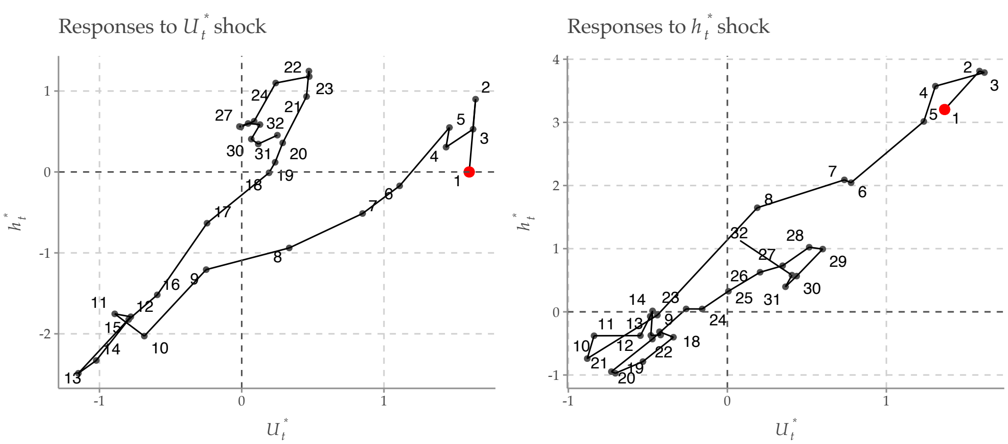
[7] To our knowledge, results from IRFs have not previously been reported in this fashion, though this visualization is clearly illustrative. It should be emphasized that standard representations—as in figure 2—include error bands, which indicate statistical significance. In contrast, this cyclical visualization does not. It also extends to a longer time horizon, where IRFs are clearly statistically insignificant. In summary, we consider this visualization useful, but urge the reader to not interpret it in isolation.

Figure 4: IRFs from the Preferred ordering’s VAR Model, Robustness Specification



Note: Solid lines indicate estimated responses, while dashed lines are 95 percent bootstrapped confidence intervals obtained via 5,000 Monte Carlo replications.

Figure 5: Cyclical Trajectories Derived from Preferred Ordering’s VAR model, Robustness Specification



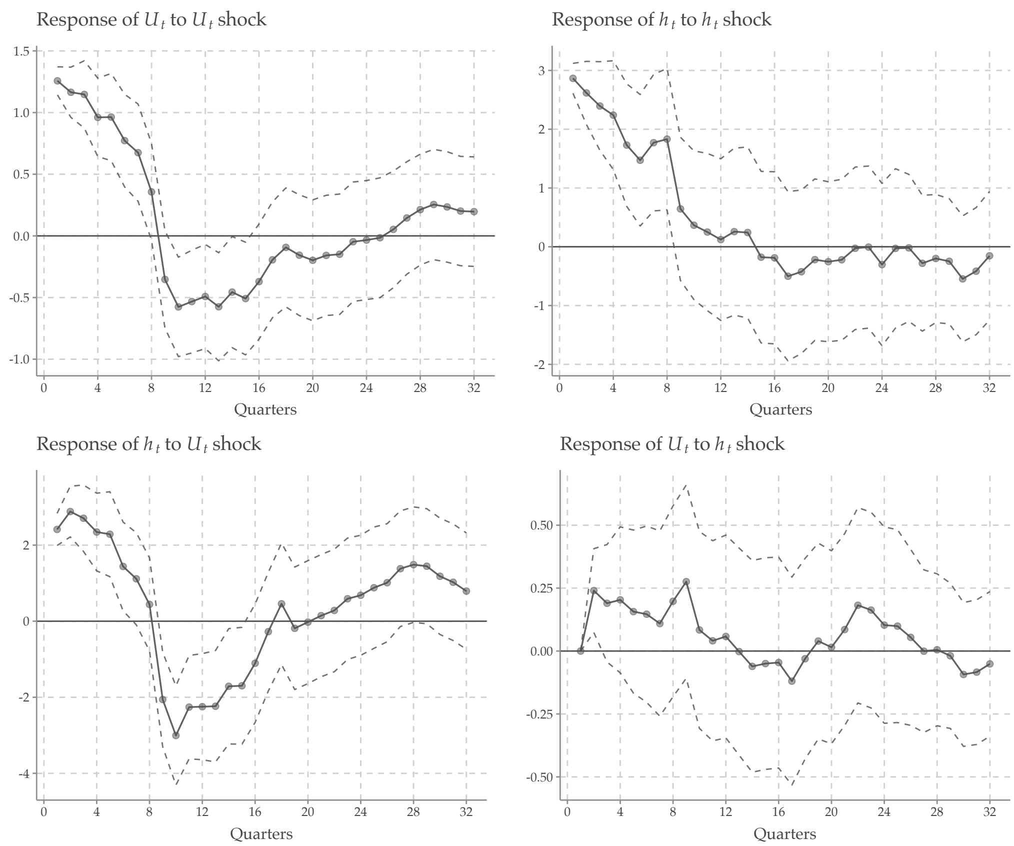
Note: Red dots denote the starting point, and labels represent 1–32 quarters ahead.

We continue our investigation by applying what we call a “second” ordering. Namely, we estimate VAR models using the $U_t \rightarrow h_t$ causal ordering, more closely related to the model in Freitas and Serrano (2015). Such specification assumes that the investment share is contemporaneously impacted by the rate of utilization, but not vice-versa. Figure 6 shows the IRFs with the baseline variables, following the same layout as in the previous figures. We observe a positive response of h_t to an exogenous shock to the rate of utilization for the first seven quarters, then a negative impact between the eighth and sixteenth quarters. On the other hand, the rate of capacity utilization shows only a marginal positive response to an investment share shock in the second quarter.

We can evaluate the cyclical trajectories from the previous set of plots in figure 7. We observe a clear clockwise pattern for the responses to the rate of capacity shock, while there is no clear behavior when considering the responses to investment share shocks in the U-h plane.

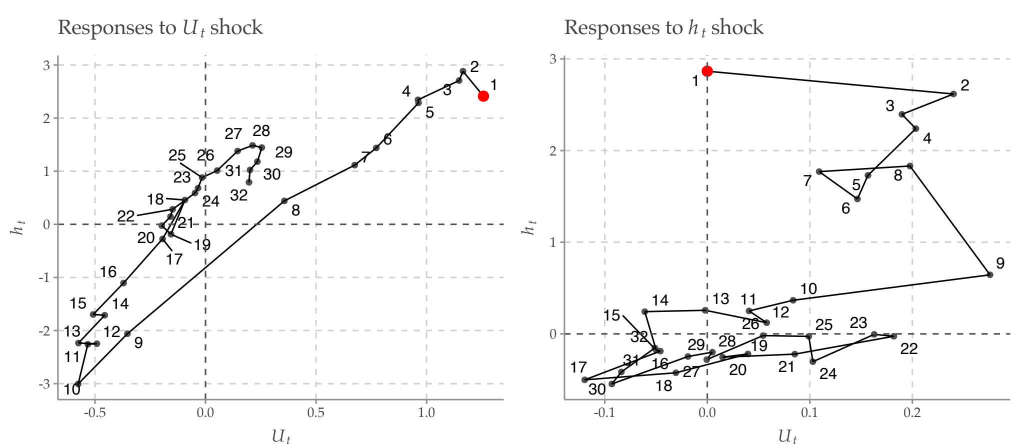
Finally, we study the dynamic interactions between our variables with the robustness specification. Figure 8 presents similar results as before, with the difference that there is no statistically significant response of the rate of capacity utilization to an investment share shock. And this is reflected in figure 9, where we observe a clockwise pattern in the left panel, while there is no cyclical behavior in the right panel.

Figure 6: IRFs from the Second Ordering's VAR Model



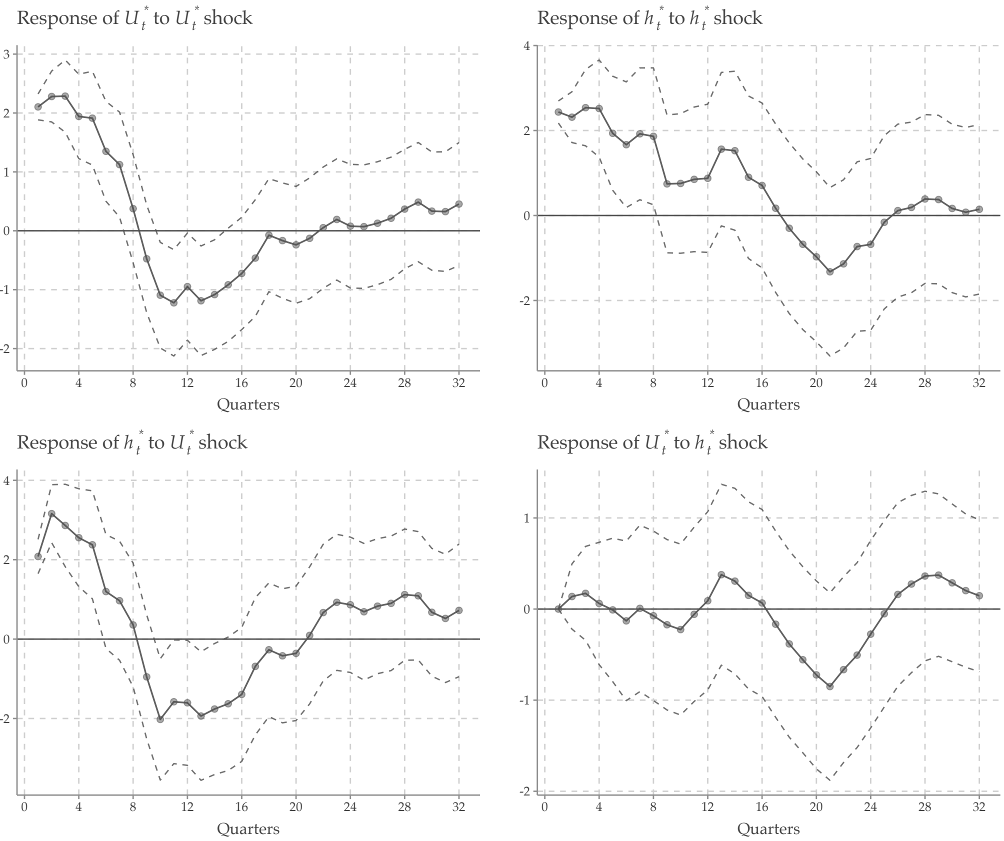
Note: Solid lines indicate estimated responses, while dashed lines are 95 percent bootstrapped confidence intervals obtained via 5,000 Monte Carlo replications.

Figure 7: Cyclical Trajectories Derived from Second Ordering's VAR Model



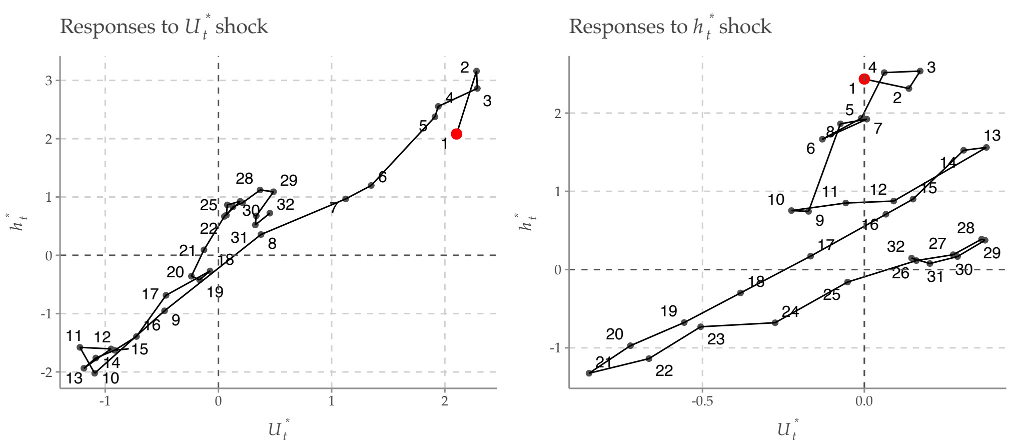
Note: Red dots denote the starting point, and labels represent 1–32 quarters ahead.

Figure 8: IRFs from the Second Ordering's VAR Model, Robustness Specification.



Note: Solid lines indicate estimated responses, while dashed lines are 95 percent bootstrapped confidence intervals obtained via 5,000 Monte Carlo replications.

Figure 9: Cyclical Trajectories Derived from Second Ordering's VAR Model, Robustness Specification



Note: Red dots denote the starting point, and labels represent 1–32 quarters ahead.

Further, we have also run VAR models in both orderings and variable specifications with covariates in first differences and in levels. VAR models in first-differences do not show significant IRFs in either specification, and also no clear cyclical patterns. The VAR models in levels show broadly the same patterns for IRF results as with the Hamilton filter, but cyclical patterns are less pronounced. These results are available upon request.

We summarize our results as follows. Based on standard Keynesian priors, our "preferred" ordering identified the opposite sign pattern as the one predicted by Freitas and Serrano (2015): while the investment share shows a significant negative response to an exogenous shock to the rate of capacity utilization, the latter is positive and significantly impacted by an exogenous shock to the latter. Furthermore, the first responses are usually observed after the second year, while the second scenario is immediately verified within our specified time horizon. Similar conclusions are also validated in our "second" ordering, where only the investment share is contemporaneously affected by the rate of utilization.

Our impulse-response analysis was further complemented by pairing together the responses of both system variables to the two exogenous shocks over the subsequent 32-quarter period. In the baseline specification, we infer a clockwise cycle in the U-h plane for both exogenous shocks.The robustness specification following the "preferred" ordering verified a clockwise pattern only in the context of a shock to the rate of utilization, with a brief counter clockwise cycle in later quarters. Our VAR models using the "second" ordering have also confirmed clockwise trajectories for rate of capacity shocks, while there is no clear pattern for shocks to the investment share.

5 CONCLUDING REMARKS

This paper provides an empirical assessment of the Sraffian supermultiplier (SSM) theory. Based on an important formalization of the theory (Freitas and Serrano 2015), we show that SSM predicts: (i) utilization to lead investment over the cycle, and (ii) cyclical convergence (or complex roots) to arise under virtually any parametrization. We implement a simple test of these predictions and show that SSM theory fails this test: investment leads the cycle. This finding is consistent with (post-)Keynesian theories of growth and cycles.

The central problem of SSM theory is the exogenous rate of growth of autonomous expenditures. This rate of growth (z) can not create capacity and must be autonomous from any other variable. If not for the first assumption, it would in fact become induced. If not for the second assumption, it would systematically vary with said variables either over the course of the cycle or in the long run—or both—and could thus not drive the system. Most commonly offered candidates for z—such as consumer durables, residential investment, or government expenditures—tend not to be independent of financial conditions, and feedback in an important fashion into growth and cycles themselves.

In summary, many questions regarding the design of SSM theory remain unanswered. Recent enthusiasm regarding it as a solution to post-Keynesian worries over the nature of the long run appears unwarranted.

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